Solutions Manual
to accompany

APPLIED STRENGTH OF MATERIALS

Fourth Edition

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CHAPTER 1 Basic Concepts in Strength of Materials

1-1 TO 1-15 ANSWERS IN TEXT.

1-16 $W = m \cdot g = 1800\,kg \cdot 9.81\,m/s^2 = 17658\,kg{\cdot}m/s^2 = 17.7 \times 10^3\,N$
$W = 17.7\,kN$

1-17 TOTAL WT. $= mg = 4000\,kg \cdot 9.81\,m/s^2 = 39.24\,kN$
EACH FRONT WHEEL: $F_F = (\frac{1}{2})(0.40)(39.24\,kN) = 7.85\,kN$
EACH REAR WHEEL: $F_R = (\frac{1}{2})(0.60)(39.24\,kN) = 11.77\,kN$

1-18 LOADING = TOTAL FORCE/AREA
TOTAL FORCE $= 6800\,kg \cdot 9.81\,m/s^2 = 66.7\,kN$
AREA $= (5.0\,m)(3.5\,m) = 17.5\,m^2$
LOADING $= 66.7\,kN / 17.5\,m^2 = 3.81\,kN/m^2 = 3.81\,kPa$

1-19 FORCE = WT $= m \cdot g = 25\,kg \cdot 9.81\,m/s^2 = 245\,N$
k = SPRING SCALE $= 4500\,N/m = F/\Delta L$
$\Delta L = \frac{F}{K} = \frac{245\,N}{4500\,N/m} = 0.0545\,m = 54.5 \times 10^{-3}\,m = 54.5\,mm$

1-22 $W = 17.7\,kN = 17700\,N \times 0.2248\,LB/N = 3980\,LB$

1-23 $F_F = 7.85\,kN = 7850\,N \times 0.2248\,LB/N = 1765\,LB$
$F_R = 11.77\,kN = 11770\,N \times 0.2248\,LB/N = 2646\,LB$

1-24 LOADING $= 3.81\,kPa = 3.81 \times 10^3 \frac{N}{m^2} \times \frac{0.2248\,LB}{N} \times \frac{1\,m^2}{(3.28\,ft)^2} = 79.6\,\frac{LB}{FT^2}$

1-25 $F = 245\,N \cdot 0.2248\,LB/N = 55.1\,LB$
$K = 4500 \frac{N}{m} \times \frac{0.2248\,LB}{N} \times \frac{1\,m}{39.37\,in} = 25.7\,LB/IN$
$\Delta L = \frac{F}{K} = \frac{55.1\,LB}{25.7\,LB/IN} = 2.14\,IN$

1-26 $m = \frac{w}{g} = \frac{2750\ LB}{32.2\ FT/S^2} = 85.4\ \frac{LB\cdot S^2}{FT} = \underline{85.4\ SLUGS}$

1-27 $m = \frac{w}{g} = \frac{12800\ LB}{32.2\ FT/S^2} = 398\ \frac{LB\cdot S^2}{FT} = \underline{398\ SLUGS}$

1-29 $p = 1200\ psi \times 6.895\ kPa/psi = \underline{8274\ kPa}$

1-30 $\sigma = 21600\ psi \times 6.895\ kPa/psi = 149{,}000\ kPa = \underline{149\ MPa}$

1-31 $S_u = 14000\ psi \times 6.895\ kPa/psi = 96{,}500\ kPa = \underline{96.5\ MPa}$

$S_u = 76000\ psi \times 6.895\ kPa/psi = 524\ 000\ kPa = \underline{524\ MPa}$

1-32 $n = 1750\ \frac{REV}{MIN} \times \frac{2\pi\ RAD}{REV} \times \frac{1\ MIN}{60\ S} = \underline{183\ RAD/S}$

1-33 $A = 14.1\ in^2 \times \frac{(25.4\ mm)^2}{IN^2} = \underline{9097\ mm^2}$

1-34 $y = 0.08\ IN \times 25.4\ mm/IN = \underline{2.03\ mm}$

1-35 DIMENSIONS: $18\ IN \times 25.4\ mm/IN = 457\ mm$

$12\ IN \times 25.4\ mm/IN = 305\ mm$

$AREA = (18\ IN)^2 = \underline{324\ IN^2}$

$AREA = (457\ mm)^2 = \underline{2.09 \times 10^5\ mm^2}$

VOLUME $= V =$ AREA $\times$ HEIGHT

$V = 324\ IN^2 \times 12\ IN = \underline{3888\ IN^3}$

$V = (1.5\ FT)^2 \times 1.0\ FT = \underline{2.25\ FT^3}$

$V = (2.09 \times 10^5\ mm^2) \times 305\ mm = \underline{6.37 \times 10^7\ mm^3}$

$V = (0.457\ m)^2 \times 0.305\ m = 0.0637\ m^3 = \underline{6.37 \times 10^{-2}\ m^3}$

1-36 $A = \pi D^2/4 = \pi(0.505\ IN)^2/4 = \underline{0.200\ IN^2}$

$A = 0.200\ IN^2 \times \frac{(25.4\ mm)^2}{IN^2} = \underline{129\ mm^2}$

1-37 $\sigma = \frac{F}{A} = \frac{3200\ N}{\pi D^2/4} = \frac{3200\ N}{\pi(10\ mm)^2/4} = 40.7\ N/mm^2 = \underline{40.7\ MPa}$

1-38 $\sigma = \frac{F}{A} = \frac{20 \times 10^3\ N}{(10)(30)\ mm^2} = 66.7\ N/mm^2 = \underline{66.7\ MPa}$

1-39 $\sigma = \frac{F}{A} = \frac{860\ LB}{(0.40\ IN)^2} = \underline{5375\ psi}$

1-40 $\sigma = \frac{F}{A} = \frac{1850\ LB}{\pi(0.375\ IN)^2/4} = \underline{16750\ psi.}$

1-41 LOAD ON SHELF $= W = mg = 1840\,kg \cdot 9.81\,m/s^2 = 18050\,N$

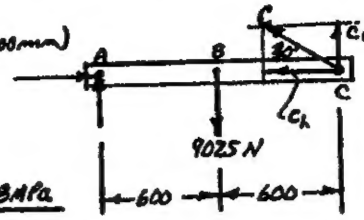
$W/2 = 9025\,N$ ON EACH SIDE

$\Sigma M_A = 0 = (9025\,N)(600\,mm) - C_V(1200\,mm)$

$C_V = 4512\,N$

$C = C_V/\sin 30° = 9025\,N$

$\sigma = \frac{P}{A} = \frac{C}{A} = \frac{9025\,N}{\pi(12\,mm)^2/4} = \underline{79.8\,MPa}$

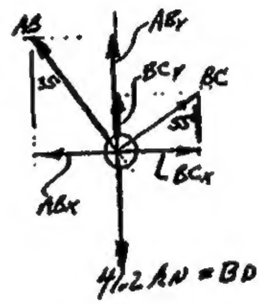


1-42 $\sigma = \frac{P}{A} = \frac{70000\,LB}{\pi(8\,in)^2/4} = \underline{1393\,psi}$

1-43 $\sigma = \frac{P}{A} = \frac{29\,500\,LB/3}{(3.5\,in)^2} = \underline{803\,psi}$

1-44 $\sigma = \frac{P}{A} = \frac{3500\,N}{(8.0\,mm)^2} = \underline{54.7\,MPa}$

1-45 $w = mg = 4200\,kg \cdot 9.81\,m/s^2 = 41.2\,kN$



$AB_X = AB \sin 35°$

$AB_Y = AB \cos 35°$

$BC_X = BC \sin 55°$

$BC_Y = BC \cos 55°$

$\Sigma F_X = 0 = AB_X - BC_X$

$0 = AB \sin 35° - BC \sin 55°$

$AB = BC \cdot \frac{\sin 55°}{\sin 35°} = 1.428\,BC$

$\Sigma F_Y = 0 = AB_Y + BC_Y - 41.2\,kN = AB \cos 35° + BC \cos 55° - 41.2\,kN$

$0 = (1.428\,BC)\cos 35° + BC \cos 55° - 41.2\,kN$

$41.2\,kN = BC[1.170 + 0.574] = 1.743\,BC$

$BC = 41.2\,kN/1.743 = 23.63\,kN$

$AB = 1.428\,BC = 33.75\,kN$

STRESS IN ROD AB: $\sigma_{AB} = \frac{AB}{A} = \frac{33.75 \times 10^3\,N}{\pi(20\,mm)^2/4} = \underline{107.4\,MPa}$

STRESS IN ROD BC: $\sigma_{BC} = \frac{BC}{A} = \frac{23.63 \times 10^3\,N}{\pi(20\,mm)^2/4} = \underline{75.2\,MPa}$

STRESS IN ROD BD: $\sigma_{BD} = \frac{BD}{A} = \frac{41.2 \times 10^3\,N}{\pi(20\,mm)^2/4} = \underline{131.1\,MPa}$

1-46 $F = 0.01097\, m R n^2 = (0.01097)(0.40)(0.60)(3000)^2\ N$

$F = 23695\ N$

$A = \pi(16\,mm)^2/4 = 201\ mm^2$

$\sigma = \dfrac{F}{A} = \dfrac{23695\,N}{201\,mm^2} = \underline{118\,MPa}$

1-47 $A = (30\ mm)^2 = 900\,mm^2$

FOR AB: $F_{AB} = (110 - 40 + 80)\,kN = 150\,kN$

$\sigma_{AB} = \dfrac{F_{AB}}{A} = \dfrac{150\times10^3\,N}{900\,mm^2} = \underline{167\,MPa}$ TENSION

FOR BC: $F_{BC} = 110 - 40 = 70\,kN$

$\sigma_{BC} = \dfrac{F_{BC}}{A} = \dfrac{70\times10^3\,N}{900\,mm^2} = \underline{77.8\,MPa}$ TENSION

FOR CD: $F_{CD} = 110\,kN$

$\sigma_{CD} = \dfrac{F_{CD}}{A} = \dfrac{110\times10^3\,N}{900\,mm^2} = \underline{122\,MPa}$ TENSION

1-48 AREAS: A-C; $A_1 = \pi(25)^2/4 = 491\,mm^2$

C-D; $A_2 = \pi(16)^2/4 = 201\,mm^2$

FOR AB: $F_{AB} = -9.65 - 12.32 + 4.45 = -17.52\,kN$

$\sigma_{AB} = \dfrac{F_{AB}}{A_1} = \dfrac{-17.52\times10^3\,N}{491\,mm^2} = \underline{-35.7\,MPa}$ COMPR.

FOR BC: $F_{BC} = -9.65 - 12.32 = -21.97\,kN$

$\sigma_{BC} = \dfrac{F_{BC}}{A_1} = \dfrac{-21.97\times10^3\,N}{491\,mm^2} = \underline{-44.7\,MPa}$ COMPR.

FOR CD: $F_{CD} = -9.65\,kN$

$\sigma_{CD} = \dfrac{F_{CD}}{A_2} = \dfrac{-9.65\times10^3\,N}{201\,mm^2} = \underline{-48.0\,MPa}$ COMPR.

1-49 $A = \pi[(1.90)^2 - (1.61)^2]/4 = 0.799\ in^2$ ($1\tfrac{1}{2}$ IN PIPE-APP. A-12)

FOR BC: $\sigma_{BC} = \dfrac{F_{BC}}{A} = \dfrac{2500\,LB}{0.799\,in^2} = \underline{3129\ psi}$ TENSION

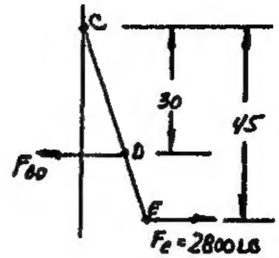
FOR AB: $F_{AB} = 2500 + 2(8000\cos 30^\circ) = 16356\,LB$

$\sigma_{AB} = \dfrac{F_{AB}}{A} = \dfrac{16356\,LB}{0.799\,in^2} = \underline{20\,471\ psi}$ TENSION

1-50 $\Sigma M_C = 0 = 2800(45) - F_{BD}(30)$

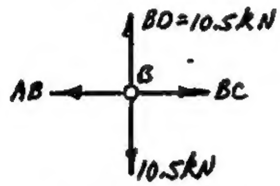
$F_{BD} = 4200\ \text{LB}$

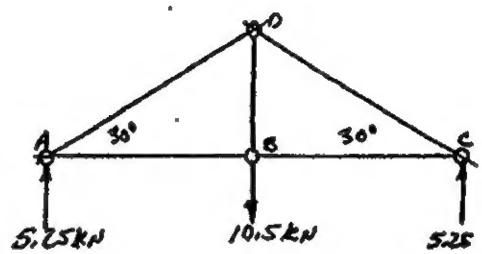
$\sigma_{BD} = \dfrac{F_{BD}}{A} = \dfrac{4200\ \text{LB}}{(2.0)(0.65)\ \text{IN}^2} = \underline{3231\ \text{psi}}$ TENSION



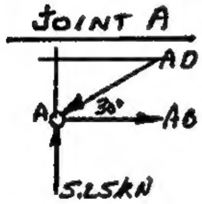
1-51

JOINT B





JOINT A



$AD \sin 30° = 5.25\ \text{kN}$

$AD = 10.5\ \text{kN} = CD$

$AB = AD \cos 30° = 9.09\ \text{kN} = BC$

STRESSES:

AB, BC: $\sigma_{AB} = \sigma_{BC} = \dfrac{9.09 \times 10^3\ \text{N}}{(12)(30)\ \text{mm}^2} = \underline{25.3\ \text{MPa}}$ TENSION

BD: $\sigma_{BD} = \dfrac{10.5 \times 10^3\ \text{N}}{(2)(10)(30)\ \text{mm}^2} = \underline{17.5\ \text{MPa}}$ TENSION

AD, CD: $A = (30)^2 - (20)^2 = 500\ \text{mm}^2$

$\sigma_{AD} = \sigma_{CD} = \dfrac{-10.5 \times 10^3\ \text{N}}{500\ \text{mm}^2} = \underline{-21.0\ \text{MPa}}$ COMPRESSION

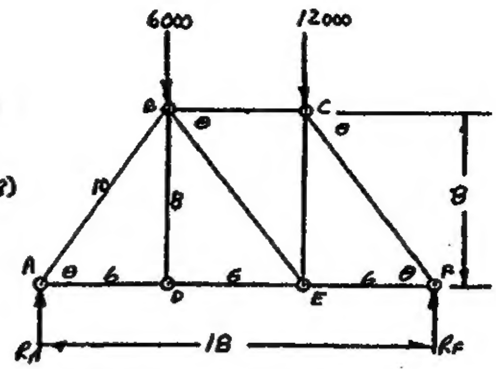
1-52

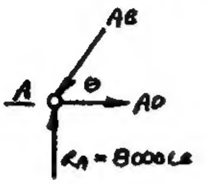
$\Sigma M_A = 0 = 6000(6) + 12000(12) - R_F(18)$

$R_F = 10000$ LB

$\Sigma M_F = 0 = 12000(6) + 6000(12) - R_A(18)$

$R_A = 8000$ LB

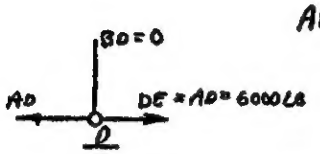


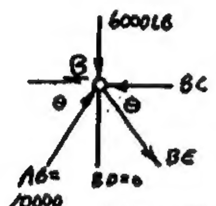


$R_A = AB \sin\theta = AB(0.8)$

$AB = R_A/0.8 = 8000/0.8 = 10000$ LB COMP.

$AD = AB \cos\theta = 10000(0.6) = 6000$ LB TENS.



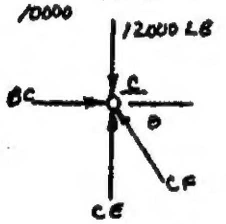


$BE \sin\theta + 6000 - AB \sin\theta = 0$

$BE = \dfrac{AB \sin\theta - 6000}{\sin\theta} = \dfrac{10000(0.8) - 6000}{0.8} = 2500$ LB TENS.

$BC = AB \cos\theta + BE \cos\theta = 10000(0.6) + 2500(0.6) =$

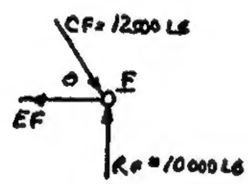
$BC = 7500$ LB COMP.



$BC = CF \cos\theta$

$CF = BC/\cos\theta = 7500/0.6 = 12500$ LB COMP.

$CE = 12000 - CF \sin\theta = 12000 - 12500(0.8) = 2000$ LB C



$EF = CF \cos\theta = 12500(0.6) = 7500$ LB TENS.

AREAS OF MEMBERS: (APP. A5, A6)

AD, DE, EF – $2(0.484) = 0.968$ IN^2

BD, BE, CE – 0.484 IN^2

AB, BC, CF – $2(1.21) = 2.42$ IN^2

NOTE: COMPRESSION MEMBERS MUST BE CHECKED FOR COLUMN BUCKLING

STRESSES:

$\sigma_{AD} = \sigma_{DE} = 6000/0.968 = {}^{+}6198$ psi

$\sigma_{EF} = 7500/0.968 = {}^{+}7748$ psi

$\sigma_{BD} = 0$

$\sigma_{BE} = 2500/0.484 = {}^{+}5165$ psi

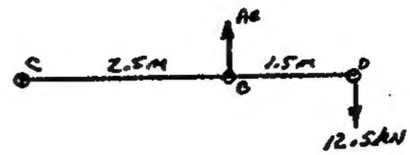
$\sigma_{CE} = 2000/0.484 = {}^{-}4132$ psi

$\sigma_{AB} = -10000/2.42 = -4132$ psi

$\sigma_{BC} = -7500/2.42 = -3099$ psi

$\sigma_{CF} = -12500/2.42 = -5165$ psi

__1-53__



$\Sigma M_C = 0 = (2.5)(4.0) - AB(2.5)$

$AB = 20\ kN$

$\sigma = \dfrac{20 \times 10^3 N}{(20)^2 mm^2} = \underline{50\ MPa}$

__1-54__ $A = \pi(0.505)^2/4 = 0.200\ IN^2$

$\sigma = F/A = 12\,600\ LB/0.200\ IN^2 = \underline{63\,000\ psi}$

__1-55__ $A = (2.65)(1.40) + 2[(1.40)(0.5)(\tfrac{1}{2})] = 4.41\ IN^2$

$\sigma = F/A = (52000\ LB/4.41\ IN^2) = \underline{11\,791\ psi}$

__1-56__ $A = (80)(40) - (60)(15) + \pi(40)^2/4 = 3557\ mm^2$

$\sigma = F/A = 640 \times 10^3 N/3557\ mm^2 = \underline{180\ MPa}$

__1-57__ DIRECT SHEAR – SINGLE SHEAR

$A_S = [\pi(12.0)^2/4]\ mm^2 = 113\ mm^2$

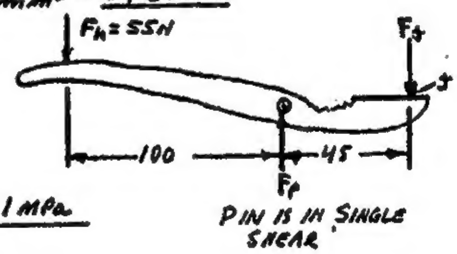
$\tau = F/A_s = 16.5 \times 10^3 N/113\ mm^2 = \underline{146\ MPa}$

__1-58__ $\Sigma F_y = 0 = 55(145) - F_p(45)$

$F_p = 177\ N$

$A_s = \pi(3.0)^2/4 = 7.07\ mm^2$

$\tau = F_p/A_s = 177\ N/7.07\ mm^2 = \underline{25.1\ MPa}$



__1-59__ FROM PROB 1-46: $F = 23\,695\ N$

$A_s = 2[\pi(10)^2/4] = 157\ mm^2$ DOUBLE SHEAR

$\tau = F/A_s = 23\,695\ N/157\ mm^2 = \underline{151\ MPa}$

1-60 $A_s = (3.0)(3.5) = 10.5\ in^2$

$\tau = F/A_s = (1800\ lb)/(10.5\ in^2) = \underline{171\ psi}$

1-61 $A_s = [2(35) + \pi(8)](5.0) = 475.7\ mm^2$

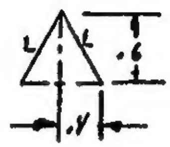
$\tau = F/A_s = 38.6 \times 10^3\ N / 475.7\ mm^2 = \underline{81.1\ MPa}$

1-62 $L = \sqrt{.4^2 + .6^2} = 0.721\ in.$

$A_s = [2(1.60) + \pi(0.8)/2 + 2(0.721)]\,0.194$

$A_s = 1.144\ in^2$

$\tau = F/A_s = 45000\ lb/1.144\ in^2 = \underline{39324\ psi}$

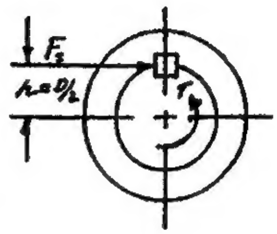


1-63 $T = F_s \cdot R$

$F_s = T/R = \dfrac{95\ N\cdot m}{35\ mm/2} \cdot \dfrac{10^3\ mm}{m} = 5429\ N$

$A_s = b \cdot L = (10)(22) = 220\ mm^2$

$\tau = F_s/A_s = 5429\ N/220\ mm^2 = \underline{24.7\ MPa}$



1-64 $F_s = T/R = 8000\ lb\cdot in/1.0\ in = 8000\ lb$

$A_s = b \cdot L = (0.50)(2.25) = 1.125\ in^2$

$\tau = F_s/A_s = 8000\ lb/1.125\ in^2 = \underline{7111\ psi}$

1-65 PIN DOUBLE SHEAR; $A_s = 2[\pi(0.5)^2/4] = 0.393\ in^2$

$\tau = F/A_s = 20000\ lb/0.393\ in^2 = \underline{50\,930\ psi}$

COLLAR SHEAR COLLAR FROM CONNECTOR BODY

$A_s = \pi d t = \pi(0.875)(0.1875) = 0.5154\ in^2$

$\tau = F/A_s = 20000\ lb/0.5154\ in^2 = \underline{38\,800\ psi}$

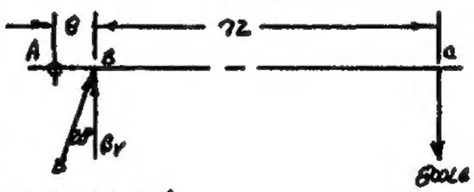
1-66 $\Sigma M_A = 0 = 800(80) - B_y(8)$

$B_y = 8000\ lb$

$B = B_y/\cos 20° = 8513\ lb$

$A_s = 2(\pi(0.375)^2/4) = 0.221\ in^2$

$\tau = B/A_s = 8513\ lb/0.221\ in^2 = \underline{38\,540\ psi}$



1-67 $A_s = (40)(12) = 480\ mm^2$

$\tau = F/A_s = 88 \times 10^3\ N/480\ mm^2 = \underline{183\ MPa}$

1-68 $A_s = (40)(120) = 4800\ mm^2$

$\tau = F/A_s = 88.2 \times 10^3\ N / 4800\ mm^2 = \underline{18.4\ MPa}$

1-69 $A_s = \pi D t = \pi(12)(8) = 301.6\ mm^2$

$\tau = F/A_s = 22.3 \times 10^3 N / 301.6\ mm^2 = \underline{73.9\ MPa}$

1-70 $A_s = 2[\pi(12)^2/4] = 226.2\ mm^2$ TWO RIVETS–SINGLE SHEAR

$\tau = F/A_s = 10.2 \times 10^3 N / 226.2\ mm^2 = \underline{45.1\ MPa}$

1-71 $A_s = 4[\pi(12)^2/4] = 452.4\ mm^2$ TWO RIVETS–DOUBLE SHEAR

$\tau = F/A_s = 10.2 \times 10^3 N / 452.4\ mm^2 = \underline{22.55\ MPa}$

BEARING STRESS

1-72 A) W6×15 ON STEEL PLATE: $A_b = 4.43\ in^2$ (APP. A-7)

$\sigma_b = F/A_b = 26000\ LB / 4.43\ in^2 = \underline{5869\ PSI}$

B) STEEL PLATE ON CONCRETE: $A_b = (12)^2 = 144\ in^2$

$\sigma_b = F/A_b = 26000\ LB / 144\ in^2 = \underline{181\ PSI}$

C) CONCRETE PIER ON CONCRETE FOOTING: $A_b = (18)^2 = 324\ in^2$

$\sigma_b = F/A_b = 26000\ LB / 324\ in^2 = \underline{80.2\ PSI}$

D) CONCRETE FOOTING ON SOIL: $A_b = (36)^2 = 1296\ in^2$

$\sigma_b = F/A_b = 26000\ LB / 1296\ in^2 = \underline{20.1\ PSI}$

1-73 a) PIPE ON FLOOR: $A_b = \frac{\pi}{4}(2.375^2 - 2.067^2) = 1.075\ in^2$ (APP. A-12)

$\sigma_b = F/A_b = 2350\ LB / 1.075\ in^2 = \underline{2187\ PSI}$

b) 2.375 IN DIA. ROUND PLATE : $A_b = 2.375^2\left(\frac{\pi}{4}\right) = 4.43\ in^2$

$\sigma_b = F/A_b = 2350\ LB / 4.430\ in^2 = \underline{530\ PSI}$

1-74 a) BOLT HEAD ON WASHER: $A_b = A_{HEX} - A_{SD}$ (SEE APP. A-1)

$A_b = 0.866(0.75)^2 - \pi(0.562)^2/4 = 0.239\ in^2$

$\sigma_b = F/A_b = 385\ LB / 0.239\ in^2 = \underline{1610\ PSI}$

b) WASHER ON WOOD: $A_b = \frac{\pi}{4}(1.375^2 - 0.562^2) = 1.237\ in^2$

$\sigma_b = F/A_b = 385\ LB / 1.237\ in^2 = \underline{311\ PSI}$

1-75 DATA FROM PROB. 1-64: $F = 8000\,LB$, $L = 2.25\,IN$, $h = 0.375\,IN.$

$\sigma_b = F/A_b = 8000\,LB/(2.25)(0.375/2)\,IN^2 = \underline{18\,963\,PSI}$

1-76 DATA FROM PROB. 1-65: $F = 20\,000\,LB$, FIG. 1-38

a) PIN/TUBE: $A_b = D_p(D-d) = (0.50)(1.25-0.875) = 0.1875\,IN^2$

$\sigma_b = F/A_b = 20\,000\,LB/0.1875\,IN^2 = \underline{106\,700\,PSI}$ (VERY HIGH)

b) COLLAR/TUBE: $A_b = \frac{\pi}{4}(D^2-d^2) = \frac{\pi}{4}(1.25^2-0.875^2) = 0.626\,IN^2$

$\sigma_b = F/A_b = 20000\,LB/0.626\,IN^2 = \underline{31\,950\,PSI}$

1-77 FROM FIG. 1-43: $A_b = 2(d)(t) = 2(12)(15) = 360\,mm^2$

$\sigma_b = F/A_b = 10.2\times10^3\,N/360\,mm^2 = \underline{28.3\,MPa}$

1-78 FROM FIG 1-44:

a) ON MIDDLE PART: $A_b = 2\,d\,t_1 = (2)(12)(15) = 360\,mm^2$

$\sigma_b = F/A_b = 10.2\times10^3\,N/360\,mm^2 = \underline{28.3\,MPa}$

b) ON OUTER PARTS: $A_b = 4\,d\,t_2 = (4)(12)(10) = 480\,mm^2$

$\sigma_b = F/A_b = 10.2\times10^3\,N/480\,mm^2 = \underline{21.25\,MPa}$

1-79 FIG. 1-47: $A_b = (10)(6) + \frac{1}{2}(\frac{\pi}{4})(10)^2 = 99.3\,mm^2$

$\sigma_b = F/A_b = 535\,N/99.3\,mm^2 = \underline{5.39\,MPa}$

ONLY THOSE PROBLEMS REQUIRING NUMERICAL DATA ARE SHOWN.

2-14 S_u = 90 ksi (621 MPa); S_y = 60 ksi (414 MPa); 25% ELONG.
BECAUSE % ELONGATION > 5%, IT IS DUCTILE. (APP. A-13)

2-15 1020 HR: 36 % ELONGATION – GREATER DUCTILITY
1040 HR: 25 % ELONGATION (APP. A-13)

2-16 AISI 1141 OQT 700: HIGH SULFUR ALLOY STEEL WITH 0.41% CARBON, QUENCHED IN OIL, TEMPERED AT 700°F. (APP. A-13)

2-17 YES. S_y = 172 ksi @ OQT 700, S_y = 129 ksi @ OQT 900
BY INTERPOLATION $S_y \approx$ 150 ksi @ OQT 800. (APP. A-13)

2-18 E = 30 × 10^6 psi (207 GPa) FOR ALL CARBON AND ALLOY STEELS. (APP. A-13)

2-19 WT = DENSITY × VOLUME = (0.283 LB/IN^3)(1.0)(4.0)(14.5) IN^3 = $\underline{16.4\ LB}$
(APP. A-13) VALUE OF LB_m = VALUE OF LB FORCE (WT.)

2-20 VOLUME = AREA × LENGTH = $\frac{\pi}{4}(50)^2 \times 250 = 4.909 \times 10^5\ mm^3$
STEEL BAR
MASS = $\frac{7680\ kg}{m^3} \times \frac{4.909 \times 10^5\ mm^3}{1} \times \frac{1\ m^3}{(10^3\ mm)^3} = 3.77\ kg$
(APP. A-13)
WT = $m \cdot g = 3.77\ kg \cdot 9.81\ m/s^2 = 36.98\ kg \cdot m/s^2 = \underline{36.98\ N}$

2-21 MAGNESIUM WOULD BECAUSE IT HAS A LOWER E.
E_{MG} = 45 GPa; E_{Ti} = 114 GPa; Ti IS STIFFER. (APP. A-14)

2-23 ALLOY OF ALUMINUM WITH SILICON AND MAGNESIUM.
HEAT TREATED TO T6 TEMPER.

2-28 (APP. A-17)

	S_u	S_y	E	DENSITY
6061-O	18 ksi	8 ksi	10×10^6 psi	0.10 LB/IN^3
6061-T4	35 ksi	21 ksi	"	"
6061-T6	45 ksi	40 ksi	"	"

2-29 S_{ut} = 40 ksi; S_{uc} = 140 ksi (APP. A-16)

2-31 BENDING σ_d = 1450 psi; TENSION σ_d = 850 psi; COMP. 1000 psi PARALLEL TO GRAIN, 385 psi PERPENDICULAR TO GRAIN; SHEAR τ_d = 95 psi (APP. A-18)

2-32 2000 TO 7000 psi (SECTION 2-10)

2-44 Graphite fibers.

2-45 S-glass, quartz fibers, tungsten fibers coated with silicon carbide.

2-51

Material	*Specific strength (in)*	*Ratio to AISI 1020*
Graphite/Epoxy (High Strength)	4.86×10^6	25.0
Aramid/Epoxy Composite	4.00×10^6	20.6
Boron/Epoxy Composite	3.60×10^6	18.5
Graphite/Epoxy (Ultra-hi mod)	2.76×10^6	14.2
Glass/Epoxy Composite	1.87×10^6	9.63
Titanium Ti-6Al-4V	1.00×10^6	5.15
AISI 5160 OQT 700 Steel	0.929×10^6	4.78
Aluminum 7075-T6	0.822×10^6	4.23
Aluminum 6061-T6	0.459×10^6	2.36
AISI 1020 HR Steel	0.194×10^6	1.00

2-52

Material	*Specific modulus (in)*	*Ratio to AISI 1020*
Graphite/Epoxy (Ultra-hi mod)	8.28×10^8	7.81
Boron/Epoxy Composite	4.00×10^8	3.77
Graphite/Epoxy (High Strength)	3.45×10^8	3.25
Aramid/Epoxy Composite	2.20×10^8	2.07
AISI 1020 HR Steel	1.06×10^8	1.00
AISI 5160 OQT 700 Steel	1.06×10^8	1.00
Titanium Ti-6Al-4V	1.03×10^8	0.97
Aluminum 6061-T6	1.02×10^8	0.96
Aluminum 7075-T6	0.99×10^8	0.93
Glass/Epoxy Composite	0.66×10^8	0.62

2-60 $V_m = 1 - V_f = 1.0 - 0.60 = 0.40$

2-61 See Equation (2-5).

2-62 See Equations (2-6), (2-7), (2-8), (2-9).

2-63 Given: $V_f = 0.50$; Fibers are high strength carbon-PAN; Matrix is Epoxy
See Table 2-9 for data. $V_m = 1 - V_f = 1.0 - 0.50 = 0.50$
Use Equation (2-5): $s_{uc} = s_{uf} V_f + \sigma_m' V_m$

Strain at which fibers would fail: $\varepsilon_f = s_{uf} / E_f = (820\times 10^3 \text{ psi})/(40\times10^6 \text{ psi})$

$\varepsilon_f = 0.0205$

Stress in matrix at this strain: $\sigma_m' = E_m\, \varepsilon = (0.56\times10^6 \text{ psi})(0.0205) = 11\,480 \text{ psi}$

Then: $s_{uc} = (820\times 10^3 \text{ psi})(0.50) + (11\,480 \text{ psi})(0.50) = \underline{415\times10^3 \text{ psi}}$

Modulus of elasticity: $E_c = E_f V_f + E_m V_m = (40\times10^6)(0.5) + (0.56\times10^6)(0.50)$

$\underline{E_c = 20.3\times10^6 \text{ psi}}$

Specific weight: $\gamma_c = \gamma_f V_f + \gamma_m V_m = (0.065)(0.50) + (0.047)(0.50)$

$\underline{\gamma_c = 0.056 \text{ lb/in}^3}$

2-64 Given: $V_f = 0.50$; Fibers are high modulus carbon; Matrix is Epoxy
See Table 2-9 for data. $V_m = 1 - V_f = 1.0 - 0.50 = 0.50$
Use Equation (2-5): $s_{uc} = s_{uf} V_f + \sigma_m' V_m$

Strain at which fibers would fail: $\varepsilon_f = s_{uf} / E_f = (325\times 10^3 \text{ psi})/(100\times10^6 \text{ psi})$

$\varepsilon_f = 0.00325$

Stress in matrix at this strain: $\sigma_m' = E_m\, \varepsilon = (0.56\times10^6 \text{ psi})(0.00325) = 1820 \text{ psi}$

Then: $s_{uc} = (325\times 10^3 \text{ psi})(0.50) + (1820 \text{ psi})(0.50) = \underline{163\times10^3 \text{ psi}}$

Modulus of elasticity: $E_c = E_f V_f + E_m V_m = (100\times10^6)(0.5) + (0.56\times10^6)(0.50)$

$\underline{E_c = 50.3\times10^6 \text{ psi}}$

Specific weight: $\gamma_c = \gamma_f V_f + \gamma_m V_m = (0.078)(0.50) + (0.047)(0.50)$

$\underline{\gamma_c = 0.0625 \text{ lb/in}^3}$

2-65 Given: $V_f = 0.50$; Fibers are aramid; Matrix is Epoxy
See Table 2-9 for data. $V_m = 1 - V_f = 1.0 - 0.50 = 0.50$
Use Equation (2-5): $s_{uc} = s_{uf} V_f + \sigma_m' V_m$

Strain at which fibers would fail: $\varepsilon_f = s_{uf} / E_f = (500\times 10^3 \text{ psi})/(19\times10^6 \text{ psi})$

$\varepsilon_f = 0.0263$

Stress in matrix at this strain: $\sigma_m' = E_m\, \varepsilon = (0.56\times10^6 \text{ psi})(0.0263) = 14\,740 \text{ psi}$

Then: $s_{uc} = (500\times 10^3 \text{ psi})(0.50) + (14\,740 \text{ psi})(0.50) = \underline{257\times10^3 \text{ psi}}$

Modulus of elasticity: $E_c = E_f V_f + E_m V_m = (19\times10^6)(0.5) + (0.56\times10^6)(0.50)$

$\underline{E_c = 9.78\times10^6 \text{ psi}}$

Specific weight: $\gamma_c = \gamma_f V_f + \gamma_m V_m = (0.052)(0.50) + (0.047)(0.50)$

$\underline{\gamma_c = 0.0495 \text{ lb/in}^3}$

Solutions to Problems 2-66 to 2-67: Some data approximated from Figure P2-66.
Most accurate values are for Ultimate strength (b.)and % elongation (f).
Elastic limit (d.) estimated between proportional limit (c.) and yield strength (a.)
Modulus of elasticity (e.) computed from (Δ stress / Δ strain). Data are approximated
Materials found from Appendixes A-13 through A-17 matching s_u, s_y, % Elongation, and E

2-66
a. s_y = 73 ksi - Offset
b. s_u = 83 ksi
c. s_p = 60 ksi
d. s_{el} = 67 ksi
e. E = 10.0x10^6 psi
f. 11% Elongation
g. Ductile
h. Aluminum
I. 7075-T6

2-67
a. s_y = 173 ksi Yield point
b. s_u = 187 ksi
c. s_p = 162 ksi
d. s_{el} = 168 ksi
e. E = 29.0x10^6 psi
f. 15% Elongation
g. Ductile
h. Steel
I. AISI 4140 OQT 900

2-68
a. s_y = 62 ksi Offset
b. s_u = 75 ksi
c. s_p = 50 ksi
d. s_{el} = 56 ksi
e. E = 16.7x10^6 psi
f. 15% Elongation
g. Ductile
h. Copper Alloy
I. C54400 Bronze-hard

2-69
a. s_y = 49 ksi - Yield point
b. s_u = 65 ksi
c. s_p = 46 ksi
d. s_{el} = 48 ksi
e. E = 26.5x10^6 psi
f. 36% Elongation
g. Ductile
h. Steel
I. AISI 1020 CD

2-70
a. No s_y - Brittle
b. s_u = 55 ksi
c. s_p = 50 ksi
d. s_{el} = 53 ksi
e. E = 20.0x10^6 psi
f. 0.5% Elongation
g. Brittle
h. Cast Iron
I. ASTM A48 Grade 60

2-71
a. s_y = 53 ksi - Offset
b. s_u = 59 ksi
c. s_p = 31 ksi
d. s_{el} = 42 ksi
e. E = 12.0x10^6 psi
f. 5.0% Elongation
g. Borderline Brittle/Ductile
h. Zinc
I. Cast ZA-12

2-72
a. s_y = 35 ksi - Yield point
b. s_u = 57 ksi
c. s_p = 30 ksi
d. s_{el} = 27 ksi
e. E = 26x10^6 psi
f. 21% Elongation
g. Ductile
h. Structural Steel
I. ASTM A36

2-73
a. s_y = 19 ksi - Offset
b. s_u = 40 ksi
c. s_p = 14 ksi
d. s_{el} = 17 ksi
e. E = 6x10^6 psi
f. 5% Elongation
g. Borderline Brittle/Ductile
h. Magnesium
I. ASTM AZ 63A-T6

2-74
a. $s_y = 155$ ksi - Offset
b. $s_u = 170$ ksi
c. $s_p = 142$ ksi
d. $s_{el} = 149$ ksi
e. $E = 16.5 \times 10^6$ psi
f. 8% Elongation
g. Ductile
h. Titanium
I. 6Al-4V

2-75
a. $s_y = 40$ ksi - Offset
b. $s_u = 45$ ksi
c. $s_p = 30$ ksi
d. $s_{el} = 35$ ksi
e. $E = 10.0 \times 10^6$ psi
f. 17% Elongation
g. Ductile
h. Aluminum
I. 6061-T6

2-76
a. $s_y = 80$ ksi - Offset
b. $s_u = 90$ ksi
c. $s_p = 62$ ksi
d. $s_{el} = 71$ ksi
e. $E = 26 \times 10^6$ psi
f. 15% Elongation
g. Ductile
h. Stainless Steel
I. AISI 430 full hard

2-77
a. $s_y = 80$ ksi - Offset
b. $s_u = 95$ ksi
c. $s_p = 55$ ksi
d. $s_{el} = 68$ ksi
e. $E = 26 \times 10^6$ psi
f. 2.0% Elongation
g. Brittle, but does yield
h. Malleable Iron
I. ASTM A220 Grade 80002

CHAPTER 3 Design of Members Under Direct Stresses

3-1 $\sigma = P/A = \frac{8.50 \times 10^3 N}{\pi (10 mm)^2/4} = 108 MPa = \sigma_d = S_y/2$

REQ'D $S_y = 2\sigma_d = 2(108 MPa) = \underline{216 MPa}$

ALUMINUM 2014-T4 HAS $S_y = 290 MPa$ (APPENDIX A-17)

3-2 $\sigma = P/A = \frac{20.0 \times 10^3 N}{(10)(30) mm^2} = 66.7 MPa = \sigma_d = S_u/8$

REQD $S_u = 8\sigma_d = 8(66.7 MPa) = \underline{533 MPa}$ PLUS GOOD DUCTILITY

AISI 1141 ANNEALED HAS $S_u = 600 MPa$; 26% ELONGATION. (A-13)

3-3 $\sigma = P/A = 1720 LB/(0.401 IN)^2 = 10750 PSI = \sigma_d = S_u/8$

REQD $S_u = 8\sigma_d = 8(10750) = \underline{86000 PSI}$ PLUS GOOD DUCTILITY

AISI 1040 WQT 1300 HAS $S_u = 87 ksi$ & 32% ELONGATION (A-13)

3-4 $\sigma = P/A = \frac{1850 LB}{\pi (0.375 IN)^2/4} = 16750 psi = \sigma_d = 0.60 S_y$ (AISC)

REQD $S_y = \sigma_d/0.60 = 16750 psi/0.60 = \underline{27900 psi}$

ASTM A36 STRUCTURAL STEEL HAS $S_y = 36000 PSI$ (A-15)

3-5 $\sigma = P/A = \frac{5200 LB}{5.25 IN^2} = \underline{990 psi \text{ TOO HIGH}}$; $\sigma_{ALLOW} = 575 psi$ (A-18)

(A-4)

3-6 a) NO. 1 GRADE DOUGLAS FIR HAS $\sigma_{ALLOW} = 1050 PSI$ (A-18)

b) TO USE NO. 2 GRADE SOUTHERN PINE: $\sigma_{ALLOW} = 575 psi$

REQD AREA $= \frac{P}{\sigma} = \frac{5200 LB}{575 LB/IN^2} = 9.04 IN^2$; USE 2X8 OR 4X4 OR TWO 2X4.

3-7 $\sigma = P/A$; $A = \frac{P}{\sigma} = \frac{6400 LB}{12000 LB/IN^2} = 0.533 IN^2 = \pi D^2/4$

REQD $D = \sqrt{4A/\pi} = \sqrt{4(0.533 IN^2)/\pi} = \underline{0.824 IN}$

SPECIFY 7/8 IN (0.875 IN) OR 1.00 IN. DIA.

3-8 TOTAL MASS = 1150 + 6350 = 7500 kg; 1875 kg ON EACH STRAP.

$F = m \cdot g = 1875 kg \cdot 9.81 m/s^2 = 18394 N$

REQD $A = \frac{P}{\sigma_d} = \frac{18394 N}{70 N/mm^2} = 263 mm^2 = (w)(8 mm)$

REQD $w = 263 mm^2/8 mm = \underline{32.8 mm}$

SPECIFY $w = 35.0 mm$

3-9 FORCE ON SHELF $= 1840\,kg \cdot 9.81\,m/s^2 = 18050\,N$: $9025\,N/SIDE$

$C_Y = A_Y = 9025\,N/2 = 4513\,N$

$C = C_Y/\sin 20° = 13\,194\,N$

C = FORCE IN ROD

REQD $A = \frac{C}{\sigma_d} = \frac{13194\,N}{110\,N/mm^2} = 120\,mm^2 = \pi D^2/4$

REQD $D = \sqrt{4A/\pi} = \sqrt{4(120\,mm^2)/\pi} = \underline{12.4\,mm}$

$\underline{\text{SPECIFY } D = 14.0\,mm}$

3-10 $\sigma = \frac{P}{A} = \frac{70\,000\,LB}{\pi(8.0\,IN)^2/4} = 1393\,PSI = \frac{1}{4} \times$ RATED STRENGTH (SEC. 2-10)

REQ'D RATED STRENGTH $= 4(1393) = 5570\,PSI$

SPECIFY $\underline{\text{6000 PSI RATED STRENGTH}}$

3-11 LOAD ON EACH BLOCK $= 29\,500\,LB/3 = 9833\,LB$

$\sigma = \frac{P}{A} = \frac{9833\,LB}{12.25\,IN^2} = 803\,PSI$

IF COMPRESSION IS PERPENDICULAR TO GRAIN, NO SUITABLE WOOD LISTED.

IF PARALLEL TO GRAIN: NO. 1 SOUTH. PINE $- \sigma_{ALL.} = 850\,PSI$ (A-18)

NO. 2 HEMLOCK $- \sigma_{ALL.} = 800\,PSI$

NO. 2 DOUGLAS FIR $- \sigma_{ALL.} = 1000\,PSI$

3-12 $\sigma_{ALLOW.} = \frac{\text{RATED STRENGTH}}{4} = \frac{20.7\,MPa}{4} = 5.18\,MPa$ (SEC. 2-10)

REQD $A = \frac{P}{\sigma} = \frac{1.50 \times 10^6\,N}{5.18\,N/mm^2} = 2.90 \times 10^5\,mm^2 = \pi D^2/4$

REQD $D = \sqrt{4A/\pi} = 607\,mm$; SPECIFY $\underline{700\,mm\ DIA.}$

3-13 $S_u = 483\,MPa = \sigma = P/A$; $P = \sigma \cdot A$ (A-17)

$A = \pi(12^2 - 10^2)/4 = 34.56\,mm^2$

$P = \sigma \cdot A = 483\,\frac{N}{mm^2} \cdot 34.56\,mm^2 = 16.7 \times 10^3\,N = \underline{16.7\,kN}$

3-14 $A = (40\,mm)^2 = 1600\,mm^2$; $\sigma_{ALLOW.} = 1.69\,MPa \perp$ GRAIN (A-18)

$\sigma_{ALLOW.} = 5.52\,MPa \parallel$ GRAIN (#2 HEMLOCK)

$P = \sigma \cdot A = (1.69\,N/mm^2)(1600\,mm^2) = \underline{2.70\,kN - \perp \text{ GRAIN}}$

$P = \sigma \cdot A = (5.52\,N/mm^2)(1600\,mm^2) = \underline{8.83\,kN - \parallel \text{ GRAIN}}$

3-15 $\sigma_d = 0.60\,S_y = 0.60(50\,ksi) = 30.0\,ksi = P/A$ (AISC) (A-15) $S_y = 50\,ksi$

REQD $A = P/\sigma_d = \frac{4000\,LB}{30\,000\,LB/IN^2} = 0.133\,IN^2 = \pi D^2/4$ IF $D < 0.75\,IN$

REQD $D = \sqrt{4A/\pi} = \sqrt{4(0.133\,IN^2)/\pi} = 0.412\,IN$

SPECIFY $\underline{D = 7/16\,IN.\ (0.4375\,IN.)}$ OR $\underline{0.500\,IN.}$ (OK $- D < 0.75\,IN.$)

3-16 $A = (2.65)(1.40) + 2\left[\frac{1}{2}(1.40)(0.5)\right] = 4.41\ IN^2$

$\sigma = P/A = \dfrac{52\,000\ LB}{4.41\ IN^2} = \underline{11\,791\ PSI} = \sigma_d = S_{uc}/N$

$N = \dfrac{S_{uc}}{\sigma} = \dfrac{80\,000\ PSI}{11\,791\ PSI} = \underline{6.78}$

3-17 FOR SHOCK LOADING - DUCTILE METALS $\sigma_d = S_{uc}/12$

$\sigma_d = 1650\ MPa/12 = 137.5\ MPa$ (A-16)

REQD $A = \dfrac{P}{\sigma_d} = \dfrac{135 \times 10^3\ N}{137.5\ N/mm^2} = 982\ mm^2 = BH = B(2B) = 2B^2$

REQD $B = \sqrt{A/2} = \sqrt{982/2} = 22.2\ mm$; $H = 44.4\ mm$

SPECIFY $\underline{B = 25.0\ mm;\ H = 50.0\ mm}$

3-18 $\sigma_d = T.S./8 = 9{,}000\ PSI/8 = 1125\ PSI = P/A$ (A-19)

REQ'D $A = \dfrac{P}{\sigma_d} = \dfrac{110\ LB}{1125\ LB/IN^2} = 0.0978\ IN^2 = (0.20)(w)$

REQ'D $w = A/0.20 = 0.0978\ IN^2/0.20\ IN = 0.489\ IN$

SPECIFY $\underline{w = 0.500\ IN}$

3-19 $A = (80)(40) - (60)(15) + \pi(40)^2/4 = 3557\ mm^2$

$\sigma = \dfrac{P}{A} = \dfrac{640 \times 10^3\ N}{3557\ mm^2} = 180\ MPa = \sigma_d$

FOR DUCTILE METALS: $\sigma_d = S_y/2$

REQ'D. $S_y = 2(180) = 360\ MPa$

POSSIBLE METALS: AISI 1040 HR, $S_y = 414\ MPa$ (A-13)

AISI 4140 ANNEALED, $S_y = 414\ MPa$ (A-13)

SOFT - C54400 BRONZE, $S_y = 393\ MPa$ (A-14)

ALUMINUM 2014-T6, $S_y = 414\ MPa$ (A-17)

AISI 1020 CD, $S = 441\ MPa$ (A-13)

3-20 SEE PROB. 1-48: $\sigma_{MAX} = -48.0\ MPa$ COMPRESSION

$\sigma_d = 0.6\,S_y = 0.6(248\ MPa) = 149\ MPa$ (AISC) (A-15)

OK FOR COMPRESSIVE STRESS.

COLUMN BUCKLING SHOULD BE CHECKED.

3-21 $\sigma = 50\ MPa$ IN MEMBER AB - SEE PROB 1-53.

$\sigma_d = S_u/8$; REQ'D $S_u = 8\sigma = 8(50) = 400\ MPa$

ALUMINUM 2014-T4, $S_u = 427\ MPa$; 20% ELONG. (A-17)

3-22 $A = 0.938\ in^2$; $\sigma_d = 0.60\ S_y = 0.60(36000\ psi) = 21600\ psi$

$P_{allow.} = \sigma_d \cdot A = (21600\ lb/in^2)(0.938\ in^2) = \underline{20260\ lb}$

BEARING STRESS

3-23 FORCE = 20 kN = 12.5 kN (4.0 m/2.5 m)

a) TENSION IN MEMBER 1 AT PIN HOLES:

$A_t = 2[(20-12)(14)] = 224\ mm^2$

$\sigma = \frac{P}{A} = \frac{20 \times 10^3\ N}{224\ mm^2} = 89.3\ MPa$ TOO HIGH

FOR 6061-T4: $\sigma_d = S_y/2 = 145\ MPa/2 = 72.5\ MPa$ (A-17)

b) BEARING AT THE PIN - MEMBER 1

$\sigma_b = \frac{P}{A_b} = \frac{20 \times 10^3\ N}{(12)(14)(2)\ mm^2} = 59.5\ MPa$

FOR 6061-T4; $\sigma_{bd} = 0.65\ S_y = 0.65(145\ MPa) = 94.3\ MPa$ OK

c) BEARING AT THE PIN - MEMBER 2

$\sigma_b = \frac{P}{A_b} = \frac{20 \times 10^3\ N}{(12)(20)\ mm^2} = 83.3\ MPa$

FOR 2014-T4; $\sigma_{bd} = 0.65\ S_y = 0.65(290\ MPa) = 189\ MPa$ OK

(EQ. 3-12)

d) PIN IN BEARING: - MEMBER 2

$\sigma_{b_{MAX}} = 83.3\ MPa$ AT MEMBER 2

FOR 2014-T6; $\sigma_d = 0.65\ S_y = 0.65(414\ MPa) = 269\ MPa$ OK

e) PIN IN SHEAR - DOUBLE SHEAR

$A_s = \frac{\pi}{4}(12)^2(2) = 226\ mm^2$

$\tau = P/A_s = 20 \times 10^3\ N / 226\ mm^2 = 88.4\ MPa$

FOR 2014-T6; $\tau_d = \frac{0.5\ S_y}{2} = \frac{0.5(414\ MPa)}{2} = 103\ MPa$ OK

3-24 W = 90 kN TOTAL; 45 kN ON TWO LEGS; 22.5 kN ON EACH LEG

a) STEEL PLATE: $\sigma_b = \frac{22.5 \times 10^3\ N}{(0.10\ m)^2} = 2.25\ MPa$

(EQ. 3-8)

FOR A36 STEEL: $\sigma_{bd} = 0.9\ S_y = 0.9(248\ MPa) = 223\ MPa$ OK

b) TOP OF CONCRETE: $A_b = 2(0.20\ m)^2 = 0.080\ m^2$ (TWO LEGS)

$\sigma_b = \frac{45 \times 10^3\ N}{0.08\ m^2} = 0.563\ MPa$

CONCRETE: 2000 psi = 2.0 ksi × 6.895 MPa/ksi = 13.79 MPa = σ_c

(TABLE 3-6) — $\sigma_{bd} = 0.35\ \sigma_c \sqrt{A_2/A_1} = 0.35(13.79\ MPa)\sqrt{\frac{0.30\ m^2}{0.08\ m^2}} = 9.35\ MPa$ OK

c) SOIL: $\sigma_b = \frac{45 \times 10^3\ N}{(.8)(2.5)\ m^2} = 22.5\ kPa$

ON COMPACT GRAVEL $\sigma_{bd} = 380\ kPa$ OK (TABLE 3-6)

3-25 ON SOFT ROCK: $\sigma_{bd} = 480\ kPa = 480 \times 10^3\ N/m^2$ (TABLE 3-6)

REQ'D. $A = \frac{P}{\sigma_{bd}} = \frac{160 \times 10^3\ N}{480 \times 10^3\ N/m^2} = 0.333\ m^2 = S^2$

REQ'D. SIDE DIMENSION: $S = 0.577\ m$

3-26 $W_b = \frac{S_y - 13}{20}(0.66dL) = \frac{36-13}{20}(0.66)(3.00)(16.0) = \underline{36.4\ KIPS}$ (EQ. 3-9)

3-27 $W_b = \frac{46-13}{20}(0.66)(3.00)(16.0) = \underline{52.3\ KIPS}$

3-28 a) $W_b = \frac{36-13}{20}(0.66)(5.00)(8.00) = \underline{30.36\ KIPS}$

b) $W_b = \frac{46-13}{20}(0.66)(5.00)(8.00) = \underline{43.56\ KIPS}$

3-29 LOAD ON EACH FOOT $= F = 10\,000/4 = 2500\ LB$

$A_b = 1.59\ IN^2$ (TABLE A-9 $2 \times 2 \times 1/4$)

$\sigma_b = F/A_b = 2500\ LB / 1.59\ IN^2 = \underline{1572\ PSI}$

REQD $A_b = F/\sigma_{bd} = 2500/400 = 6.25\ IN^2$; USE SQUARE PLATE

SIDE $= \sqrt{A_b} = \sqrt{6.25\ IN^2} = \underline{2.50\ IN}$

3-30 ALLOWABLE REACTION = W_b FROM EQ. 3-9.

$W_b = \frac{S_y - 13}{20}(0.66)(d)(L)$ U.S. CUSTOMARY UNITS

$S_y = 36\ KSI$

$d = 2(200\ mm)(1.0\ IN/25.4\ mm) = 15.75\ IN.$

$L = (150\ mm)(1.0\ IN/25.4\ mm) = 5.91\ IN.$

$W_b = \frac{36-13}{20}(0.66)(15.75)(5.91) = 70.6\ KIPS$

$W_b = (70.6\ KIPS)(4.448\ kN/KIP) = \underline{314\ kN}$

OR - USING EQ 3-10 FOR SI UNITS

$d = 2h = 2(200\ mm) = 400\ mm$, $L = 150\ mm$

$S_y = 248\ MPa$

$W_b = (248 - 89.6)(3.3 \times 10^{-5})(400)(150) = \underline{314\ kN}$

SHEARING STRESS

3-31 $T = 5000$ LB-IN

FORCE ON SIDE OF KEY $= F = T/R = 5000/1.125 = 4444$ LB

SHEAR: $\tau = F/A_s = F/bL$

LET $\tau = \tau_d = \frac{0.5S_y}{2} = \frac{(0.5)(64000\text{ PSI})}{2} = 16000$ PSI

$$\text{REQ'D. } L = \frac{F}{b\,\tau_d} = \frac{4444 \text{ LB}}{(0.50\text{ IN})(16000\text{ LB/IN}^2)} = \underline{0.556 \text{ IN}}$$

BEARING: $\sigma = \frac{F}{A_b} = \frac{F}{(h/2)(L)}$

LET $\sigma = \sigma_{bd} = 0.9S_y = 0.9(64000\text{ PSI}) =$ [illegible]00 PSI

$$\text{REQ'D. } L = \frac{F}{(h/2)\,\sigma_{bd}} = \frac{4444}{(0.50/2)(57600\text{ LB/IN}^2)} = \underline{0.309 \text{ IN.}}$$

3-32 $\tau = F/A_s = F/(8\text{ IN})(a)$; LET $\tau = \tau_d = 6000$ PSI

$$\text{REQ'D } a = \frac{F}{(8\text{ IN})(\tau_d)} = \frac{21000 \text{ LB}}{(8\text{ IN})(6000\text{ LB/IN}^2)} = \underline{0.438 \text{ IN}}$$

3-33 SHEAR: TWO PINS, EACH IN DOUBLE SHEAR; $A_s = 4[\pi D^2/4] = \pi D^2$

$\tau = F/A_s$; FOR STATIC LOAD, $\tau = \tau_d = 0.5S_y/2 = \frac{0.5(82000)}{2} = 20500$ PSI

REQ'D. $A_s = F/\tau_d = \frac{42000 \text{ LB}}{20500 \text{ LB/IN}^2} = 2.049 \text{ IN}^2 = \pi D^2$

REQ'D. $D = \sqrt{A_s/\pi} = \sqrt{2.049 \text{ IN}^2/\pi} = \underline{0.808 \text{ IN.}}$

SPECIFY $D = 1.00$ IN.

CHECK BEARING: $\sigma_{bd} = (0.9)(S_y) = 0.9(82000\text{ PSI}) = 73800$ PSI

$\sigma_b = \frac{F}{A_b} = \frac{F}{(D)(2L)}$

$$\text{REQ'D. } L = \frac{F}{(D)(2)(\sigma_{bd})} = \frac{42000 \text{ LB}}{(1.0\text{ IN})(2)(73800\text{ LB/IN}^2)} = \underline{0.285 \text{ IN}}$$

VERY SMALL

3-34 $\tau_d = 0.5S_y/6 = (0.5)(565\text{ MPa})/6 = 47.1$ MPa

$\tau = F/A_s = P/2[\pi D^2/4]$

$$F_{ALLOW} = \tau_d \cdot A_s = 47.1 \frac{N}{mm^2} \cdot \frac{2\pi(16\text{ mm})^2}{4} = \underline{18.9 \text{ kN}}$$

3-35 FROM PROB. 1-59: $F = 23695$ N; $\tau = 151$ MPa ON PIN.

LET $\tau = \tau_d = \frac{S_{us}}{8} \cong \frac{0.82 S_u}{8}$; REQ'D. $S_u = \frac{8\tau}{0.82} = \frac{8(151)}{0.82} = 1473$ MPa

POSSIBLE STEEL: AISI 4140 OQT 700, $S_u = 1593$ MPa; 12% ELONG.

3-36 $A_s = (\pi D)(t) = \pi(20)(8)\text{ mm}^2 = 503\text{ mm}^2$

$\tau = S_{us} \cong 0.82 S_u = 0.82(448\text{ MPa}) = 367\text{ MPa} = 367\text{ N/mm}^2$

REQ'D. $F = \tau \cdot A_s = 367 \frac{N}{mm^2} \cdot 503\text{ mm}^2 = \underline{185 \text{ kN}}$

3-37 $\tau = S_{us} = 165$ MPa - GIVEN IN APP. A17

REQ'D. $F = \tau \cdot A_s = 165 \frac{N}{mm^2} \cdot 503\text{ mm}^2 = \underline{83 \text{ kN}}$

3-38 $\tau = S_{us} \cong 0.90\,S_u = 0.90(331\,MPa) = 298\,MPa$ COPPER

REQ'D. $F = \tau \cdot A_s = \frac{298\,N}{mm^2} \cdot 503\,mm^2 = \underline{150\,kN}$

3-39 $\tau = S_{us} \cong 0.82\,S_u = 0.82(621\,MPa) = 509\,MPa$

REQ'D. $F = \tau \cdot A_s = \frac{509\,N}{mm^2} \cdot 503\,mm^2 = \underline{256\,kN}$

3-40 $F = \tau \cdot A_s$: LET $\tau = S_{us} \cong 0.82\,S_u = 0.82(448\,MPa) = 367\,MPa$

$A_s = [2(35\,mm) + \pi(8\,mm)][5.0\,mm] = 476\,mm^2$

$F = (367\,N/mm^2)(476\,mm^2) = \underline{175\,kN}$

3-41 $F = \tau \cdot A_s$: LET $\tau = S_{us} = 16\,ksi$ (FROM APP. A-17)

$A_s = 1.144\,in^2$ (SEE PROB. 1-62)

$F = (16\,000\,lb/in^2)(1.144\,in^2) = \underline{18\,300\,lb}$

3-42 $A_s = (3.5\,in)(3.0\,in) = 10.5\,in^2$

$\tau = \frac{F}{A_s} = \frac{1800\,lb}{10.5\,in^2} = 171\,psi$ (UNSAFE)

MAXIMUM ALLOWABLE SHEAR STRESS LISTED IN TABLE A-18 IS __95 PSI.__

3-43 $F = \tau \cdot A_s$: LET $\tau = S_{us} \cong 0.82\,S_u = 0.82(97\,000\,psi) = 79\,540\,psi$

$F = \tau \cdot A_s = (79\,540\,lb/in^2)(7.5\,in \cdot 0.105\,in) = \underline{62\,650\,lb}$

3-44 LET $\tau = S_{us} \cong 0.82(S_u) = 0.82)(263\,000\,psi) = 215\,660\,psi$

$F = \tau \cdot A_s = (215\,660\,lb/in^2)(7.5 \cdot 0.105)\,in^2 = \underline{169\,800\,lb}$

3-45 LET $\tau = S_{us} \cong 0.82\,S_u = 0.82(185\,000\,psi) = 151\,700\,psi$

$F = \tau \cdot A_s = (151\,700\,lb/in^2)(7.5 \cdot 0.105)\,in^2 = \underline{119\,500\,lb}$

3-46 C36000 BRASS: LET $\tau = S_{us} \cong 0.9(68000\,psi) = 61200\,psi$

$F = \tau \cdot A_s = (61200\,lb/in^2)(7.5 \cdot 0.105)\,in^2 = \underline{48\,200\,lb}$

3-47 ALUM. 5154-H32: LET $\tau = S_{us} = 152\,000\,psi$

$F = \tau \cdot A_s = (152\,000\,lb/in^2)(7.5 \cdot 0.105)\,in^2 = \underline{119\,700\,lb}$

3-48 1. FIND F_2:

$\Sigma M_0 = 0 = (F_1 \cos 30)(30) + (F_1 \sin 30)(40)$

$+ (F_2 \sin 10°)(10) - (F_2 \cos 10°)(25)$

$0 = 165.5 - 22.88\,F_2$

$\underline{F_2 = 7.23\,kN}$

2. FIND RESULTANT F ON PIN

$R_p = [(F_{1x} + F_{2x})^2 + (F_{1y} + F_{2y})^2]^{1/2}$

$R_p = [(3.12 + 7.12)^2 + (1.8 - 1.26)^2]^{1/2}$

$\underline{R_p = 10.7\,kN}$

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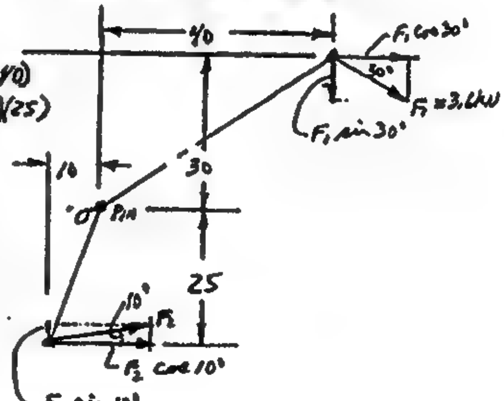


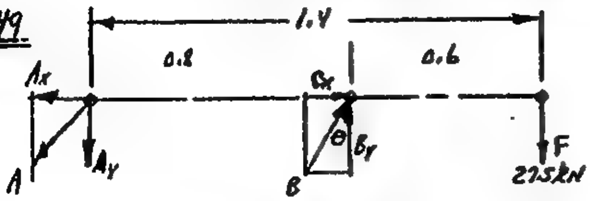
3-48 (CONTINUED)

3. DESIGN SHEAR STRESS: $\tau_d = \frac{S_{ns}}{8} \cong \frac{0.95_u}{8} = \frac{(0.90)(1482\ MPa)}{8} = 167\ MPa$

4. REQ'D $A_s = \frac{R}{\tau_d} = \frac{10.7 \times 10^3\ N}{167\ N/mm^2} = 64.2\ mm^2 = \frac{\pi D^2}{4}$

5. REQ'D. $D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(64.2)}{\pi}} = \underline{9.04\ mm}$

3-49



$\tan\theta = 0.8/1.50$

$\theta = 28.1°$

$\Sigma M_A = 0 = 27.5(1.4) - B_y(0.8)$

$B_y = 48.125\ kN$

$A_y = B_y - F = 20.625\ kN$

$\underline{B = B_y/\cos\theta = 54.5\ kN}$

$B_x = B\sin\theta = 25.7\ kN$

$A_x = B_x = 25.7\ kN$

$\underline{A = \sqrt{A_x^2 + A_y^2} = 32.95\ kN}$

$\underline{C = B = 54.5\ kN}$

$\tau_d = \frac{0.5\ S_y}{2} = \frac{0.5(441\ MPa)}{2} = 110\ MPa$

PIN A: REQ'D. $A_s = \frac{A}{\tau_d} = \frac{32.95 \times 10^3\ N}{110\ N/mm^2} = 299\ mm^2 = \left[\frac{\pi D_A^2}{4}\right]2 = \frac{\pi D_A^2}{2}$

REQ'D. $D_A = \sqrt{2A/\pi} = \sqrt{2(299)/\pi} = \underline{13.8\ mm}$

PINS B AND C: REQ'D. $A_s = \frac{54.5 \times 10^3\ N}{110\ N/mm^2} = 494\ mm^2 = \frac{\pi D_B^2}{2}$

REQ'D. $D_B = D_C = \sqrt{2A_s/\pi} = \sqrt{2(494)/\pi} = \underline{17.7\ mm}$

3-50 FORCES ON JOINTS FOUND FROM PROBLEM 1-51.

JOINT A: $F_s = AD = 10.5\ kN$ = JOINT C

JOINT B: $F_s = \sqrt{10.5^2 + 9.09^2} = 13.9\ kN$ = JOINT D

$\tau_d = \frac{0.5\ S_y}{2} = \frac{(0.5)(345)}{2} = 86.3\ MPa$

REQ'D. $A_s = \frac{F_s}{\tau_d} = \frac{10.5 \times 10^3\ N}{86.3\ N/mm^2} = 121.7\ mm^2 = \frac{\pi D^2}{4}$

$D = \sqrt{4A_s/\pi} = \underline{12.4\ mm}$ FOR JOINTS A AND C

REQ'D. $A_s = \frac{F_s}{\tau_d} = \frac{13.9 \times 10^3\ N}{86.3\ N/mm^2} = 161.1\ mm^2$

$D = \sqrt{4A_s/\pi} = \underline{14.3\ mm}$ FOR JOINTS B AND D.

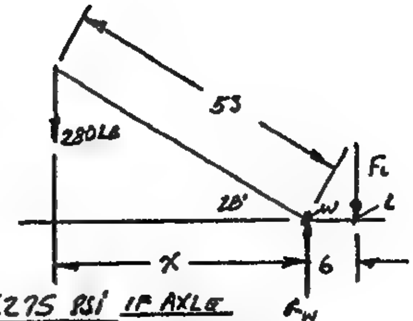
3-51 $x = 53\cos 28° = 46.8$ IN.

$\Sigma M_W = 0 = 280(46.8) - F_L(6)$

$\underline{F_L = 2184 \text{ LB}}$

$\Sigma M_L = 0 = 280(52.8) - R_W(6)$

$\underline{R_W = 2464 \text{ LB}}$



SHEAR STRESS ON AXLE:

$\tau = \dfrac{R_W}{A_S} = \dfrac{2464 \text{ LB}}{2(\pi(0.5)^2/4)\text{IN}^2} = \underline{6275 \text{ PSI}}$ $\underline{\text{IF AXLE IS IN DOUBLE SHEAR}}$

3-52 a) SHEAR OF PIN: $\tau = F/A_s$; LET $\tau = \tau_d = \frac{S_{us}}{8} = \frac{0.82 S_u}{8} = \frac{0.82(97)}{8} = 9.94$ KSI

$F_{ALLOW} = \tau_d \cdot A_s = 9940 \frac{LB}{IN^2} \cdot \frac{2\pi(0.63)^2}{4} IN^2 = \underline{6200 \text{ LB}}$

b) BEARING: $\sigma_b = F/A_b$; LET $\sigma_b = \sigma_{bd} = 0.9 S_y = 0.9(82) = 73.8$ KSI

$F_{ALLOW} = \sigma_{bd} \cdot A_b = 73800 \frac{LB}{IN^2} \cdot (0.63)(2)(0.38) IN^2 = \underline{35334 \text{ LB}}$

c) TENSION: $\sigma = \frac{F}{A_t} K_t$; LET $\sigma = \sigma_d = \frac{S_u}{8} = \frac{97}{8} = 12.125$ KSI

$A_t = (1.50 - 0.63)(2)(0.38) = 0.661 IN^2$

K_t FROM APP. A-21-4 CURVE B; $d/w = 0.63/1.50 = 0.42$; $K_t = 2.83$

$F_{ALLOW} = \frac{\sigma_d \cdot A_t}{K_t} = \frac{(12125 \text{ LB/IN}^2)(0.661 IN^2)}{2.83} = \underline{2832 \text{ LB}}$

3-53 IMPACT: $\tau_d = \frac{S_y}{12} = \frac{90 KSI}{12} = 7.50$ KSI $= 7500$ PSI

$A_s = 2\pi D^2/4 = \pi D^2/2$

REQD. $A_s = F/\tau_d = \frac{500 LB}{7500 LB/IN^2} = 0.0667 IN^2 = \pi D^2/2$

REQD. $D = \sqrt{\frac{2 A_s}{\pi}} = \sqrt{\frac{2(0.0667)}{\pi}} = \underline{0.206 IN}$; SPECIFY $\underline{D = 0.250 IN.}$

3-54 $F_{ALLOW.}$ ON EACH BOLT: $F = A_s \tau_d = \frac{\pi(1.25)^2 IN^2}{4} \cdot \frac{6000 LB}{IN^2} = 7363$ LB

TORQUE $= F \cdot$ RADIUS $= \frac{7363 LB}{BOLT} \cdot 8$ BOLTS $\cdot 4.50$ IN $= \underline{2.65 \times 10^5 \text{ LB-IN}}$

3-55 $F = 25$ kN REPEATED. AISI 4140 OQT 1100 $S_u = 1014$ MPa

$\sigma_d = S_u/N$; LET $\sigma_{MAX} = \sigma_d$: THEN $N = S_u/\sigma_{MAX}$

HOLE: $d/D = 10/25 = 0.40$: $K_{tg} = 5.10$ (A-21-5)

$\sigma_g = \frac{F}{A} = \frac{F}{\pi D^2/4} = \frac{25000 N}{\pi(25)^2/4 \; mm^2} = 50.9$ MPa

$\sigma_{MAX} = K_{tg} \cdot \sigma_g = (5.10)(50.9) = 260$ MPa

$N = S_u/\sigma_{MAX} = 1014/260 = \underline{3.90}$ – LOW – SHOULD BE > 8.

FILLET: $D/d = 25/20 = 1.25$: $r/d = 2/20 = 0.10$: $K_t = 1.77$ (A-21-2)

$\sigma_{NOM} = \frac{F}{A} = \frac{F}{\pi d^2/4} = \frac{25000 N}{\pi(20)^2/4 \; mm^2} = 79.6$ MPa

$\sigma_{MAX} = K_t \sigma_{NOM} = (1.77)(79.6) = 141$ MPa

$N = S_u/\sigma_{MAX} = 1014/141 = \underline{7.20 - LOW}$

3-56 $r/d = \frac{0.50}{6.0} = 0.083$; $D/d = \frac{9}{6} = 1.50$; THEN $\underline{K_t = 1.95}$ (A-21-2)

$\sigma_{MAX} = K_t\,\sigma_{NOM} = \frac{(1.95)(900\,N)}{\pi(6\,mm)^2/4} = \underline{62.1\ MPa}$

3-57 ASSUME CIRCULAR GROOVES: APPENDIX A-21-1

$\frac{r}{d_g} = \frac{6\,mm}{75\,mm} = 0.08$; $\frac{d}{d_g} = \frac{85}{75} = 1.13$; THEN $\underline{K_t = 2.35}$

$\sigma_{MAX} = \frac{K_t F}{A} = \frac{2.35(36\times10^3\,N)}{\pi(75)^2/4\ mm^2} = \underline{19.15\ MPa}$

3-58 STRESS AT HOLE: $a/D = \frac{0.150}{1.00} = 0.50$; APP. A-21-5 $K_{tg} = 6.3$

$\sigma_{MAX} = \frac{K_t F}{A_g} = \frac{6.3(F)}{\pi(1.0)^2/4} = 8.02\,F$

STRESS AT FILLET: LET $K_t = 1.7$ AND $\sigma_{MAX} = 8.02F$

$\sigma_{MAX} = \frac{K_t F}{A} = \frac{1.7F}{A} = 8.02F$

REQ'D. $A = \frac{1.7\cancel{F}}{8.02\cancel{F}} = 0.212\ IN^2 = \pi d^2/4$

REQ'D $d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.212)}{\pi}} = \underline{0.519\ IN}$

THEN $D/d = 1.00/0.519 = 1.93$

FOR $K_t = 1.7$ AND $D/d = 1.93$; $r/d = 0.17$

THEN $r = 0.17d = 0.17(0.519) = \underline{0.088\ IN.}$

ADDITIONAL PROBLEMS – DIRECT TENSION OR COMPRESSION

3-59 $F = 125\,kN$. $A = (32)(40)\,mm^2 = 1280\,mm^2$. SPECIFY MATERIAL.

a) $\sigma_{MAX} = F/A = \dfrac{125\times10^3 N}{1280\,mm^2} = 97.7\,MPa$

b) CASE B: LET $N = 8$; $\sigma_d = s_u/8$; REQ'D $s_u = 8\,\sigma_{MAX}$

$s_u = 8(97.7\,MPa) = 781\,MPa$

SPECIFY AISI 4140 OQT 1300, $s_u = 814\,MPa$, 23% ELONGATION

GOOD DUCTILITY. OTHER MATERIALS AVAILABLE IN TABLE A-13.

3-60 $F = 125\times10^3 N/2 = 6250\,N$ PER ROD.

SPECIFY AISI 1040 H.R. $S_u = 621\,MPa$, 25% ELONGATION GOOD DUCTILITY

CASE: C DETERMINE DIAMETER

$\sigma_d = S_u/12$ SHOCK. $\sigma = F/A$. REQ'D $A = F/\sigma_d$

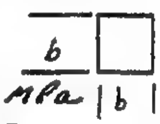
$\sigma_d = 621\,MPa/12 = 51.75\,MPa = 51.75\,N/mm^2$

$A = \dfrac{F}{\sigma_d} = \dfrac{6250\,N}{51.75\,N/mm^2} = 120.8\,mm^2 = \pi D^2/4$

REQ'D $D = \sqrt{4A/\pi} = \sqrt{4(120.8)/\pi} = 12.4\,mm = D_{MIN}$

SPECIFY $D = 14.0\,mm$ – PREFERRED BASIC SIZE

3-61 $F = 4.0\,kN$. REPEATED. CASE C. $\sigma_d = S_u/8$



AISI 1040 CD. $S_u = 669\,MPa$. $\sigma_d = 669/8 = 83.6\,MPa$

REQ'D $A = \dfrac{F}{\sigma_d} = \dfrac{4.0\times10^3 N}{83.6\,N/mm^2} = 47.8\,mm^2 = b^2$ [SQUARE]

$b_{min} = \sqrt{A} = \sqrt{47.8\,mm^2} = 6.9\,mm$. SPECIFY $b = 7.0\,mm$

3-62 $A = (30)(20) = 600\,mm^2$. $\sigma = F/A$. $F_{MAX} = \sigma_d A$ (REPEATED)

AISI 4140 OQT 1300. $S_u = 814\,MPa$, 23% ELONGATION – DUCTILE

$\sigma_d = S_u/8 = 814\,MPa/8 = 102\,MPa$

$F_{MAX} = \sigma_d \cdot A = \dfrac{102\,N}{mm^2}\cdot 600\,mm^2 = 61{,}200\,N = 61.2\,kN = F_{MAX}$

3-63 $F = 8.25\,kN$ REPEATED. CASE B – SPECIFY MATERIAL.

$\sigma_{MAX} = K_t\,\sigma_{nom} = K_t \cdot \dfrac{F}{\pi d_g^2/4} = K_t \dfrac{8.25\times10^3 N}{\pi(22\,mm)^2/4} = K_t(21.7\,MPa)$

APP A21-1: $r/d_g = 4.0\,mm/22\,mm = 0.182$; $D/d_g = 30/22 = 1.36$; $K_t = 1.65$

$\sigma_{MAX} = K_t \cdot \sigma_{nom} = 1.65(21.7\,MPa) = 35.8\,MPa$

LET $\sigma_{MAX} = \sigma_d = S_u/8$. REQ'D $S_u = 8\sigma_d = 8(35.8\,MPa) = 286\,MPa$

SPECIFY ALUMINUM 6061-T6. $S_u = 310\,MPa$, 17% ELONG.

OR ANY STEEL FROM TABLE A-13 COULD BE USED.

3-64 $M = 2800\,kg$. $W = mg = 2800\,kg \cdot 9.81\,m/s^2 = 27468\,N \cdot s^2/m = 27.5\,kN$

$\sigma = F/A$. REQ'D $A = F/\sigma_d$. $\sigma_d = S_u/12$ IMPACT

6061-T4 ALUMINUM. $S_u = 241\,MPa$. $\sigma_d = 241/12 = 20.08\,MPa$

$$A_{min} = \frac{F}{\sigma_d} = \frac{27.5 \times 10^3\,N}{20.08\,N/mm^2} = 1369\,mm^2 = \pi D^2/4$$

$\underline{D_{min}} = \sqrt{4A/\pi} = \sqrt{4(1369\,mm^2)/\pi} = \underline{41.7\,mm}$. $\underline{\text{SPECIFY } D = 45\,mm}$

3-65 HEMLOCK, GRADE 3 WOOD. $\sigma_{ALLOW.} = 500\,psi$ COMP. PARALLEL TO GRAIN

$\sigma = F/A$. $F_{MAX} = \sigma_{ALL.} \cdot A = 500\,LB/IN^2 \cdot (3.50)(7.50)\,IN^2 = \underline{13{,}125\,LB = F_{MAX}}$

3-66 AISI 1141 OQT 900. $S_u = 1007\,MPa$. $\sigma_d = S_u/12$ SHOCK. $F = 45.0\,kN$

$\sigma_d = 1007\,MPa/12 = 83.9\,MPa$. $\sigma = F/A$. $A_{min} = F/\sigma_d$

$$A_{min} = \frac{45.0 \times 10^3\,N}{83.9\,N/mm^2} = 536\,mm^2 = \pi D^2/4. \quad D = \sqrt{4A/\pi} = \sqrt{\frac{4(536\,mm^2)}{\pi}}$$

$\underline{D_{min} = 26.1\,mm}$, SPECIFY $\underline{D = 28.0\,mm}$ BASIC SIZE

3-67 FIND REPEATED TENSILE LOAD. AISI 4140 OQT 1300, $S_u = 118\,ksi$

$\sigma_d = S_u/8 = 118\,ksi/8 = 14.75\,ksi$. 1-IN PIPE: $A = 0.494\,IN^2$

$\sigma = \frac{F}{A}$. $F_{MAX} = \sigma_d A = (14750\,lb/in^2)(0.494\,in^2) = \underline{7287\,lb = F_{MAX}}$

3-68 $F = 550\,kN$ SHOCK. (a) SPECIFY DUCTILE IRON 60-40-18 FOR DUCTILITY.

$S_u = 414\,MPa$, 18 % ELONGATION. $\sigma_d = S_u/12 = 414\,MPa/12 = 34.5\,MPa$

$$\sigma = F/A. \quad \text{REQ'D } A = \frac{F}{\sigma_d} = \frac{550 \times 10^3\,N}{34.5\,N/mm^2} = 15{,}942\,mm^2 = \pi D^2/4$$

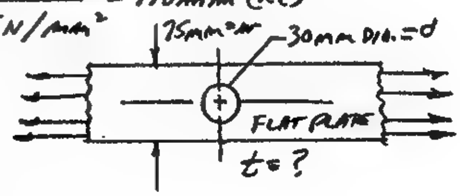
$\underline{D_{MIN}} = \sqrt{4A/\pi} = \sqrt{4(15942\,mm^2)/\pi} = \underline{142\,mm}$. $\underline{\text{SPECIFY } D = 160\,mm}$

3-69 $F = 16.5\,kN$ REPEATED. ASTM A242. $S_u = 483\,MPa$. $\sigma_d = S_u/12 = \frac{483\,MPa}{12} = 40.25\,MPa$

$$\text{FIND } A.\ \sigma_{MAX} = K_t(F/A).\ A_{MIN} = \frac{F(K_t)}{\sigma_d} = \frac{16.5 \times 10^3\,N(K_t)}{40.25\,N/mm^2} = 410\,mm^2(K_t)$$

$d/w = 30/75 = 0.40$; $K_t = 2.23$

APP A21-4 CURVE A.



$A_{MIN} = 410\,mm^2(2.23) = 914\,mm^2 = wt$

$$\underline{t_{MIN}} = \frac{A_{MIN}}{w} = \frac{914\,mm^2}{75\,mm} = \underline{12.2\,mm}$$

SPECIFY $\underline{t = 14\,MM}$ $\underline{\text{PREFERRED BASIC SIZE}}$

3-70 FIGURE P3-70. AISI 1141 OQT 1100. $S_u = 800$ MPa. $\sigma_d = S_u/8$ REPEATED LOAD.
$\sigma_{max} = K_t F/A$. $F_{ALLOW} = \sigma_d A/K_t$ $\sigma_d = 100$ MPa
MIDDLE SECTION: $A = (10\text{ mm})(6\text{ mm}) = 60\text{ mm}^2$. $\frac{r}{h} = \frac{1.5\text{ mm}}{10\text{ mm}} = 0.15$
APP. A21-3. $H/h = 16\text{ mm}/10\text{ mm} = 1.60$. $K_t = 1.90$
$$F_{ALLOW} = \frac{\sigma_d \cdot A}{K_t} = \frac{(100\text{ N/mm}^2)(60\text{ mm}^2)}{1.90} = 3158\text{ N}$$
AT PIN: APP. A21-4: CURVE B. $w = 16$ mm, $d = 6$ mm. $\frac{d}{w} = \frac{6}{16} = 0.375$
$K_t = 3.15$. $A = (w-d)t = (16-6)(6) = 60\text{ mm}^2$
$$F_{ALLOW} = \frac{\sigma_d \cdot A}{K_t} = \frac{(100\text{ N/mm}^2)(60\text{ mm}^2)}{3.15} = 1967\text{ N} = F_{ALLOW}$$

3-71 SPECIFY MATERIAL. $\sigma_{max} = K_t F/A$. $\sigma_d = S_u/12$ SHOCK
AT HOLE: APP A21-5 CURVE A. $d/D = 12/30 = 0.40$. $K_{tg} = 5.0$
$A = \frac{\pi D^2}{4} = \frac{\pi(30\text{ mm})^2}{4} = 707\text{ mm}^2$ GROSS AREA
LET $\sigma_{MAX} = \sigma_d = \frac{S_u}{12} = \frac{K_t F}{A}$, REQ.D $S_u = \frac{12 K_t F}{A} = \frac{12(5.0)(12.6\times10^3\text{ N})}{707\text{ mm}^2}$
REQ'D. $S_u = 1070$ MPa
AT FILLET: APP A21-2. $r/d = 1.2\text{ mm}/18\text{ mm} = 0.067$. $D/d = 30/18 = 1.67$
$K_t = 2.10$. $A = \pi d^2/4 = \frac{\pi(18\text{ mm})^2}{4} = 254\text{ mm}^2$
$$\text{REQ'D. } S_u = \frac{12 K_t F}{A} = \frac{12(2.10)(12.6\times10^3\text{ N})}{254\text{ mm}^2} = 1248\text{ MPa}$$
STRESS AT FILLET GOVERNS: $S_{u_{MIN}} = 1248$ MPa WITH GOOD DUCTILITY
SPECIFY: AISI 4140 OQT 900. $S_u = 1289$ MPa; 15% ELONGATION

DIRECT SHEAR STRESS

3-72 COMPUTE FORCE REQUIRED TO PUNCH OUT THE SHAPE IN FIGURE P3-72. $\tau = F/A$: $F = \tau A$: LET $\tau = S_{us}$
AISI 1020 CD. $S_u = 75$ ksi. $S_{us} = 0.82 S_u = 0.82(75) = 61.5$ ksi
A = SHEAR AREA = PERIMETER × THICKNESS = $p \cdot t$: $t = 0.085$ IN
$p = 1.0 + 1.0 + 2.0 + \sqrt{1^2+1^2} + 0.50 + \pi(0.5)/2 = 6.70$ IN
$A = p \cdot t = (6.70)(0.085) = 0.569\text{ IN}^2$.
$F = S_{us} \cdot A = (61.5\times10^3\text{ lb/in}^2)(0.569\text{ in}^2) = 35\,020\text{ lb} = F$

3-73 $F = \tau A = S_{us} \cdot A$. 6061-T4. $S_{us} = 24$ ksi - APP. A-17. $t = 0.10$ IN
$p = 4(1.25) + 3(0.5) + \pi(1.50)/2 = 8.86\text{ IN}$; $A = p \cdot t = 0.886\text{ IN}^2$
$F = S_{us} \cdot A = (24\times10^3\text{ lb/in}^2)(0.886\text{ in}^2) = 21\,260\text{ lb} = F$

3-74 $F = \tau \cdot A = S_{us} \cdot A$. $S_{us} = 110$ MPa (ALUM. 3003-H18) . $A = p \cdot t$, $t = 3.0$ mm

$p = 30 + 60 + \sqrt{20^2 + 40^2} + \sqrt{10^2 + 40^2} = 176$ mm . $A = 527.9$ mm^2

$F = S_{us} \cdot A = (110\ N/mm^2)(527.9\ mm^2) = \underline{58.06\ kN = F}$

3-75 $F = \tau \cdot A = S_{us} \cdot A$. AISI 1040 CD . $S_u = 669$ MPa . $S_{us} = .82 S_u = 549$ MPa

$A = p \cdot t$, $t = 1.60$ mm . $p = 50 + 30 + 2(20) + \pi(20)/2 = 151.4$ mm

$A = (151.4\ mm)(1.60\ mm) = 242\ mm^2$. $\underline{F} = 549 \frac{N}{mm^2} \cdot 242\ mm^2 = \underline{133\ kN}$

3-76 $F = \tau \cdot A = S_{us} \cdot A$. AISI 1080 OQT 900 . $S_u = 1234$ MPa

$S_{us} = 0.82 S_u = 0.82(1234\ MPa) = \underline{1012\ MPa}$

$A = p \cdot t$. $t = 0.80$ mm . $p = 60 + 2\sqrt{15^2 + 15^2} + 22 + \pi(4) = 137$ mm

$A = (137\ mm)(0.80\ mm) = 109.6\ mm^2$. $F = (1012\ N/mm^2)(109.6\ mm^2)$

$\underline{F = 110.9\ kN}$

3-77 $F = \tau \cdot A = S_{us} \cdot A$. ALUM. 3003-H12 . $S_{us} = 83$ MPa

$A = p \cdot t$, $t = 1.40$ mm .

$p = 2(90) + 2(18) + 3[\pi(6)] + 2[2(12) + \pi(6)] = 358$ mm

$A = p \cdot t = (358)(1.40) = 502\ mm^2$

$F = S_{us} \cdot A = (83\ N/mm^2)(502\ mm^2) = \underline{41.6\ kN = F}$

3-78 $F = \tau \cdot A = S_{us} \cdot A$. ALUM 5154-H38 . $S_{us} = 193$ MPa

$A = p \cdot t$. $t = 2.00$ mm . $p_{OUTSIDE} = 2(128) + 2(50) = 356$ mm

$p_{INSIDE} = \pi(15) + 2[2(15) + 2(10)] = 147.1$ mm

TOTAL $p = p_o + p_i = 503.1$ mm . $A = p \cdot t = (503.1)(2.0) = 1006\ mm^2$

$F = S_{us} \cdot A = (193\ N/mm^2)(1006\ mm^2) = \underline{194.2\ kN = F}$

3-79 DATA FROM PROB. 3-78

<u>FIRST STEP:</u> $p = p_{INSIDE} = 147.1$ mm . $A_1 = p \cdot t = (147.1)(2.0) = 294.2\ mm^2$

$F_1 = S_{us} \cdot A_1 = (193\ N/mm^2)(294.2\ mm^2) = \underline{56.8\ kN = F_1}$

<u>2ND STEP:</u> $p = p_{OUTSIDE} = 356$ mm . $A_2 = (356)(2.0) = 712\ mm^2$

$F_2 = S_{us} \cdot A_2 = (193\ N/mm^2)(712\ mm^2) = \underline{137.4\ kN = F_2} = \underline{ANSWER}$

BEARING STRESS

3-80 $\sigma_b = \dfrac{F_b}{A_b}$. $F_b = 28\,500\ lb/4_{LEGS} = 7125\ lb/LEG$. $A = 1.44\ IN^2$

$\sigma_b = 7125\ lb/1.44\ IN^2 = \underline{4948\ psi = \sigma_b}$

3-81 CONCRETE FLOOR. $\sigma_c = 3000\ psi$, $\sigma_{bd} = 0.35\,\sigma_c \sqrt{A_2/A_1}$

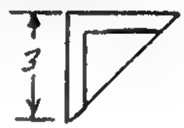
BUT $\sigma_{bd\,MAX} = 0.70\,\sigma_c$ BECAUSE $A_2/A_1 >> 4$. $F_b = 7125\ lb$

$\sigma_{bd} = 0.70(3000) = 2100\ psi = F_b/A_b$. REQ'D $A_b = F_b/\sigma_{bd}$

$A_{b\,MIN} = 7125\ lb / 2100\ lb/in^2 = 3.39\ IN^2$

TRIANGULAR AREA: $A = \frac{1}{2}(3)(3) = 4.50\ IN^2$

WELD PAD TO BOTTOM OF EACH LEG.



3-82 $A_b = (3.50)(7.50) = 26.25\ IN^2$, $F_b = \sigma_{bd} A_b$. $\sigma_{bd} = 55\ psi$

$F_b = (55\ lb/IN^2)(26.25\ IN^2) = \underline{1444\ lb = F_b\ \text{ALLOWABLE}}$

3-83 4-IN SCH.40 PIPE, $A_b = 3.174\ IN^2$, $\sigma_{bd} = 0.70\,\sigma_c$ ON LARGE FLOOR

$\sigma_{bd} = 0.70(4000\ psi) = 2800\ psi$

$F_b = \sigma_{bd} \cdot A_b = (2800\ lb/IN^2)(3.174\ IN^2) = \underline{8\,887\ lb = F_b\ \text{ALLOWABLE}}$

3-84 DATA FROM PROB 3-83. $\sigma_{bd} = 2800\ psi = F_b/A_b$

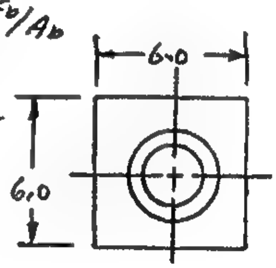
$F_b = 10(8887\ lb) = 88{,}870\ lb$

REQ'D. $A_b = F_b/\sigma_{bd} = \dfrac{88\,870\ lb}{2800\ lb/in^2} = 31.74\ IN^2$

TRY SQUARE PLATE: $A_b = b^2$.

$\underline{b_{MIN}} = \sqrt{A_{b\,MIN}} = \sqrt{31.74\ IN^2} = \underline{5.63\ IN}$

USE $\underline{b = 6.0\ IN\ \text{SQUARE PLATE}}$.



PROBLEMS WITH TWO OR MORE KINDS OF DIRECT STRESS

3-85 FIG. P3-85. RIVETED PLATES. ASSUME STATIC LOAD

6061-T6 PLATES: $S_y = 40\,ksi$, $S_u = 45\,ksi$, 17% ELONGATION

2014-T4 RIVETS: $S_{us} = 38\,ksi$, $S_y = 42\,ksi$

a) SHEAR OF RIVETS: $\tau_d = S_{us}/4 = 38\,ksi/4 = 9.50\,ksi = F/A_s$

$A_s = 2(\pi(0.5\,in)^2/4) = 0.393\,in^2$ TWO CROSS SECTIONS

$F = \tau_d \cdot A_s = \left(9.5\times10^3 \frac{lb}{in^2}\right)(0.393\,in^2) = 3730\,lb$

b) TENSILE STRESS ON PLATE: $\sigma_d = S_y/3 = 40\,ksi/3 = 13.3\,ksi$

$\sigma = F/A$, $A = [3.0\,in - 2(0.5\,in)]\,0.375\,in = 0.75\,in^2$

$F = \sigma_d \cdot A = \left(13\,333 \frac{lb}{in^2}\right)(0.75\,in^2) = 10000\,lb$

c) BEARING AT RIVETS/HOLES: $\sigma_b = \frac{F}{A_b} = \sigma_{bd} = 0.65\,S_y$

$\sigma_{bd} = 0.65(40\,ksi) = 26\,ksi$ ON PLATE

$A_b = 2(D)(t) = (0.50)(.375)(2) = 0.375\,in^2$ - PROJECTED AREA

$F = \sigma_{bd} \cdot A_b = \left(26\,000 \frac{lb}{in^2}\right)(0.375\,in^2) = 9750\,lb$

SHEAR STRESS GOVERNS: $F_{allow} = 3730\,lb$

3-86 DATA FROM PROBLEM 3-85.

a) SHEAR OF RIVETS: $\tau_d = 9500\,lb/in^2 = F/A_s$

$A_s = 3(\pi(0.375)^2/4) = 0.3313\,in^2$

$F = \tau_d \cdot A_s = (9500\,lb/in^2)(0.3313\,in^2) = 3148\,lb$

b) TENSILE STRESS IN PLATE: $\sigma_d = 13\,333\,psi = F/A$

$A = [3.0 - 3(0.375)](0.375) = 0.703\,in^2$

$F = \sigma_d \cdot A = (13{,}333\,lb/in^2)(0.703\,in^2) = 9375\,lb$

c) BEARING: $\sigma_{bd} = 26\,000\,lb/in^2$ ON PLATE

$A_b = 3[D \cdot t] = 3[(0.375)(0.375)] = 0.422\,in^2$

$F = \sigma_{bd} \cdot A_b = (26\,000\,lb/in^2)(0.422\,in^2) = 10{,}969\,lb$

SHEAR STRESS GOVERNS: $F_{allow} = 3148\,lb$

3-87 DATA FROM PROBLEM 3-85.

a) SHEAR OF RIVETS: $\tau_d = 9500\ lb/in^2 = F/A_s$

$A_s = 4[\pi(0.375)^2/4] = 0.4418\ in^2$

$F = \tau_d \cdot A_s = (9500\ lb/in^2)(0.4418\ in^2) = \underline{4197\ lb}$

b) TENSION ON PLATE: $\sigma_d = 13\,333\ lb/in^2 = F/A$

$A = [3.0 - 2(0.375)]\,0.375 = 0.844\ in^2$

$F = \sigma_d \cdot A = (13\,333\ lb/in^2)(0.844\ in^2) = \underline{11\,250\ lb}$

c) BEARING AT RIVETS:

ON PLATES: $\sigma_{bd} = 0.65(40\ ksi) = 26\ ksi$

$A_b = 2(0.25)(0.375)(2) = 0.375\ in^2$

$F_b = \sigma_{bd} \cdot A_b = (26000\ lb/in^2)(0.375\ in^2) = \underline{9750\ lb}$

ON RIVETS: $\sigma_{bd} = 0.65(42\ ksi) = 27.3\ ksi$

$A_b = 2(0.375)(0.375) = 0.281\ in^2$

$F_b = (27300\ lb/in^2)(0.281\ in^2) = \underline{7678\ lb}$

SHEAR OF RIVETS GOVERNS: $\underline{F_{ALLOW} = 4197\ lb}$

3-88 REPEATED FORCE.

TENSION IN LINK A: $\sigma_d = s_u/8 = 147\ ksi/8 = 18.375\ ksi$ (AISI 4140 OQT 1100)

$\sigma = \dfrac{K_t F}{A_{NET}}$; $F = \dfrac{\sigma_d A_{NET}}{K_t} = \dfrac{(18375\ lb/in^2)(1.5-0.75)(1.25)\ in^2}{2.60}$

APP A 21-4, CURVE B: $d/w = 0.75/1.5 = 0.50$. $K_t = 2.60$

$\underline{F_{ALLOW} = 6626\ lb}$

SHEAR STRESS IN PIN: $\tau_d = s_y/8 = 97\ ksi/8 = 12.125\ ksi$ (AISI 1141 OQT 1100)

$\tau = \dfrac{F}{A_s}$: $F = \tau_d \cdot A_s = \left(12125\ \dfrac{lb}{in^2}\right)\left(2[\pi(0.75)^2/4]\right) = \underline{10{,}713\ lb}$

BEARING STRESS AT PIN: $\sigma_{bd} = 0.90\,s_y = (0.90)(97\ ksi) = 87.3\ ksi$

$A_b = (1.25\ in)(0.75\ in) = 0.9375\ in^2$

$\sigma_b = F/A_b$: $F = \sigma_{bd} \cdot A_b = (87300\ lb/in^2)(0.9375\ in^2) = \underline{81843\ lb}$

TENSION IN LINK GOVERNS: $\underline{F_{ALLOW} = 6626\ lb}$

3-89 FORCES IN MEMBERS: AB = 2465 LB (T), AC = 1925 LB (C)
BC = 1375 LB (T), BD = 1200 LB (T), CE = 650 LB (C),
CD = 750 LB (C), DE = 961 LB (T).

SUPPORT FORCES: A_y = 1540 LB ↓, B_y = 2640 LB ↑, B_x = 100 LB ←

MATERIAL: ASTM A36 STRUCTURAL STEEL, S_y = 36 KSI
ASSUME STATIC LOAD. $\sigma_d = S_y/2 = 18$ KSI

REQ'D AREA: $A_{MIN} = F/\sigma_d$

MEMBER	F (LB)	σ_d (KSI)	A_{MIN} (IN²)	SQUARE b_{MIN}	ROUND D_{MIN}	THREAD
AB	2465	18	0.137	.374	.418	1/2-13
BC	1375	18	0.0764	.276	.312	3/8-16
BD	1200	18	0.0667	.258	.291	3/8-16
DE	961	18	0.0534	.231	.261	3/8-16

ALTERNATIVE DESIGNS:

SQUARE ROD: $A = b^2$; $b_{MIN} = \sqrt{A}$

ROUND ROD: $A = \pi D^2/4$; $D_{MIN} = \sqrt{4A/\pi}$

THREADED ROD: LET A_{MIN} < TENSILE STRESS AREA OF THREAD FROM APP A-3, COARSE THREADS

FOR THREADED ROD, ATTACH TO CLEVIS AT EACH END. DESIGN PIN FOR CLEVIS FOR SAFE SHEAR STRESS.

AT B: PIN JOINT ATTACHED TO FRAME

AT A: PROVIDE ROLLER ON PIN THAT BEARS ON FRAME.

AT C AND E: PROVIDE AN ADDITIONAL CLEVIS FROM WHICH TO ATTACH LOADS.

NOTE: COMPRESSION MEMBERS MUST BE DESIGNED WITH COLUMN BUCKLING ANALYSIS. SEE CHAPTER 14.

3-90 FORCES IN MEMBERS:

AB = 4687 N (T)	BE = 0	BF = 2241 N (T)	CG = 1097 N (T)
AD = 1400 N (T)	DE = 2300 N (C)	CF = 800 N (C)	FG = 500 N (C)
BD = 2597 N (C)	BC = 750 N (T)	EF = 2300 N (C)	

SUPPORT FORCES: A_y = 1400 N ↑, A_x = 4683 N ←, D_x = 4488 N →

DESIGNS COULD BE SIMILAR TO PROBLEM 3-89.
FORCES ARE GENERALLY SMALLER. SMALL WIRES MAY BE USED FOR TENSION MEMBERS. COMPRESSION MEMBERS MUST BE DESIGNED FOR BUCKLING. SEE CHAPTER 14.

3-91 FORCE ANALYSIS:

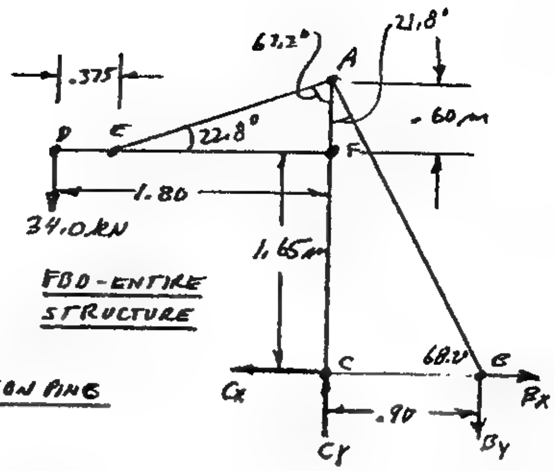
USING FBD OF ENTIRE STRUCTURE:

$\Sigma M_c = 0 = (34.0\,kN)(1.8\,m) - B_y(.90)$

$B_y = (34)(1.8)/.9 = 68.0\,kN \downarrow$

$\Sigma F_y = 0 = 34.0 + 68.0 - C_y$

$C_y = 102\,kN \uparrow$



FBD - ENTIRE STRUCTURE

AB IS A TWO-FORCE MEMBER

$AB_y = B_y = 68.0\,kN = AB\cos 21.8°$

$AB = 68/\cos 21.8° = \underline{73.2\,kN \text{ TENSION}}$

$AB_x = AB \sin 21.8° = 27.2\,kN$ ⌊ = FORCE ON PIN B

$B_x = AB_x = 27.2\,kN$

$\Sigma F_x = 0 = B_x - C_x$

$C_x = B_x = 27.2\,kN$

FORCE ON PIN C $= \sqrt{C_x^2 + C_y^2} = \sqrt{27.2^2 + 102^2} = 105.6\,kN$

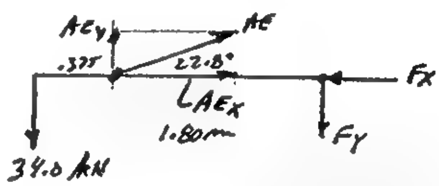
FBD OF BOOM:

$\Sigma M_F = 0 = (34.0\,kN)(1.80\,m) - AE_y(1.425)$

$AE_y = (34)(1.8)/1.425 = 42.95\,kN$

$\underline{AE} = AE_y/\sin 22.8° = \underline{110.8\,kN \text{ TENSION}}$

$AE_x = AE\cos 22.8° = 102.2\,kN$



FBD OF BOOM

$\Sigma F_x = 0 = AE_x - F_x$

$F_x = AE_x = 102.2\,kN$

$\Sigma M_E = 0 = (34.0\,kN)(.375) - F_y(1.425)$

$F_y = (34)(.375)/1.425 = 8.95\,kN$

FORCE ON PIN F $= \sqrt{F_x^2 + F_y^2} = \sqrt{102.2^2 + 8.95^2} = 102.6\,kN$

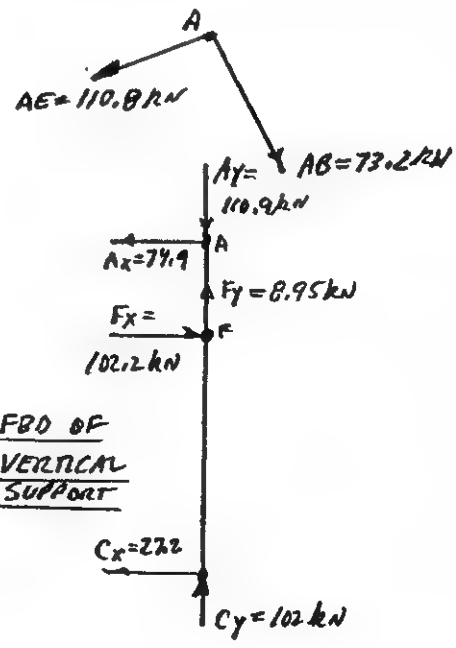
PIN A:

$A_y = AE_y + AB_y = (110.8)\cos 67.2° + (73.2)\cos 21.8°$

$A_y = 42.95 + 68.0 = 110.9\,kN \downarrow$

$A_x = AB_x - AE_x = 73.2 \sin 21.8° - (110.8)\sin 67.2$

$A_x = 27.18 - 102.1 = -74.9\,kN \leftarrow$



FBD OF VERTICAL SUPPORT

SUMMARY OF RESULTS:

a) FORCES IN WIRES: $AE = 110.8\,kN$; $AB = 73.2\,kN$

c) SHEARING FORCE IN EACH PIN:

PIN A: $F_A = \sqrt{74.9^2 + 110.9^2} = 134\,kN$

PIN F: $F_F = \sqrt{102.2^2 + 8.95^2} = 102.6\,kN$

PIN C: $F_C = \sqrt{27.2^2 + 102^2} = 105.6\,kN$

PIN E: $F_E = AE = 110.8\,kN$

PIN D: $F_D = 34.0\,kN$

[NEXT PAGE FOR PARTS b, d, e.]

3-91 (CONTINUED)

b) DESIGN OF RODS: $AE = 110.8\ kN$, $AB = 73.2\ kN$, MODERATE SHOCK.

$\sigma_d = S_u/12$. SPECIFY AISI 4140 OQT 900. $S_u = 1289\ MPa$, 15% ELONG.

HIGH STRENGTH, GOOD DUCTILITY.

$\sigma_d = 1289\ MPa/12 = 107.4\ MPa = 107.4\ N/mm^2$

LET $\sigma_d = \sigma_{MAX} = F/A$. THEN REQ'D $A = \frac{F}{\sigma_d}$

FOR AE: $A = \frac{F}{\sigma_d} = \frac{110.8 \times 10^3\ N}{107.4\ N/mm^2} = 1031\ mm^2 = \pi D^2/4$

REQ'D $D = \sqrt{4A/\pi} = \sqrt{4(1031\ mm^2)/\pi} = 36.2\ mm$

SPECIFY $D = 40\ mm$ - PREFERRED BASIC SIZE. APP. 2

FOR AB: $A = \frac{F}{\sigma_d} = \frac{73.2 \times 10^3\ N}{107.4\ N/mm^2} = 681.6\ mm^2 = \pi D^2/4$

REQ'D $D = \sqrt{4A/\pi} = \sqrt{4(681.6\ mm^2)/\pi} = 29.5\ mm$

SPECIFY $D = 30.0\ mm$ - PREFERRED BASIC SIZE

d) DESIGN OF PINS: ALL PINS TO BE IN DOUBLE SHEAR USING A CLEVIS-TYPE CONNECTION. [SEE FIG. 3-7]

FROM TABLE 3-4: $\tau_d = S_{ys}/6 = S_y/12$

SPECIFY AISI 4140 OQT 900. $S_y = 1193\ MPa$, 15% ELONG.

$\tau_d = S_y/12 = 1193\ MPa/12 = 99.4\ MPa = 99.3\ N/mm^2$

LET $\tau_d = \tau_{MAX} = F/A_s$. THEN REQ'D $A_s = F/\tau_d$

PIN A: $F_A = 134\ kN$. REQ'D. $A_s = \frac{F}{\tau_d} = \frac{134 \times 10^3\ N}{99.3\ N/mm^2} = 1348\ mm^2$

$A_s = 2A = 2\ \pi D^2/4 = \pi D^2/2$. REQ'D $D_A = \sqrt{2A_s/\pi}$

$D_{MIN} = \sqrt{\frac{2(1348\ mm^2)}{\pi}} = 29.3\ mm$. SPECIFY $D = 30.0\ mm$

PIN F: $F = 102.6\ kN$. $A_s = \frac{F}{\tau_d} = \frac{102.6 \times 10^3}{99.3} = 1032\ mm^2$

$D_{MIN} = \sqrt{2A_s/\pi} = \sqrt{\frac{2(1032\ mm^2)}{\pi}} = 25.6\ mm$; $D_F = 28\ mm$

PIN C: $F = 105.6\ kN$. $A_s = \frac{105.6 \times 10^3}{99.3} = 1062\ mm^2$

$D_{MIN} = \sqrt{\frac{2(1062)}{\pi}} = 26.0\ mm$. SPECIFY $D_C = 28.0\ mm$

PIN E: $F = 110.8\ kN$. $A_s = \frac{110.8 \times 10^3}{99.3} = 1115\ mm^2$

$D_{MIN} = \sqrt{\frac{2(1115)}{\pi}} = 26.6\ mm$. SPECIFY $D_E = 28.0\ mm$

PIN D: $F = 34.0\ kN$. $A_s = \frac{34 \times 10^3}{99.3} = 342.4\ mm^2$

$D_{MIN} = \sqrt{\frac{2(342.4)}{\pi}} = 14.8\ mm$. SPECIFY $D = 16\ mm$

[NEXT PAGE FOR BEARING STRESS]

3-91 (CONTINUED)

BEARING STRESS ON PINS: [SEE FIG 3-7 FOR DESIGN OF JOINT]

$$\sigma_b = \frac{F}{A_b} = \frac{F}{D \cdot t_1} \quad \text{AND } t_2 \geq t_1/2$$

$$\text{REQ'D. } t_1 = \frac{F}{D\,\sigma_{bd}}$$

$\sigma_{bd} = 0.90\,s_y$ EQ. 3-8 FOR STEEL.

$s_y = 1193$ MPa - AISI 4140 OQT 900 FOR PINS

MATERIAL FOR MATING PARTS MUST BE AT LEAST AS STRONG.

$\sigma_{bd} = 0.90(1193\text{ MPa}) = 1074\text{ MPa} = 1074\text{ N/mm}^2$

PIN A: $D = 30.0$ mm. $F = 134$ kN

$$t_{1\,min} = \frac{134 \times 10^3\text{ N}}{(30\text{ mm})(1074\text{ N/mm}^2)} = \underline{4.16\text{ mm}}$$

THE REQ'D THICKNESS IS QUITE SMALL. IT IS HIGHLY LIKELY THAT ACTUAL DIMENSIONS FOR t_1 AND t_2 ARE MUCH LARGER FOR OTHER STRESS CONDITIONS.

PINS F, C, AND E: ALL HAVE $D = 28$ mm. $F_E = 110.8$ kN

$$t_{1\,MIN} = \frac{110 \times 10^3\text{ N}}{(28\text{ mm})(1074\text{ N/mm}^2)} = \underline{3.66\text{ mm}} \quad \text{PIN E}$$

THIS IS ALSO VERY SMALL. PINS F AND C HAVE SLIGHTLY LOWER FORCES, SO REQ'D t IS SIMILAR.

NOTE: IF BOOM OR COLUMN ARE MADE FROM A MATERIAL WITH LOWER STRENGTH (SUCH AS STRUCTURAL STEEL), BEARING STRESS CALCULATIONS MUST BE REDONE.

PIN D: $F_D = 34.0$ kN. $D = 16$ mm

$$t_{1\,MIN} = \frac{34.0 \times 10^3\text{ N}}{(16\text{ mm})(1074\text{ N/mm}^2)} = \underline{1.98\text{ mm SMALL}}$$

4-1 $\sigma = 800$ PSI; $E = 1.40 \times 10^6$ PSI; $L = 6.0$ FT $\times 12$ IN/FT $= 72$ IN.

$$\delta = \frac{\sigma L}{E} = \frac{800 \text{ PSI}(72 \text{ IN})}{1.40 \times 10^6 \text{ PSI}} = \underline{0.041 \text{ IN}}$$

4-2 $A = (0.75)(12) = 9\ mm^2$

a) ABS; $E = 2.5$ GPa $= 2500$ MPa $= 2500\ N/mm^2$

$$\delta = \frac{FL}{EA} = \frac{(90\ N)(375\ mm)}{(2500\ N/mm^2)(9\ mm^2)} = \underline{1.50\ mm}$$

b) PHENOLIC; $E = 17.2$ GPa $= 17200$ MPa $= 17200\ N/mm^2$

$$\delta = \frac{FL}{EA} = \frac{(90)(375)}{(17200)(9)} = \underline{0.218\ mm}$$

4-3 $D_o = 2.50$ IN; $D_i = 2.50 - 2(0.085) = 2.33$ IN; $E = 10.6 \times 10^6$ PSI

$A = \pi(D_o^2 - D_i^2)/4 = 0.645\ IN^2$

$$\delta = \frac{FL}{EA};\quad F = \frac{\delta EA}{L} = \frac{(0.005 \text{ IN})(10.6 \times 10^6 \text{ PSI})(0.645 \text{ IN}^2)}{14.5 \text{ IN}} = \underline{2357 \text{ LB}}$$

$$\sigma = \frac{F}{A} = \frac{2357 \text{ LB}}{0.645 \text{ IN}^2} = \underline{3655 \text{ PSI}} \quad \text{OK; } s_y = 42000 \text{ PSI}$$

4-4 1) WEIGH IT. DENSITY OF ALUMINUM – 0.10 LB/IN³ (WT. DENSITY)
DENSITY OF MAGNESIUM – 0.066 LB/IN³

2) EXERT A KNOWN LOAD AND MEASURE DEFORMATION.
E FOR ALUMINUM – 10×10^6 PSI
E FOR MAGNESIUM – 6.5×10^6 PSI

4-5 a) AISI 1020 HR STEEL: STRENGTH: $\sigma_d = \frac{s_u}{8} = \frac{448}{8} = 56.0$ MPa

REQ'D. $A = \dfrac{F}{\sigma_d} = \dfrac{3500\ N}{56.0\ N/mm^2} = 62.5\ mm^2 = \pi D^2/4$

REQ'D. $D = \sqrt{4A/\pi} = \sqrt{4(62.5)/\pi} = 8.92$ mm

DEFORMATION: $E = 207$ GPa $= 207000$ MPa $= 207000\ N/mm^2$

REQ'D. $A = \dfrac{FL}{E\delta} = \dfrac{(3500\ N)(630\ mm)}{(207000\ N/mm^2)(0.12\ mm)} = 88.8\ mm^2 = \dfrac{\pi D^2}{4}$

REQ'D $D = \sqrt{4A/\pi} = \underline{10.63\ mm}$ GOVERNS

b) AISI 4140 OQT 700 STEEL IS MUCH STRONGER THAN AISI 1020 HR.
THEREFORE: DEFORMATION GOVERNS. E IS SAME.
REQ'D. $D = 10.63$ mm

c) ALUM. 6061-T6: STRENGTH: $\sigma_d = \frac{s_u}{8} = \frac{310\text{ MPa}}{8} = 38.75$ MPa

REQ'D. $A = F/\sigma_d = 3500\ N / 38.75\ N/mm^2 = 90.32\ mm^2$

REQ'D. $D = \sqrt{4A/\pi} = 10.7$ mm

DEFORMATION: $E = 69$ GPa $= 69000$ MPa $= 69000\ N/mm^2$

REQ'D. $A = \dfrac{FL}{E\delta} = \dfrac{(3500)(630)}{(69000)(0.12)} = 266.3\ mm^2$ GOVERNS

REQ'D $D = \sqrt{4A/\pi} = \underline{18.4\ mm}$

(CONT. NEXT PAGE)

4-5 (CONTINUED) MASS = VOL × DENSITY = A × L × DENSITY

STEEL: $MASS = (88.8\,mm^2)(630\,mm)(7680\,kg/m^3) \times \frac{1\,m^3}{(10^3\,mm)^3} = \underline{0.430\,kg}$

ALUM: $MASS = (266.3)(630)(2770)/10^9 = \underline{0.465\,kg}$

4-6 $A = \pi D^2/4 = \pi(12)^2/4 = 113.1\,mm^2$

$$\delta = \frac{FL}{EA} = \frac{(17.0\times10^3\,N)(220\,mm)}{(207000\,N/mm^2)(113.1\,mm^2)} = \underline{0.160\,mm}$$

4-7 a) Ti-6Al-4V: $E = 114\,GPa = 114000\,MPa = 114000\,N/mm^2$

$$\delta = \frac{FL}{EA} = \frac{(5000\,N)(1250\,mm)}{(114000\,N/mm^2)(64\,mm^2)} = \underline{0.857\,mm}$$

b) AISI 501 OQT 1000 STEEL: $E = 200\,GPa = 200000\,N/mm^2$

$$\delta = \frac{FL}{EA} = \frac{(5000)(1250)}{(200000)(64)} = \underline{0.488\,mm}$$

4-8 REQ'D. $A = \frac{F}{\sigma_d} = \frac{35000\,LB}{21600\,LB/IN^2} = 1.620\,IN^2$; Use $\underline{L4\times4\times1/4}$; $A = 1.94\,IN^2$

$$\delta = \frac{FL}{AE} = \frac{(35000\,LB)(13.0\,FT)(12\,IN/FT)}{(1.94\,IN^2)(29\times10^6\,LB/IN^2)} = \underline{0.097\,IN}$$

4-9 ELONGATION: $\delta = \frac{FL}{EA} = \frac{(450\,LB)(8.40\,IN)}{(30\times10^6\,LB/IN^2)(.25)(.125)\,IN^2} = \underline{0.0040\,IN}$

COMPRESSION: $\delta = \frac{FL}{EA} = \frac{(50)(8.40)}{(30\times10^6)(.25)(.125)} = \underline{0.00045\,IN}$

4-10 LOWER SECTION: $\delta_1 = \frac{F_1L_1}{EA} = \frac{(5000)(10)}{(30\times10^6)(.50)} = 0.0033\,IN$

MID SECTION: $\delta_2 = \frac{F_2L_2}{EA} = \frac{(7000)(15)}{(30\times10^6)(.50)} = 0.0070\,IN$

TOP SECTION: $\delta_3 = \frac{F_3L_3}{EA} = \frac{(10500)(25)}{(30\times10^6)(0.50)} = 0.0175\,IN$

TOTAL: $\delta_T = \delta_1 + \delta_2 + \delta_3 = \underline{0.0278\,IN}$

4-11 $A = \pi(D_o^2 - D_i^2)/4 = \pi(1.25^2 - 1.126^2)/4 = 0.2314\,IN^2$

$$F = \frac{\delta EA}{L} = \frac{(0.050\,IN)(10\times10^6\,LB/IN^2)(0.2314\,IN^2)}{36.0\,IN} = 3214\,LB$$

$\sigma = F/A = 3214\,LB/0.2314\,IN^2 = \underline{13\,900\,PSI}$ TOO HIGH

$\sigma_d = \frac{S_u}{8} = \frac{45000\,PSI}{8} = 5625\,PSI$

4-12 $A = \pi(D_o^2 - D_i^2)/4 = \pi(0.75^2 - 0.563^2)/4 = 0.1928\,IN^2$

$\sigma = F/A = 2500\,LB/0.1928\,IN^2 = 12964\,PSI = \sigma_d = S_y/2;\ S_y = 25\,930\,PSI$

ALUMINUM 3003-H18 HAS $S_y = 27000\,PSI$

$$\delta = \frac{FL}{EA} = \frac{(2500\,LB)(8.75\,FT)(12\,IN/FT)}{(10\times10^6\,LB/IN^2)(0.1928\,IN^2)} = \underline{0.136\,IN}$$

4-13 $A = \frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}(56^2 - 48^2) = 653\ mm^2$

$$\delta = \frac{FL}{EA} = \frac{(18.2\times10^3 N)(40\ mm)}{(69\,000\ N/mm^2)(653\ mm^2)} = \underline{0.016\ mm}$$

$$\sigma = \frac{F}{A} = \frac{18.2\times10^3 N}{653\ mm^2} = \underline{27.9\ MPa}$$

4-14 $A = \pi D^2/4 = \pi(0.375)^2/4 = 0.1104\ IN^2$

$$\sigma = \frac{F}{A} = \frac{1600\ LB}{0.1104\ IN^2} = \underline{14\,500\ PSI}$$

$$\delta = \frac{FL}{EA} = \frac{(1600\ LB)(135\ FT)(12\ IN/FT)}{(30\times10^6\ LB/IN^2)(0.1104\ IN^2)} = \underline{0.782\ IN}$$

4-15 $A = (30\ mm)^2 = 900\ mm^2$; $E = 114\ GPa = 114\,000\ N/mm^2$

$F_{AB} = 110 + 80 - 40 = 150\ kN$ TENSION

$F_{BC} = 110 - 40 = 70\ kN$ TENSION

$F_{CD} = 110\ kN$ TENSION

$$\delta_T = \delta_{AB} + \delta_{BC} + \delta_{CD} = \frac{F_{AB}L}{EA} + \frac{F_{BC}L}{EA} + \frac{F_{CD}L}{EA}$$

$$\delta_T = \frac{(150\times10^3 N)(250\ mm)}{(114\,000\ N/mm^2)(900\ mm^2)} + \frac{(70\times10^3)(250)}{(114\,000)(900)} + \frac{(110\times10^3)(250)}{(114\,000)(900)}$$

$$\delta_T = 0.365\ mm + 0.171\ mm + 0.268\ mm = \underline{0.804\ mm}$$

4-16 $E = \frac{FL}{\delta A} = \frac{(10\,000\ LB)(10.0\ IN)}{(0.023\ IN)(\pi(0.75)^2/4)\ IN^2} = \underline{9.84\times10^6\ PSI}$

PROBABLY ALUMINUM: $E \cong 10.0\times10^6\ PSI$

4-17 $A_{AB} = A_{BC} = \frac{\pi(25)^2}{4} = 491\ mm^2$; $A_{CD} = \frac{\pi(16)^2}{4} = 201\ mm^2$

$F_{AB} = -9.65 - 12.32 + 4.45 = -17.52\ kN$ (COMP.); $L_{AB} = 80\ mm$

$F_{BC} = -9.65 - 12.32 = -21.97\ kN$ (COMP.); $L_{BC} = 100\ mm$

$F_{CD} = -9.65\ kN$ (COMP.); $L_{CD} = 120\ mm$

$E = 5.9\ GPa = 5900\ N/mm^2$

$$\delta_{AD} = \frac{F_{AB}L_{AB}}{EA_{AB}} + \frac{F_{BC}L_{BC}}{EA_{BC}} + \frac{F_{CD}L_{CD}}{EA_{CD}}$$

$$\delta_{AD} = \frac{(-17.52\times10^3 N)(80\ mm)}{(5900\ N/mm^2)(491\ mm^2)} + \frac{(-21.97\times10^3)(100)}{(5900)(491)} + \frac{(-9.65\times10^3)(120)}{(5900)(201)}$$

$$\delta_{AD} = -0.4838\ mm - 0.7584\ mm - 0.9765\ mm = \underline{-2.22\ mm}$$

SHORTER

4-18 $\sigma_c = F/A_c = 64000\,\text{LB}/50.27\,\text{IN}^2 = 1273\,\text{PSI}$; LET $\sigma_d > 4\sigma_c = 5092\,\text{PSI}$;

USE $\sigma_d = 6000$ PSI RATED: THEN $E = 4.7\times10^6$ PSI (SEC 2-10)

$A_c = \pi(8)^2/4 = 50.27\,\text{IN}^2$; $A_T = 6.36\,\text{IN}^2$ (APP. A-9)

$$\delta = \delta_c + \delta_T = \frac{(64000\,\text{LB})(3.0\,\text{FT})(12\,\text{IN/FT})}{(4.7\times10^6\,\text{LB/IN}^2)(50.27\,\text{IN}^2)} + \frac{(64000)(8.6)(12)}{(30\times10^6)(6.36)}$$

$\delta = 0.0098\,\text{IN} + 0.0346\,\text{IN} = \underline{0.0444\,\text{IN}}$

4-19 a) $\delta = \dfrac{FL}{EA} = \dfrac{(120\,\text{LB})(10.5\,\text{FT})(12\,\text{IN/FT})}{(17\times10^6\,\text{LB/IN}^2)(0.00322\,\text{IN}^2)} = \underline{0.276\,\text{IN.}}$

$A = \pi D^2/4 = \pi(0.064)^2/4 = 0.00322\,\text{IN}^2$

$\sigma = F/A = 120\,\text{LB}/0.00322\,\text{IN}^2 = 37300\,\text{PSI}$; CLOSE TO $S_y = 44000\,\text{PSI}$

b) $\sigma = \dfrac{F}{A} = 200\,\text{LB}/0.00322\,\text{IN}^2 = 62\,200\,\text{PSI}$: THIS EXCEEDS THE ULTIMATE STRENGTH OF THE WIRE. IT SHOULD BREAK.

4-20 $\sigma = F/A = 25\,\text{LB}/(0.006)(0.75)\,\text{IN}^2 = \underline{5556\,\text{PSI}}$

$\delta = \dfrac{FL}{EA} = \dfrac{(25\,\text{LB})(25\,\text{FT})(12\,\text{IN/FT})}{(30\times10^6\,\text{LB/IN}^2)(0.0045\,\text{IN}^2)} = \underline{0.0556\,\text{IN}}$

4-21 $F = \sigma_{allow}\cdot A = (550\,\text{LB/IN}^2)(12.25\,\text{IN}^2) = \underline{6737\,\text{LB}}$

$\delta = \dfrac{FL}{EA} = \dfrac{(6737\,\text{LB})(10.75\,\text{FT})(12\,\text{IN/FT})}{(1300000\,\text{LB/IN}^2)(12.25\,\text{IN}^2)} = \underline{0.055\,\text{IN}}$

4-22 $A = (200^2 - 150^2) = 17500\,\text{mm}^2$

$F = \sigma\cdot A = (-200\,\text{N/mm}^2)(17500\,\text{mm}^2) = -3.5\times10^6\,\text{N} = \underline{-3.50\,\text{MN}}$

$\delta = \dfrac{\sigma L}{E} = \dfrac{(-200\,\text{N/mm}^2)(1800\,\text{mm})}{(165\,000\,\text{N/mm}^2)} = \underline{-2.18\,\text{mm}}$

4-23 $A = \pi D^2/4 = \pi(3.0)^2/4 = 7.07\,\text{mm}^2$: (ASSUME MASS OF PLATE IS SMALL)

$F = \dfrac{\delta EA}{L} = \dfrac{(6.0\,\text{mm})(110\,000\,\text{N/mm}^2)(7.07\,\text{mm}^2)}{3600\,\text{mm}} = \underline{1296\,\text{N}}$

$\sigma = F/A = 1296\,\text{N}/7.07\,\text{mm}^2 = \underline{183\,\text{MPa}}$ (LESS THAN S_y)

$m = W/g = 1296\,\text{N}/9.81\,\text{m/s}^2 = 132\,\text{N}\cdot\text{s}^2/\text{m} = \underline{132\,\text{kg}}$

4-24 $\sigma = 50\,\text{MPa}$ (PROB 1-53): $\delta = \sigma L/E = \dfrac{(50\,\text{N/mm}^2)(1250\,\text{mm})}{69\,000\,\text{N/mm}^2} = \underline{0.906\,\text{mm}}$

4-25 $\delta = \alpha L(\Delta t) = (6.0\times10^{-6}\,{}^\circ\text{F}^{-1})(80\,\text{FT})(140^\circ\text{F})\times(12\,\text{IN})/\text{FT} = \underline{0.806\,\text{IN}}$

4-26 $\delta = \alpha L(\Delta t) = (11.3\times10^{-6}\,{}^\circ\text{C}^{-1})(12\,000\,\text{mm})(77^\circ\text{C}) = \underline{10.4\,\text{mm}}$

4-27 $\sigma = E\alpha(\Delta t) = (207\times10^9\,\text{PA})(11.3\times10^{-6}\,{}^\circ\text{C}^{-1})(77^\circ\text{C}) = \underline{180\,\text{MPa}}$ HIGH!

4-28 $\delta = \alpha L(\Delta t) = (11.3\times10^{-6}\,{}^\circ\text{C}^{-1})(625\,\text{mm})(156^\circ\text{C}) = \underline{1.10\,\text{mm}}$

4-29 a) $\delta = \alpha L(\Delta t) = (11.3\times10^{-6}\,{}^\circ\text{C}^{-1})(625\,\text{mm})(65^\circ\text{C}) = \underline{0.459\,\text{mm}}$

b) $\sigma = E\alpha(\Delta t) = (207\times10^9\,\text{PA})(11.3\times10^{-6}\,{}^\circ\text{C}^{-1})(91^\circ\text{C}) = \underline{213\,\text{MPa}}$

4-30 $\delta = \alpha L(\Delta t) = (6.0\times10^{-6}\,{}^\circ\text{F}^{-1})(140\,\text{FT})(80^\circ\text{F})(12\,\text{IN/FT}) = \underline{0.806\,\text{IN.}}$

THIS IS THE <u>TOTAL</u> WIDTH. EACH END COULD BE 0.41 IN.

4-31 DECK COULD EXPAND BY A TOTAL OF 0.50 IN WITHOUT STRESS.

REQ'D. $\Delta t = \dfrac{\delta}{\alpha L} = \dfrac{0.50\,IN}{(6.0\times10^{-6}\,°F^{-1})(140\,FT)(12\,IN/FT)} = 49.6\,°F$

$t_2 = t_1 + \Delta t = 30 + 49.6 = 79.6\,°F$

REMAINING $\Delta t = 110\,°F - 79.6\,°F = 30.4\,°F$

$\sigma = E\alpha(\Delta t) = (3.8\times10^6\,PSI)(6.0\times10^{-6}\,°F^{-1})(30.4\,°F) = \underline{693\,PSI}$

4-32 $\Delta t = 110\,°F - 60\,°F = 50\,°F$

$\delta = \alpha L(\Delta t) = (6.0\times10^{-6}\,°F^{-1})(140\times12\,IN)(50\,°F) = \underline{0.504\,IN}$

4-33 $\delta = \pi(55.300) - \pi(55.100) = 0.2\pi$ mm (CHANGE IN CIRCUMFERENCE)

$\Delta t = \dfrac{\delta}{\alpha L} = \dfrac{0.2\pi\,mm}{(16.9\times10^{-6}\,°C^{-1})(\pi)(55.100\,mm)} = 214.8\,°C$

$t_2 = t_1 + \Delta t = 20 + 214.8 = \underline{234.8\,°C}$

4-34 FOR FIRST PART OF COOLING, RING IS UNRESTRAINED UNTIL ITS DIAMETER GETS TO 55.200 mm. $\delta = 55.300 - 55.200 = 0.100$ mm

$\Delta t = \dfrac{\delta}{\alpha L} = \dfrac{0.100\,mm}{(16.9\times10^{-6}\,°C^{-1})(55.200\,mm)} = 107.2\,°C$

$t_2 = t_1 - \Delta t = 234.8\,°C - 107.2\,°C = 127.6\,°C$

ADDITIONAL $\Delta t = 127.6 - 20 = 107.6\,°C$

$\sigma = E\alpha(\Delta t) = (193\times10^9\,Pa)(16.9\times10^{-6}\,°C^{-1})(107.6\,°C) = \underline{351\,MPa}$

4-35 $\delta_{BRASS} = \alpha L(\Delta t) = (20.5\times10^{-6}\,°C^{-1})(4200\,mm)(75\,°C) = \underline{6.46\,mm}$

$\delta_{CS} = \alpha L(\Delta t) = (10.4\times10^{-6}\,°C^{-1})(4500\,mm)(75\,°C) = \underline{3.51\,mm}$

4-36 $\delta = \alpha L(\Delta t) = (6.5\times10^{-6}\,°F^{-1})(40\times12\,IN)(190\,°F) = \underline{0.593\,IN}$

4-37 INITIAL EXPANSION OF 0.10 mm IS UNRESTRAINED.

REQ'D $\Delta t = \dfrac{\delta}{\alpha L} = \dfrac{0.10\,mm}{(25.2\times10^{-6}\,°C^{-1})(250\,mm)} = 15.9\,°C$

$t_2 = t_1 + \Delta t = 20\,°C + 15.9\,°C = 35.9\,°C$

ADD. $\Delta t = 70\,°C - 35.9\,°C = 34.1\,°C$ – RESTRAINED

$\sigma = E\alpha\Delta t = (45\times10^9\,Pa)(25.2\times10^{-6}\,°C^{-1})(34.1\,°C) = \underline{38.7\,MPa}$

4-38 $\sigma = E\alpha\Delta t = (207\times10^9\,Pa)(11.3\times10^{-6}\,°C^{-1})(90\,°C) = \underline{211\,MPa}$

4-39 $\delta_1 = 10.505 - 10.500 = 0.005$ IN UNRESTRAINED

$\Delta t_1 = \dfrac{\delta}{\alpha L} = \dfrac{0.005\,IN}{(13.0\times10^{-6}\,°F^{-1})(10.500\,IN)} = 36.6\,°F$

$t_2 = t_1 + \Delta t_1 = 75 + 36.6 = 111.6\,°F$

ADD. $\Delta t = 400 - 111.6 = 288.4\,°F$ – RESTRAINED

$\sigma = E\alpha(\Delta t) = (10\times10^6\,PSI)(13.0\times10^{-6}\,°F^{-1})(288.4\,°F) = \underline{37\,500\,PSI}$

FOR 6061-T4, $S_u = 35\,000$ PSI, <u>BAR SHOULD FAIL.</u>

ALSO, BECAUSE BAR IS IN COMPRESSION <u>IT MAY BUCKLE.</u>

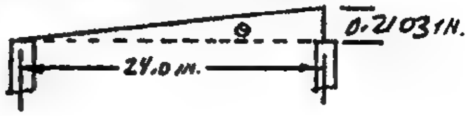
4-40 $\delta_P = \alpha L(\Delta t) = (53.0\times10^{-6}\,°F^{-1})(30.0\,in)(212-65)°F = 0.2337\,in.$

$\delta_{TI} = \alpha L(\Delta t) = (5.3\times10^{-6}\,°F^{-1})(30.0\,in)(147\,°F) = 0.0234\,in$

$\delta_P - \delta_{TI} = 0.2103\,in$

$\tan\theta = \frac{0.2103}{24} = 0.00876$

$\underline{\theta = 0.502\,deg.}$



4-41 $\delta = \alpha L(\Delta t) = (6.3\times10^{-6}\,°F^{-1})(25\,FT)(12\,IN/FT)(68-(-15))°F = \underline{0.157\,IN}$

4-42 TOTAL $\delta_T = 0.50\,mm = 0.00050\,m. \quad = \delta_S + \delta_B$

$\delta_T = \alpha_S L_S(\Delta t) + \alpha_B L_B(\Delta t) = \Delta t(\alpha_S L_S + \alpha_B L_B)$

$\Delta t = \frac{\delta_T}{\alpha_S L_S + \alpha_B L_B} = \frac{0.0005\,m}{(10.4\times10^{-6}\,°C^{-1})(2.800\,m) + (20.5\times10^{-6}\,°C^{-1})(1.900\,m)}$

$\Delta t = 7.35°C$; $t_2 = t_1 + \Delta t = 20 + 7.35 = \underline{27.35°C}$

4-43 ADDED $\sigma = E\alpha\,\Delta t = (193\times10^9\,Pa)(16.9\times10^{-6}\,°C^{-1})(35°C) = 114\,MPa$

FINAL $\sigma = 40 + 114 = \underline{154\,MPa}$

4-44 WHEN HEATED, WIRE WOULD RELAX.

$\Delta t = \frac{\sigma}{E\alpha} = \frac{40\times10^6\,Pa}{(193\times10^9\,Pa)(16.9\times10^{-6}\,°C^{-1})} = 12.3\,°C$

$t_2 = t_1 + \Delta t = 20°C + 12.3°C = \underline{32.3\,°C}$

4-45 $A_S = 150^2 - 130^2 = 5600\,mm^2$; $A_C = 130^2 = 16900\,mm^2$

$\sigma_C = \frac{F E_C}{A_S E_S + A_C E_C} = \frac{(900\times10^3\,N)(32.4\,GPa)}{(5600\,mm^2)(207\,GPa) + (16900\,mm^2)(32.4\,GPa)} = \underline{17.1\,MPa}$

$\sigma_S = \sigma_C E_S/E_C = (17.1\,MPa)(207\,GPa)/32.4\,GPa = \underline{109\,MPa}$

4-46 $A_S = 10.4\,in^2$ (APP A-9); $A_C = (5.0\,in)^2 = 25.0\,in^2$

$E_S = 30\times10^6\,psi$; $E_C = 4.7\times10^6\,psi$ (SEC 2-10)

LET $\sigma_C = \sigma_{dC} = 1500\,psi$; THEN $\sigma_S < \sigma_{dS} = 21600\,psi$

$F = \frac{\sigma_{dC}[A_S E_S + A_C E_C]}{E_C} = \frac{1500\,psi\,[(10.4)(30\times10^6) + 25(4.7\times10^6)]}{4.7\times10^6}$

$\underline{F = 137{,}000\,LB}$; CHECK $\sigma_S = \sigma_C \frac{E_S}{E_C} = 1500(30/4.7) = 9575\,psi$ OK

4-47 LET $\sigma_C = 1500\,psi$; $\sigma_S = \sigma_C \frac{E_S}{E_C} = 1500\ 30/4.7 = 9575\,psi$ - OK

$\frac{F}{\sigma_C} = \frac{A_S E_S + A_C E_C}{E_C} = \frac{A_S E_S}{E_C} + A_C = A_S\frac{30}{4.7} + A_C = 6.38A_S + A_C$

$A_S = b^2 - (b-2t)^2 = b^2 - [b-2(0.5)]^2 = b^2 - (b-1)^2 = b^2 - b^2 + 2b - 1 = 2b - 1$

$A_C = (b-1)^2 = b^2 - 2b + 1$

$\frac{F}{\sigma_C} = 6.38(2b-1) + b^2 - 2b + 1 = b^2 + 10.77b - 5.38$

$\frac{F}{\sigma_C} = \frac{500\,000}{1500} = 333.3 = b^2 + 10.77b - 5.38$

$b^2 + 10.77b - 338.7 = 0$

BY QUADRATIC EQN., $\underline{b = 13.8\,IN}$

4-48 $A_S = A_A = 2\pi(6)^2/4 = 56.55\ mm^2$

$$\sigma_A = \frac{PE_A}{A_S E_S + A_A E_A} = \frac{(11.3\times10^3 N)(69 GPa)}{(56.55 mm^2)(207 GPa) + (56.55 mm^2)(69 GPa)} = 49.5 MPa$$

$$\sigma_S = \sigma_A \cdot E_S/E_A = 49.5 MPa \cdot 207/69 = \underline{148.5 MPa}$$

4-49 $W = mg = 2265 kg \cdot 9.81 m/s^2 = 22.22\times10^3 N$; $A_S = 2A_C$

IF $\sigma_S = 552 MPa/2 = 276 MPa$; THEN $\sigma_C = \sigma_S \cdot \frac{E_C}{E_S} = 276 \cdot \frac{131}{200} = 181 MPa$

$\sigma_{C_{ALLOW}} = S_y/2 = 1379/2 = 689 MPa$ $\underline{OK}$

$$\sigma_C = \frac{PE_C}{A_S E_S + A_C E_C} = \frac{PE_C}{2A_C E_S + A_C E_C} = \frac{PE_C}{A_C[2E_S + E_C]}$$

$$A_C = \frac{PE_C}{\sigma_C[2E_S + E_C]} = \frac{(22.22\times10^3 N)(131 GPa)}{(181 N/mm^2)[2(200)+131] GPa} = 30.3\ mm^2$$

$$D = \sqrt{4A/\pi} = \sqrt{4(30.3)/\pi} = \underline{6.20\ mm = \text{WIRE DIA.}}$$

4-50 1) COMPUTE FORCE REQ'D. TO DEFLECT ALUM. ROD 0.12 mm

$$P_1 = \frac{\delta EA}{L} = \frac{(0.12 mm)(69000 N/mm^2)(15394 mm^2)}{1650.12\ mm} = 77.24 kN$$

2) ADDITIONAL LOAD AVAILABLE: $P_2 = 350 kN - 77.24 kN = 272.76 kN$

3) BOTH MEMBERS DEFLECT THE SAME AMOUNT UNDER P_2

$$\delta_A = \delta_S : \frac{P_A \cancel{L_A}}{E_A A_A} = \frac{P_S \cancel{L_S}}{E_S A_S} \quad [L_A = L_S]$$

$$P_A = P_S \frac{E_A A_A}{E_S A_S} = P_S \frac{(69 GPa)(15394 mm^2)}{(207 GPa)(3600 mm^2)} = 1.425 P_S$$

6 IN SCH 40 PIPE: $A_S = 5.581\ in^2 (25.4)^2 mm^2/in^2) = 3600\ mm^2$

4) $P_S + P_A = P_2 = 272.76 kN$

$P_S + 1.425 P_S = 2.425 P_S = 272.76 kN$

$\underline{P_S = 191.39 kN}$

$P_A = 272.76 - 191.39 = 81.37 kN$

5) TOTAL LOAD ON ALUM: $P_{AT} = P_1 + P_A = 77.24 + 81.37 = \underline{158.61 kN}$

6) STRESSES:

$$\sigma_S = \frac{P_S}{A_S} = \frac{191.39\times10^3 N}{3600 mm^2} = 53.2 MPa$$

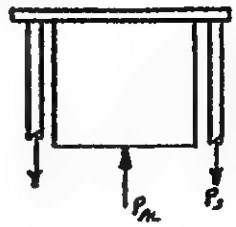
$$\sigma_A = \frac{P_{AT}}{A_A} = \frac{158.61\times10^3 N}{15394 mm^2} = \underline{10.3 MPa}$$

4-51 1) THE NUT MOVES 1.25 mm IN ONE TURN

2) THE FORCE CREATED CAUSES THE TUBE TO SHORTEN AND THE BOLTS TO GET LONGER. $\delta_{AL}+\delta_S = 1.25\text{ mm}$; AND $L_{AL}=L_S$

3) THE COMPRESSIVE FORCE IN THE TUBE EQUALS THE TENSILE FORCE IN THE BOLTS

$P_{AL}=P_S$ (ON ALL FOUR BOLTS)



4) $A_{AL}=\frac{\pi}{4}(150^2-138^2)=2714\text{ mm}^2$

$A_S = 4\pi(10^2)/4 = 314\text{ mm}^2$

5) $\frac{P_{AL}L_{AL}}{E_{AL}A_{AL}}+\frac{P_SL_S}{E_SA_S}=1.25\text{ mm}$

$\frac{P_S(450)\text{ mm}}{(69\times10^3\text{ N/mm}^2)(2714\text{ mm}^2)} + \frac{P_S(450)}{(207\times10^3)(314)} = 1.25$

$P_S[2.40\times10^{-6}+6.92\times10^{-6}] = P_S[9.32\times10^{-6}]=1.25$

$P_S = 134\text{ kN} = P_{AL}$

6) STRESSES:

$\sigma_S = \frac{P_S}{A_S} = \frac{134000\text{ N}}{314\text{ mm}^2} = \underline{427\text{ MPa}}$

$\sigma_{AL} = \frac{P_{AL}}{A_{AL}} = \frac{134000\text{ N}}{2714\text{ mm}^2} = \underline{49.4\text{ MPa}}$

4-52 $\sigma_c = \frac{PE_c}{A_sE_s+A_cE_c} = \frac{(50000\text{ lb})(2.7\times10^6\text{ psi})}{(4.43\text{ in}^2)(29\times10^6\text{ psi})+(108.7\text{ in}^2)(2.7\times10^6\text{ psi})} = \underline{320\text{ psi}}$

$A_s=4.43\text{ in}^2$; $A_c=\pi(12)^2/4-4.43=108.7\text{ in}^2$

$\sigma_s = \sigma_c\cdot E_s/E_c = (320\text{ psi})(29/2.7) = \underline{3436\text{ psi}}$

NOTE: $E_s=29\times10^6$ psi FOR STRUCTURAL STEEL

$E_c = 2.7\times10^6$ psi FOR CONCRETE WITH $\sigma_c = 2000$ psi RATED

(SEE SEC. 2-10)

4-53 NO. 2 SOUTHERN PINE. $\sigma_{c\,ALLOWABLE}=1.59\text{ MPa}$; $E = 9.0\text{ GPa}$

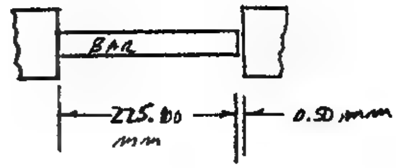
4X4 POST. $A = 7.96\times10^3\text{ mm}^2$, $L = 4.26\text{ m}$

$\sigma = \frac{F}{A}$; $\underline{F_{max}} = \sigma_{c\,ALL}\cdot A = \frac{1.59\text{ N}}{\text{mm}^2}\times 7.90\times10^3\text{ mm}^2 = \underline{12560\text{ N}} = \underline{12.56\text{ kN}}$

$\delta = \frac{FL}{EA} = \frac{(12560\text{ N})(4.25\text{ m})}{(9.0\times10^9\text{ N/m}^2)(7.90\times10^3\text{ mm}^2)}\times\frac{(10^3\text{ mm})^2}{\text{m}^2}$

$\underline{\delta} = 7.51\times10^{-4}\text{ m} = \underline{0.751\text{ mm}}$

4-54 $t_1 = 20°C$ $L_1 = 225.0$ mm
$t_{FINAL} = 205°C$. ALUM 6061-T4, $S_y = 110$ MPa



a) TEMP. AT WHICH BAR TOUCHES PLATE

$\delta = 0.50 \text{ mm} = \alpha L(\Delta t)$

$$\Delta t_1 = \frac{\delta}{\alpha L} = \frac{0.50 \text{ mm}}{(23.4 \times 10^{-6} °C^{-1})(225.0 \text{ mm})} = 95.0°C$$

$t_2 = t_1 + \Delta t = 20 + 95 = 115°C$ NO STRAIN AT THIS TEMP.

b) ADDITIONAL $\Delta t_2 = 205°C - 115°C = 90.0°C$ RESTRAINED.

$\sigma = E\alpha(\Delta t) = (69 \times 10^9 \text{ Pa})(23.4 \times 10^{-6} °C^{-1})(90.0°C)$.

$\underline{\sigma} = 145 \times 10^6 \text{ Pa} = \underline{145 \text{ MPa}}$ COMPRESSION $> S_y$ MATERIAL WOULD YIELD-FAILURE

4-55 (a) $t_1 = 20°C$. $L_1 = 2.400$ m. ALUM. 2014-T4. $L_2 = 2.405$ m

$\delta = L_2 - L_1 = 2.405 - 2.400 = 0.005 \text{ m} = 5.00 \text{ mm} = \alpha L(\Delta t)$

$$\Delta t_1 = \frac{\delta}{\alpha L_1} = \frac{0.005 \text{ m}}{(23.0 \times 10^{-6} °C^{-1})(2.400 \text{ m})} = 90.6°C$$

$\underline{t_2} = t_1 + \Delta t_1 = 20 + 91.6 = \underline{110.6°C}$

(b) INCREASE 30°C. $t_3 = 110.6°C + 30 = 140.6°C$ RESTRAINED.

$\sigma = E\alpha\Delta t_2 = (73 \times 10^9 \text{ Pa})(23.0 \times 10^{-6} °C^{-1})(30°C)$

$\underline{\sigma} = 50.4 \times 10^6 \text{ Pa} = \underline{50.4 \text{ MPa}}$ COMPRESSION

(C) $S_y = 290$ MPa FOR 2014-T4. SAFE AGAINST YIELDING.
BUT BUCKLING SHOULD BE CHECKED.

4-56 $\delta_{MAX} = 0.50$ mm. REPEATED AXIAL TENSILE LOAD.
SPECIFY MAX. PERMISSIBLE LOAD. 4140 OQT 1300. $E = 207$ GPa

a) DEFORMATION: $\delta = \frac{FL}{EA}$. THEN $F_{MAX} = \frac{\delta EA}{L}$

$A = 30 \times 20 = 600 \text{ mm}^2$, $L = 700$ mm

$$F_{MAX} = \frac{(0.50 \text{ mm})(207 \times 10^9 \text{ N})\ 600 \text{ mm}^2}{(700 \text{ mm})\ \text{m}^2} \times \frac{1 \text{m}^2}{(10^3 \text{ mm})^2}$$

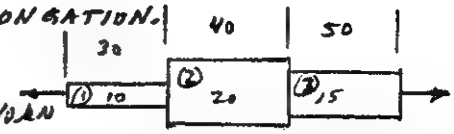
$\underline{F_{MAX}} = 88.7 \times 10^3 \text{ N} = \underline{88.7 \text{ kN}}$

(b) STRESS: $\sigma_d = S_u/8 = 814 \text{ MPa}/8 = 101.8 \text{ MPa} = F/A$

$\underline{F_{MAX}} = \sigma_d \cdot A = (101.8 \text{ N/mm}^2)(600 \text{ mm}^2) = 61\,000 \text{ N} = \underline{61.0 \text{ kN}}$

STRESS GOVERNS THE DESIGN. $\underline{F_{MAX} = 61.0 \text{ kN}}$

4-57 FIGURE P4-57. COMPUTE TOTAL ELONGATION.



① $\delta_1 = \dfrac{F_1 L_1}{E A_1} = \dfrac{(40 \times 10^3 N)(30 mm)}{(73 \times 10^3 N/mm^2)(100 mm^2)}$

$\delta_1 = 0.164 \, mm$

② $\delta_2 = \dfrac{F_2 L_2}{E A_2} = \dfrac{(40 \times 10^3)(40)}{(73 \times 10^3)(400)} = 0.0548 mm$

③ $\delta_3 = \dfrac{F_3 L_3}{E A_3} = \dfrac{(40 \times 10^3)(50)}{(73 \times 10^3)(225)} = 0.1218 mm$

$\underline{\delta_{TOTAL}} = \delta_1 + \delta_2 + \delta_3 = \underline{0.341 \, mm}$

2014-T6 ALUM.
$E = 73 GPa = 73 \times 10^9 Pa$
$E = 73 \times 10^3 MPa$
$E = 73 \times 10^3 N/mm^2$
$S_y = 414 MPa$

CHECK STRESS IN ①: $\sigma_1 = \dfrac{F_1}{A_1} = \dfrac{40 \times 10^3 N}{100 mm^2} = 400 MPa$ – CLOSE TO S_y
NOT SAFE.

4-58 AISI 1040 CD. $S_y = 82 ksi$. LET $\sigma = 0.9 S_y = 73.8 ksi = F/A$

$L = 2.00 \, IN$; $A = \dfrac{\pi D^2}{4} = \dfrac{\pi (0.505 IN)^2}{4} = 0.200 IN^2$

$E = 30 \times 10^6 psi = 30 \times 10^3 ksi$

$\delta = \dfrac{FL}{EA} = \dfrac{\sigma L}{E} = \dfrac{(73.8 ksi)(2.00 IN.)}{30 \times 10^3 ksi} = \underline{0.00492 IN} = \delta$

STRAIN $= \delta/L = 0.00492 IN / 2.00 IN = \underline{0.00246 \, IN/IN.} = \epsilon$

$F = \sigma \cdot A = (73.8 \times 10^3 LB/IN^2)(0.200 IN^2) = \underline{14760 \, LB} = F$

DISTANCE BETWEEN GAGE MARKS = 2.00 IN + δ = 2.0049 IN

PROBLEMS 59-66 FOLLOW SIMILAR SOLUTION PROCEDURE.
RESULTS SUMMARIZED ON FOLLOWING PAGE.

4-67 NYLON 6/6 PLASTIC. TENSILE STRENGTH $= S_u = 179 MPa$

LET $\sigma = 0.5 S_u = 0.5(179 MPa) = 89.5 MPa$

$L = 50 mm$. $A = t \cdot w = (12.5 mm)(16.0 mm) = 200 mm^2$

$E = 9.0 GPa = 9.0 \times 10^9 Pa \times \dfrac{1 MPa}{10^6 Pa} = 9000 MPa = 9000 N/mm^2$

$\delta = \dfrac{FL}{EA} = \dfrac{\sigma L}{E} = \dfrac{(89.5 MPa)(50 mm)}{9000 MPa} = \underline{0.497 mm} = \delta$

STRAIN $= \epsilon = \delta/L = 0.497 mm / 50 mm = \underline{0.00994 \, mm/mm} = \epsilon$

$F = \sigma \cdot A = (89.5 N/mm^2)(200 mm^2) = 17900 N = \underline{17.90 \, kN} = F$

DISTANCE BETWEEN GAGE MARKS = 50 mm + δ = $\underline{50.497 \, mm}$

PROBLEMS 68-77 FOLLOW SIMILAR SOLUTION PROCEDURE.
RESULTS SUMMARIZED ON FOLLOWING PAGE.

NOTE: DATA FROM TABLE 2-6 FOR PROBLEMS 4-71 TO 4-73 REQUIRE CONVERSIONS AS SHOWN IN LOWER TABLE ON FOLLOWING PAGE.
ALSO NOTE: E = SPECIFIC MODULUS x SPECIFIC WT.

SOLUTIONS TO PROBLEMS 4-58 TO 4-66.										
Prob. No.	Material	s_y (ksi)	$0.9*s_y$ (ksi)	E (ksi)	L (in)	A (in^2)	Elong. (in)	Length Bet. gage marks (in)	Strain (in/in)	Force (lb)
4-58	AISI 1040 CD	82	73.8	30000	2.000	0.200	0.00492	2.0049	0.00246	14760
4-59	AISI 5160 OQT 700	238	214.2	30000	2.000	0.200	0.01428	2.0143	0.00714	42840
4-60	AISI 501 OQT 1000	135	121.5	29000	2.000	0.200	0.00838	2.0084	0.00419	24300
4-61	C17000 Copper,hard	200	180.0	19000	2.000	0.200	0.01895	2.0189	0.00947	36000
4-62	AZ 63A-T6 Magn.	19	17.1	6500	2.000	0.200	0.00526	2.0053	0.00263	3420
4-63	ZA 12 Zinc	47	42.3	12000	2.000	0.200	0.00705	2.0071	0.00353	8460
4-64	ASTM A572 Gr 65	65	58.5	29000	2.000	0.200	0.00403	2.0040	0.00202	11700
4-65	Grade 4 ADI	140	126.0	24000	2.000	0.200	0.01050	2.0105	0.00525	25200
4-66	5154-H38 Alum.	39	35.1	10200	2.000	0.200	0.00688	2.0069	0.00344	7020

SOLUTIONS TO PROBLEMS 4-67 TO 4-77.										
Prob. No.	Material	s_u MPa	$0.5*s_u$ MPa	E MPa	L mm	A mm^2	Elong. mm	Length Bet. gage marks (mm)	Strain mm/mm	Force kN
4-67	Nylon 6/6	179	89.5	9000	50.0	200	0.49722	50.4972	0.00994	17.9
4-68	Polyester PET	152	76.0	11700	50.0	200	0.32479	50.3248	0.0065	15.2
4-69	Polyimide	186	93.0	22100	50.0	200	0.21041	50.2104	0.00421	18.6
4-70	Polystyrene	83	41.5	5500	50.0	200	0.37727	50.3773	0.00755	8.3
4-71	Glass/Epoxy Comp*	786	393.0	27759	50.0	200	0.7079	50.7079	0.01416	78.6
4-72	Aramid/Epoxy*	1379	689.5	75845	50.0	200	0.45455	50.4545	0.00908	137.9
4-73	Graphite/Epoxy*	1917	958.4	135590	50.0	200	0.35342	50.3534	0.00707	191.7
4-74	Unidir Lam-Long.	1380	690.0	145000	50.0	200	0.23793	50.2379	0.00476	138.0
4-75	Unidir Lam-Transv.	34	17.0	11000	50.0	200	0.07727	50.0773	0.00155	3.4
4-76	Quasi-isotropic-Long	552	276.0	55000	50.0	200	0.25091	50.2509	0.00502	55.2
4-77	Quasi-isotropic-Transv	552	276.0	55000	50.0	200	0.25091	50.2509	0.00502	55.2

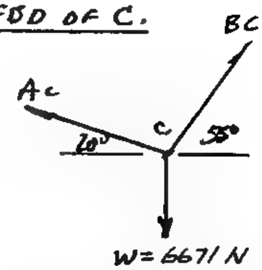
* Problems 71-73: Data conversion of units							
Prob. No.	Material From Table 2-6	s_u ksi	s_u MPa	Specific Modulus in	Specific Weight lb/in^3	Modulus E psi	Modulus E MPa
4-71	Glass/Epoxy Comp	114	786	6.60E+07	0.061	4.03E+06	27759
4-72	Aramid/Epoxy	200	1379	2.20E+08	0.050	1.10E+07	75845
4-73	Graphite/Epoxy	278	1917	3.45E+08	0.057	1.97E+07	135590

4-78 FORCE ANALYSIS: $m = 680\,kg$.

$W = mg = 680\,kg \cdot 9.81\,m/s^2 = 6671\,N$ = DOWNWARD FORCE AT C.

SINE LAW: $\dfrac{6671\,N}{\sin 75^\circ} = \dfrac{AC}{\sin 35^\circ} = \dfrac{BC}{\sin 70^\circ}$

FBD OF C.



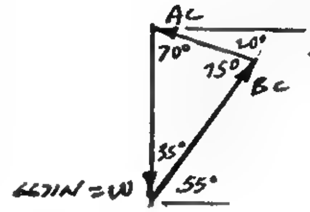
$AC = \dfrac{6671\,N(\sin 35^\circ)}{\sin 75^\circ} = 3961\,N$

$BC = \dfrac{6671\,N(\sin 70^\circ)}{\sin 75^\circ} = 6490\,N$

STRESS: ROD AREA $= A = \pi(8.00\,mm)^2/4 = 50.27\,mm^2$

$\sigma_{BC} = BC/A = 6490\,N/50.27\,mm^2 = 129\,MPa$

$\sigma_{AC} = AC/A = 3961\,N/50.27\,mm^2 = 78.8\,MPa$



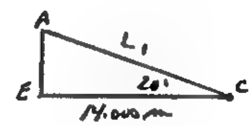
VECTOR TRIANGLE

ASSUME STATIC LOAD: SAFE FOR ANY STEEL

DEF. $\delta_{AC} = \dfrac{(AC)(L_1)}{EA}$, $\delta_{BC} = \dfrac{(BC)(L_2)}{EA}$

$L_1 = 14.000\,m/\cos 20^\circ = 14.898\,m$

$L_2 = 7.000\,m/\cos 55^\circ = 12.204\,m$

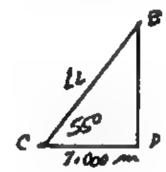


ORIGINAL VERTICAL DISTANCE FROM CEILING TO C:

$\tan 55^\circ = BD/CD$; $BD = CD \tan 55^\circ = (7.000)(\tan 55^\circ)$

$BD = 9.997\,m$

$\delta_{AC} = \dfrac{AC\,L_1}{EA} = \dfrac{(3961\,N)(14.898\,m)}{(207 \times 10^9\,N/m^2)(50.27\,mm^2)} \times \dfrac{(10^3\,mm)^2}{m^2}$



$\delta_{AC} = 5.67 \times 10^{-3}\,m = 5.67\,mm$

$\delta_{BC} = \dfrac{BC\,L_2}{EA} = \dfrac{(6490\,N)(12.204\,m)}{(207 \times 10^9\,N/m^2)(50.27\,mm^2)} \times \dfrac{(10^3\,mm)^2}{m^2} = 7.61 \times 10^{-3}\,m$

$\delta_{BC} = 7.61\,mm$

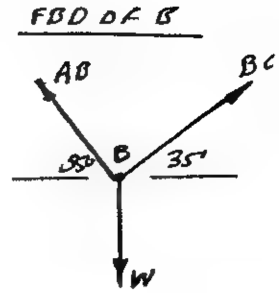
4-79 FORCE ANALYSIS. $m = 4200\ kg$.

$W = mg = (4200\ kg)(9.81\ m/s^2) = 41200 N = 41.2\ kN$

VECTOR TRIANGE IS A RIGHT TRIANGLE,

$BC = W \sin 35° = (41.2\ kN)(\sin 35°) = \underline{23.63\ kN}$

$AB = W \cos 35° = (41.2\ kN)(\cos 35°) = \underline{33.75\ kN}$



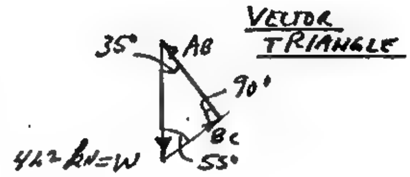
STRESS: $D = 10.0\ mm$; $A = \pi D^2/4 = 78.54\ mm^2$

$$\sigma_{AB} = \frac{F}{A} = \frac{AB}{A} = \frac{33.75 \times 10^3 N}{78.54\ mm^2} = 429.7\ MPa$$

ASSUME STATIC LOAD, $\sigma_d = S_y/2$

REQ'D $S_y = 2\sigma = 2(429.7\ MPa) = 859\ MPa$

SPECIFY AISI 4140 OQT 1100, $S_y = 903\ MPa$

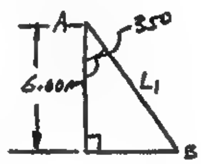


DEFORMATION:

$$\delta_{AB} = \frac{(AB)L_1}{EA}\ ; \quad \delta_{BC} = \frac{(BC)(L_2)}{EA}$$

$L_1 = 6.00\ m/\cos 35° = 7.32\ m$

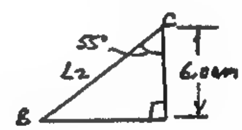
$$\delta_{AB} = \frac{(33.75 \times 10^3 N)(7.32\ m)}{(207 \times 10^9 N/m^2)(78.54\ mm^2)} \times \frac{(10^3\ mm)^2}{m^2}$$



$\delta_{AB} = 0.0152\ m = \underline{15.2\ mm}$

$$\delta_{BC} = \frac{(BC)L_2}{EA} = \frac{(23.63 \times 10^3 N)(10.46\ m)}{(207 \times 10^9 N/m^2)(78.54\ mm^2)} \times \frac{(10^3\ mm)^2}{m^2}$$

$[L_2 = 6.00\ m/\cos 55° = 10.46\ m]$



$\delta_{BC} = 0.0152\ m = \underline{15.2\ mm}$

CHAPTER 5 Torsional Shear Stress and Torsional Deflection

5-1 $\tau = \frac{Tc}{J} = \frac{(280\ N\cdot m)(10\ mm)}{\pi(20)^4/32\ mm^4} \times \frac{10^3\ mm}{m} = \underline{178\ MPa}$

5-2 $J = \frac{\pi}{32}(D_o^4 - D_i^4) = \frac{\pi}{32}(35^4 - 25^4) = 109 \times 10^3\ mm^4$

$\tau = \frac{Tc}{J} = \frac{(560\ N\cdot m)(35/2)\ mm}{109 \times 10^3\ mm^4} \times \frac{10^3\ mm}{m} = \underline{89.9\ MPa}$

5-3 $\tau = \frac{Tc}{J} = \frac{(1550\ LB\cdot IN)(1.25/2)\ IN}{\pi(1.25)^4/32\ IN^4} = \underline{4042\ psi}$

5-4 $D_i = D_o - 2t = 1.75 - 2(0.125) = 1.50\ IN$

$J = \frac{\pi}{32}(D_o^4 - D_i^4) = \frac{\pi(1.75^4 - 1.50^4)}{32} = 0.424\ IN^4$

$\tau_o = \frac{Tc}{J} = \frac{(5500\ LB\cdot IN)(1.75/2)\ IN}{0.424\ IN^4} = \underline{11\,360\ psi}$

$\tau_i = \frac{Tr_i}{J} = \frac{(5500\ LB\cdot IN)(1.50/2)\ IN}{0.424\ IN^4} = \underline{9734\ psi}$

5-5 $T = \frac{P}{n} = \frac{0.08 \times 10^3\ N\cdot m/s}{180\ rad/s} = 0.444\ N\cdot m$

$\tau = \frac{Tc}{J} = \frac{(0.440\ N\cdot m)(1.50\ mm)}{\pi(3.0)^4/32\ mm^4} \times \frac{10^3\ mm}{m} = \underline{83.8\ MPa}$

5-6 $T = \frac{P}{n} = \frac{35 \times 10^3\ N\cdot m/s}{42\ rad/s} = 833\ N\cdot m$

$J = \frac{\pi(40^4 - 25^4)}{32}\ mm^4 = 213 \times 10^3\ mm^4$

$\tau = \frac{Tc}{J} = \frac{(833\ N\cdot m)(20\ mm)}{213 \times 10^3\ mm^4} \times \frac{10^3\ mm}{m} = \underline{78.3\ MPa}$

5-7 $T = \frac{63\,000\,(P)}{n} = \frac{(63\,000)(15.0\ hp)}{240\ RPM} = 3938\ LB\cdot IN$

$\tau = \frac{Tc}{J} = \frac{3938\ LB\cdot IN\ (1.44/2)\ IN}{\pi(1.44)^4/32\ IN^4} = \underline{6716\ psi}$

$\tau_d = \frac{s_y}{2N} = \frac{101\,000\ psi}{2(6)} = \underline{8417\ psi}\ \ \underline{OK}$

5-8 FROM PROBLEM 5-7, $T = 3938$ LB·IN, $\tau_d = 8417$ PSI

$c = 1.44\text{ in.}/2 = 0.72\text{ in.}$

$$J = \frac{\pi D^4}{32} = \frac{\pi(1.44\text{ in.})^4}{32} = 0.422\text{ in}^4$$

FOR PROFILE KEYSEAT, $K_t = 2.0$

$$\tau = \frac{K_t T c}{J} = \frac{(2.0)(3938\text{ lb·in})(0.72\text{ in})}{0.422\text{ in}^4} = \underline{13\,433\text{ psi}}$$

BECAUSE $\tau > \tau_d$ DESIGN IS <u>NOT SAFE</u>.

5-9

$$T = \frac{63000(7.5\text{ hp})}{2200\text{ rpm}} = 215\text{ lb·in}$$

AT FILLET: $r/d = 0.05/0.75 = 0.066$; $D/d = 1.25/0.75 = 1.67$; $K_t = 1.55$

AT KEYSEAT: $K_t = 2.0$

$$\tau_{max} = \frac{K_t T}{Z_p} = \frac{(2.0)(215\text{ lb·in})}{\pi(0.75)^3/16\text{ in}^3} = 5190\text{ psi}$$

BLADE WOULD SEE SHOCK: $\tau_d = \dfrac{S_y}{2(6)}$

REQ'D. $S_y = 2(6)(5190) = \underline{62300\text{ psi}}$: AISI 1040 HOT ROLLED $S_y = 63$ ksi; 32% ELONG.

5-10 AT FILLET: $r/d = 0.08/1.50 = 0.053$; $D/d = 2.00/1.50 = 1.33$; $K_t = 1.58$

a) $$\tau_{max} = \frac{K_t T}{Z_p} = \frac{(1.58)(7500\text{ lb·in})}{\pi(1.50)^3/16\text{ in}^3} = \underline{17880\text{ psi}}$$

b) AT HOLE: $Z_{p\,GROSS} = \pi(2.00)^3/16 = 1.57\text{ in}^3$

$$\tau_{nom} = \frac{T}{Z_p} = \frac{7500}{1.57} = 4775\text{ psi}$$

$$K_{t\,max} = \frac{\tau_{max}}{\tau_{nom}} = \frac{17880\text{ psi}}{4775\text{ psi}} = 3.74$$

FROM APP. A-21-5 $(d/D)_{max} = 0.18$; THEN $d_{max} = 0.18(2.0) = \underline{0.360\text{ in}}$

5-11 $J = \pi(D_o^4 - D_i^4)/32 = \pi(80^4 - 60^4)/32 = 2.75 \times 10^6\text{ mm}^4$

$$\tau_{max} = \frac{Tc}{J} = \frac{(4500\text{ N·m})(40\text{ mm})}{2.75 \times 10^6\text{ mm}^4} \times \frac{10^3\text{ mm}}{\text{m}} = \underline{65.5\text{ MPa}}$$

$$\theta = \frac{TL}{GJ} = \frac{(4500\text{ N·m})(600\text{ mm})}{(26 \times 10^9\text{ N/m}^2)(2.75 \times 10^6\text{ mm}^4)} \times \frac{(10^3\text{ mm})^3}{\text{m}^3} = \underline{0.0378\text{ rad}}$$

ASSUME STEADY TORQUE: REQ'D. $S_y = 2(2)(\tau_{max})$

$S_y = 4(65.5) = \underline{262\text{ MPa}}$

6061-T6 HAS $S_y = 276$ MPa

5-12 SOLID: $J = \pi D^4/32 = \pi(50)^4/32\ mm^4 = 613.6 \times 10^3\ mm^4$

$$\tau = \frac{Tc}{J} = \frac{(850\ N\cdot m)(25\ mm)}{613.6 \times 10^3\ mm^4} \times \frac{10^3\ mm}{m} = \underline{34.6\ MPa}$$

$$\theta = \frac{TL}{GJ} = \frac{(850\ N\cdot m)(600\ mm)}{(80 \times 10^3\ N/mm^2)(613.6 \times 10^3\ mm^4)} \times \frac{(10^3\ mm)^3}{m^3} = \underline{0.0104\ rad}$$

$$MASS = (VOL)(DENS.) = A \cdot L \cdot DENS. = \frac{\pi(50)^2}{4} \times 600\ mm^3 \times \frac{7680\ kg}{m^3} \times \frac{1\ m^3}{(10^3\ mm)^3}$$

$\underline{m = 9.05\ kg}$

HOLLOW: $J = \frac{\pi}{32}(50^4 - 40^4) = 362.3 \times 10^3\ mm^4$

$$\tau = \frac{(850)(25)(10^3)}{362.3 \times 10^3} = \underline{58.7\ MPa}\ [1.69\ \text{TIMES}\ \tau_{SOLID}]$$

$$\theta = \frac{(850)(600)(10^9)}{(80 \times 10^9)(362.3 \times 10^3)} = \underline{0.0176\ rad}\ [1.69\ \text{TIMES}\ \theta_{SOLID}]$$

$$M = \frac{\pi(50^2 - 40^2)}{4} \times \frac{(600)(7680)}{10^9} = \underline{3.26\ kg}$$ [SOLID IS 2.78 TIMES HIGHER

5-13 REQ'D. $Z_p = \frac{T}{\tau_d} = \frac{1200\ N\cdot m}{45\ N/mm^2} \times \frac{10^3\ mm}{m} = 26667\ mm^3$

$$Z_p = \frac{\pi}{16} \frac{D_o^4 - D_i^4}{D_o} = \frac{\pi}{16} \frac{(1.25 D_i)^4 - D_i^4}{1.25 D_i} = 0.226\ D_i^3$$

REQ'D. $D_i = \sqrt[3]{26667/0.226} = \underline{49.0\ mm}$

$D_o = 1.25 D_i = \underline{61.3\ mm}$

5-14 $T = 63000(7.5)/240 = 1969\ LB\cdot IN$

$$\tau = \frac{T}{Z_p} = \frac{1969\ LB\cdot IN}{\pi(0.86)^3/16\ IN^3} = \underline{15764\ PSI}$$

5-15 $T = 63000(7.5)/1140 = 414\ LB\cdot IN$

REQ'D. $Z_p = \frac{T}{\tau} = \frac{414\ LB\cdot IN}{15764\ LB/IN^2} = 0.0263\ IN^3 = \pi D^3/16$

REQ'D. $D = \sqrt[3]{16 Z_p/\pi} = \sqrt[3]{16(0.0263)/\pi} = \underline{0.512\ IN}$

5-16 $T = F \cdot d = (80\ LB)(18\ IN) = 1440\ LB\cdot IN$; $Z_p = 0.6524\ IN^3$ (APP. A-12)

$\tau = T/Z_p = 1440\ LB\cdot IN / 0.6524\ IN^3 = \underline{2207\ PSI}$

5-17 $n = (1\ REV/5\ SEC)(60\ SEC/MIN) = 12.0\ RPM$; $T = (30\ LB\cdot FT)(12\ IN/FT) = 360\ LB\cdot IN$

$$P = \frac{Tn}{63000} = \frac{(360)(12)}{63000} = \underline{0.0686\ HP}$$

$\tau = T/Z_p = 360\ LB\cdot IN / \pi(0.60)^3/16\ IN^3 = \underline{8488\ PSI} = S_y/2N = S_y/8$

REQ'D $S_y = 8\tau = 8(8488) = \underline{67\,900\ PSI}$

POSSIBLE STEEL: AISI 1040 OQT 1100, $S_y = 80\ ksi$, 24% ELONG.

5-18
$$\tau = S_{ys} = S_y/2 = 441\ MPa/2 = 220\ MPa$$
$$T = \tau \cdot Z_p = \frac{220\ N}{mm^2} \times \frac{\pi (15)^3}{16} mm^3 \times \frac{m}{10^3\ mm} = \underline{146\ N \cdot m}$$

5-19
$$T = 63000(2500)/75 = 2.1 \times 10^6\ LB \cdot IN$$
$$\tau_d = \frac{S_y}{2N} = \frac{63000\ PSI}{2(6)} = 5250\ PSI$$
$$REQ'D.\ Z_p = \frac{T}{\tau_d} = \frac{2.1 \times 10^6\ LB \cdot IN}{5250\ LB/IN^2} = 400\ IN^3 = \frac{\pi}{16}\left[\frac{D_o^4 - D_i^4}{D_o}\right]$$
$$BUT\ D_i = 0.8 D_o;\ D_i^4 = 0.4096 D_o^4$$
$$Z_p = \frac{\pi}{16}\frac{(D_o^4 - 0.4096 D_o^4)}{D_o} = \frac{\pi(0.5904 D_o^4)}{16 D_o} = 0.1159 D_o^3$$
$$REQ'D.\ D_o = \sqrt[3]{\frac{Z_p}{0.1159}} = \sqrt[3]{\frac{400}{0.1159}} = \underline{15.11\ IN}$$
$$D_i = 0.8 D_o = \underline{12.09\ IN}$$

5-20
$$REQD\ Z_p = 400\ IN^3 = \pi D^3/16;\ D = \sqrt[3]{16 Z_p/\pi} = \underline{12.68\ IN}$$
$$\frac{WT_S}{WT_H} = \frac{A \cdot L \cdot DENS.}{A \cdot L \cdot DENS} = \frac{A_S}{A_H} = \frac{\pi (12.68)^2/4}{\pi(15.11^2 - 12.09^2)/4} = \underline{1.96}$$

5-21
$$T = \tau \cdot Z_p = \frac{80\ N}{mm^2} \cdot \frac{\pi (5.0)^3\ mm^3}{16} \cdot \frac{1\ m}{10^3\ mm} = \underline{1.96\ N \cdot m}$$

5-22
$$J = \pi D^4/32 = \pi(6.0)^4/32 = 127.2\ mm^4$$
$$\tau = \frac{Tc}{J} = \frac{(5.5\ N \cdot m)(3.0\ mm)}{127.2\ mm^4} \cdot \frac{10^3\ mm}{1\ m} = \underline{130\ MPa}$$
$$\theta = \frac{TL}{GJ} = \frac{(5.5\ N \cdot m)(250\ mm)}{(80 \times 10^9\ N/m^2)(127.2\ mm^4)} \times \frac{(10^3\ mm)^3}{m^3} = \underline{0.135\ RAD}\ (7.74\ deg)$$

5-23
$$\theta = \frac{TL}{GJ} = \frac{(240\ N \cdot m)(250\ mm)}{(80 \times 10^9\ N/m^2)(\pi (15)^4/32)\ mm^4} \times \frac{(10^3\ mm)^3}{m^3} = \underline{0.1509\ RAD}\ (8.65\ degs)$$

5-24
$$J = \frac{\pi}{32}(80^4 - 60^4) = 2.75 \times 10^6\ mm^4$$
$$\theta = \frac{TL}{GJ} = \frac{(2250\ N \cdot m)(1200\ mm)}{(26 \times 10^9\ N/m^2)(2.75 \times 10^6\ mm^4)} \times \frac{(10^3\ mm)^3}{m^3} = \underline{0.0378\ RAD}\ (2.16\ DEG)$$

5-25
$$\theta = \frac{TL}{GJ} = \frac{(40\ LB \cdot FT)(8\ FT)}{(11.5 \times 10^6\ LB/IN^2)(\pi (0.625)^4/32)\ IN^4} \times \frac{144\ IN^2}{FT^2} = \underline{0.267\ RAD}\ (15.3\ DEG)$$

5-26
$$\theta = (2.0\ DEG)(\pi\ RAD/180\ DEG.) = 0.0349\ RAD$$
$$J = \frac{TL}{G\theta} = \frac{(40\ LB \cdot FT)(8\ FT)}{(11.5 \times 10^6\ LB/IN^2)(0.0349\ RAD)} \times \frac{144\ IN^2}{FT^2} = 0.1148\ IN^4 = \pi D^4/32$$
$$REQ'D.\ D = \sqrt[4]{32(0.1148)/\pi} = \underline{1.04\ IN}$$

5-27 $\theta = \theta_1 + \theta_2 = \frac{(200\,N\cdot m)(400\,mm)}{(80\times10^9\,N/m^2)(15\,708\,mm^4)} \times \frac{(10^3)^2\,mm^2}{m^2} + \frac{(200)(1200)(10^6)}{(80\times10^9)(251\,300)}$

$J_1 = \pi(20)^4/32 = 15\,708\,mm^4$; $J_2 = \pi(40)^4/32 = 251\,300\,mm^4$

$\theta = 0.0637 + 0.0119 = \underline{0.0756\,RAD}$ (4.33 DEG.)

5-28 $\theta = (10.0\,DEG)(\pi\,RAD/180\,DEG) = 0.1745\,RAD$

REQD $J = \frac{TL}{G\theta} = \frac{(5.0\,N\cdot m)(150\,mm)}{(26\times10^9\,N/m^2)(0.1745)} \times \frac{10^9\,mm^3}{m^3} = 165.3\,mm^4$

REQ'D. $D = \sqrt[4]{32J/\pi} = \underline{6.40\,mm}$

$\tau = \frac{Tc}{J} = \frac{(5000\,N\cdot mm)(3.20\,mm)}{165.3\,mm^4} = \underline{96.8\,MPa}$

$N = \frac{s_y}{2(\tau)} = \frac{276\,MPa}{2(96.8\,MPa)} = \underline{1.43}$ LOW

COULD USE STRONGER ALUMINUM OR LONGER BAR

5-29 $T = \tau \cdot Z_p = \frac{250\,N}{mm^2} \times \frac{\pi(1.50)^3\,mm^3}{16} = 165.7\,N\cdot mm$

$J = \pi(1.50)^4/32 = 0.497\,mm^4$

$\theta = \frac{TL}{GJ} = \frac{(165.7\,N\cdot mm)(40\,mm)}{(48\times10^9\,N/m^2)(0.497\,mm^4)} \times \frac{10^6\,mm^2}{m^2} = \underline{0.278\,RAD}$ (15.9 DEG)

5-30 $J = \frac{\pi}{32}(18^4 - 16^4) = 3872\,mm^4$; $\theta = 40\,DEG \times \pi\,RAD/180\,DEG = 0.698\,RAD$

$T = \frac{\theta GJ}{L} = \frac{(0.698\,RAD)(43\times10^9\,N/m^2)(3872\,mm^4)}{1650\,mm} \times \frac{m^2}{10^6\,mm^2} = 70.4\times10^3\,N\cdot mm$

$\tau = \frac{Tc}{J} = \frac{(70.4\times10^3\,N\cdot mm)(9\,mm)}{3872\,mm^4} = \underline{164\,MPa}$

$N = \frac{s_y}{2\tau} = \frac{1070\,MPa}{2(164\,MPa)} = \underline{3.27}$

5-31 $J = \pi(35)^4/32 = 147.3\times10^3\,mm^4$

$T_{BC} = T_3 = 500\,N\cdot m = 500\times10^3\,N\cdot mm$; $T_{AB} = T_2 + T_3 = 1500\,N\cdot m = 1.5\times10^6\,N\cdot mm$

$G = 80\,GPa = (80\times10^9\,N/m^2)(1\,m^2/10^6\,mm^2) = 80\times10^3\,N/mm^2$

$\theta_{AC} = \theta_{AB} + \theta_{BC} = \frac{T_{AB}L_1}{GJ} + \frac{T_{BC}L_2}{GJ}$

$\theta_{AC} = \frac{(1.5\times10^6\,N\cdot mm)(500\,mm)}{(80\times10^3\,N/mm^2)(147.3\times10^3\,mm^4)} + \frac{(500\times10^3)(800)}{(80\times10^3)(147.3\times10^3)}$

$\theta_{AC} = 0.0636 + 0.0339 = \underline{0.0976\,RAD}$ (5.59 DEG) | $\theta_{AB} = 0.0636\,RAD$ (3.64 DEG.)

5-32 $\theta = (2.2\,DEG)(\pi\,RAD/180\,DEG) = 0.0384\,RAD$

REQD $J = \frac{TL}{G\theta} = \frac{(1360\,N\cdot m)(820\,mm)}{(80\times10^9\,N/m^2)(0.0384)} \times \frac{(10^3\,mm)^3}{m^3} = 363\times10^3\,mm^4 = \frac{\pi D^4}{32}$

REQD. $D = \sqrt[4]{32J/\pi} = \underline{43.9\,mm}$

$\tau = \frac{Tc}{J} = \frac{(1360\,N\cdot m)(21.95\,mm)}{363\times10^3\,mm^4} \times \frac{10^3\,mm}{m} = \underline{82.1\,MPa}$

5-33 $T = \frac{P}{n} = \frac{120\times10^3\ N{\cdot}m/s}{225\ RAD/s} = 533\ N{\cdot}m$

$J = \frac{\pi}{32}(75^4 - 55^4) = 2.208\times10^6\ mm^4$

$\tau = \frac{Tc}{J} = \frac{(533\ N{\cdot}m)(37.5\ mm)}{2.208\times10^6\ mm^4}\times\frac{10^3\ mm}{m} = \underline{9.06\ MPa}$

$\theta = \frac{TL}{GJ} = \frac{(533\ N{\cdot}m)(1525\ mm)}{(80\times10^9\ N/m^2)(2.208\times10^6\ mm^4)}\times\frac{(10^3\ mm)^3}{m^3} = \frac{0.0046\ RAD}{(0.264\ DEG)}$

5-34 $T = \frac{P}{n} = \frac{60\times10^3\ N{\cdot}m/s}{70\ RAD/s} = 857\ N{\cdot}m$

$Z_p = \pi d^3/16 = \pi(35)^3/16 = 8418\ mm^3$

$r/d = 4/35 = 0.114$; $D/d = 50/35 = 1.43$; $K_t = 1.35$ FROM APP. A-21-7

$\tau = \frac{TK_t}{Z_p} = \frac{(857\ N{\cdot}m)(1.35)}{8418\ mm^3}\times\frac{10^3\ mm}{m} = \underline{137\ MPa}$

5-35 $T = \frac{P}{n} = \frac{105\times10^3\ N{\cdot}m/s}{220\ RAD/s} = 477\ N{\cdot}m$

$Z_p = \pi d^3/16 = \pi(40)^3/16 = 12566\ mm^3$

$r/d = 6/40 = 0.150$; $D/d = 70/40 = 1.75$; $K_t = 1.29$ FROM APP. A-21-7

$\tau = \frac{TK_t}{Z_p} = \frac{(477\ N{\cdot}m)(1.29)}{12566\ mm^3}\times\frac{10^3\ mm}{m} = \underline{49.0\ MPa}$

FOR PROBLEMS 5-36 THROUGH 5-39: $\tau_d = \frac{S_y}{2N} = \frac{669\ MPa}{2(4)} = 83.6\ MPa$

OR $\tau_d = \frac{97000\ PSI}{2(4)} = 12125\ PSI$

$\tau = TK_t/Z_p$; ALLOW. $T = (\tau_d)(Z_p)/K_t$

5-36 LEFT END: $Z_p = \pi(12)^3/16 = 339.3\ mm^3$

$r/d = 2/12 = 0.167$; $D/d = 24/12 = 2.0$; $K_t = 1.27$ (A-21-

$T = \frac{(83.6\ N/mm^2)(339.3\ mm^3)}{1.27} = \underline{22.3\ N{\cdot}m}$ CRITICAL VALUE

RIGHT END: $Z_p = \pi(16)^3/16 = 804.2\ mm^3$

$r/d = 1/16 = 0.063$; $D/d = 24/16 = 1.50$; $K_t = 1.53$ (A-21-7)

$T = \frac{(83.6\ N/mm^2)(804.2\ mm^3)}{1.53} = 43.9\times10^3\ N{\cdot}mm = \underline{43.9\ N{\cdot}m}$

5-37 GROOVE $Z_p = \pi(1.20)^3/16 = 0.339\ in^3$

LEFT GROOVE: $r/d = 0.008/1.20 = 0.0067$; $D/d = 1.50/1.20 = 1.25$; $K_t \cong 3.0$ EST. (A-21-7)

RIGHT GROOVE: $r/d = 0.08/1.20 = 0.067$; $D/d = 1.25$; $K_t = 1.63$ (A-21-6)

LEFT GROOVE CRITICAL: $T = \frac{(12125\ LB/IN^2)(0.339\ IN^3)}{3.0} = \underline{1370\ LB{\cdot}IN}$

5-38 GROOVE: $Z_p = \pi(25)^3/16 = 3068\ mm^3$

$r/d = 1.50/25 = 0.060$; $D/d = 30/25 = 1.20$; $K_t = 1.66$ (A-21-6)

$T = \tau_d \cdot Z_p / K_t = (83.6\ N/mm^2)(3068\ mm^3)/1.66 = \underline{154.5 \times 10^3\ N \cdot mm = 154.5\ N \cdot m}$

FILLET: $Z_p = \pi(20)^3/16 = 1571\ mm^3$

$r/d = 1.50/20 = 0.075$; $D/d = 30/20 = 1.50$; $K_t = 1.47$ (A-21-7)

$T = (83.6)(1571)/1.47 = 89.3 \times 10^3\ N \cdot mm = \underline{89.3\ N \cdot m}$

HOLE: $Z_p = 1571\ mm^3$; $d/D = 4/20 = 0.200$; $K_{t_g} = 3.8$ (A-21-5C)

$T = (83.6)(1571)/3.8 = 34.5 \times 10^3\ N \cdot mm = \underline{34.5\ N \cdot m}$ CRITICAL

5-39 LEFT PART: $Z_p = \pi(1.25)^3/16 = 0.383\ in^3$

FILLET – $r/d = 0.188/1.25 = 0.150$; $D/d = 2.00/1.25 = 1.60$; $K_t = 1.26$ (A-21-7)

KEYSEAT: $K_t = 1.60$

$T = \frac{\tau_d \cdot Z_p}{K_t} = \frac{(12125\ LB/IN^2)(0.383\ IN^3)}{1.60} = \underline{2902\ LB \cdot IN}$ CRITICAL

OTHER PARTS OBVIOUSLY STRONGER

5-40 $\tau = T/Q$; $T = \tau Q = (50\ N/mm^2)(1664\ mm^3) = 83.2\times10^3\ N\cdot mm = \underline{83.2\ N\cdot m}$
$Q = 0.208\ a^3 = 0.208(20)^3 = 1664\ mm^3$ (FIG 5-25)

5-41 $K = 0.141\ a^4 = 0.141(20)^4 = 22.56\times10^3\ mm^4$ (FIG 5-25)
$\theta = \frac{TL}{GK} = \frac{(83.2\times10^3\ N\cdot mm)(1800\ mm)}{(80\,000\ N/mm^2)(22.56\times10^3\ mm^4)} = \underline{0.083\ RAD}$ (4.75 DEG)

5-42 $Q = 0.208\ a^3 = 0.208(1.25)^3 = 0.406\ IN^3$ (FIG 5-25)
$T = \tau Q = (7500\ LB/IN^2)(0.406\ IN^3) = 3047\ LB\cdot IN$

5-43 $K = 0.141\ a^4 = 0.141(1.25)^4 = 0.344\ IN^4$ (FIG 5-25)
$\theta = \frac{TL}{GK} = \frac{(3047\ LB\cdot IN)(48\ IN)}{(3.75\times10^6\ LB/IN^2)(0.344\ IN^4)} = \underline{0.112\ RAD}$ (6.42 DEG)

5-44 $Q = \frac{bh^2}{[3+1.8(h/b)]} = \frac{(3.0)(1.25)^2}{[3+1.8(1.25/3)]} = 1.25\ IN^3$ (FIG 5-25)
$T = \tau Q = (7500\ LB/IN^2)(1.25\ IN^3) = 9375\ LB\cdot IN$

5-45 $K = (3.0)(1.25)^3\left[\frac{1}{3} - 0.21\frac{1.25}{3.0}\left(1 - \frac{(1.25/3.0)^4}{12}\right)\right] = 1.44\ IN^4$ (FIG 5-25)
$\theta = \frac{TL}{GK} = \frac{(7500)(48)}{(3.75\times10^6)(1.44)} = \underline{0.0667\ RAD}$ (3.82 DEG.)

5-46 $K = 0.0217\ a^4 = 0.0217(30)^4 = 17.58\times10^3\ mm^4$ (FIG 5-25)
$\theta = (0.80\ DEG)(\pi\ RAD/180\ DEG.) = 0.0140\ RAD$
$T = \frac{\theta GK}{L} = \frac{(0.0140\ RAD)(26000\ N/mm^2)(17.58\times10^3\ mm^4)}{2600\ mm} = 2461\ N\cdot mm$
$= \underline{2.46\ N\cdot m}$

5-47 $Q = 0.050\ a^3 = 0.050(30)^3 = 1350\ mm^3$ (FIG 5-25)
$\tau = \frac{T}{Q} = \frac{2461\ N\cdot mm}{1350\ mm^3} = \underline{1.82\ MPa}$

5-48 CIRCULAR PART: $Z_p = \pi D^3/16 = \pi(1.75)^3/16 = 1.052\ IN^3$
$\tau = \frac{T}{Z_p} = \frac{850\ LB\cdot IN}{1.052\ IN^3} = \underline{808\ PSI}$
SHAFT WITH FLAT: $h = 1.50 - 0.875 = 0.625\ IN$; $r = D/2 = \frac{1.75}{2} = 0.875\ IN$
$h/r = 0.625/0.875 = 0.714$; $C_2 = 1.069$ (INTERPOLATION-FIG 5-25)
$Q = C_2 r^3 = (1.069)(0.875)^3 = 0.716\ IN^3$
$\tau = T/Q = 850\ LB\cdot IN/0.716\ IN^3 = \underline{1187\ PSI}$

5-49 $J = \pi(1.75)^4/32 = 0.9208\ IN^4$; $C_1 = 1.24$ (INTERPOLATION - FIG 5-25)
$K = C_1 r^4 = 1.24(0.875)^4 = 0.727\ IN^4$
$\theta = \frac{TL}{GJ} + \frac{TL}{GK} = \frac{(850)(20)}{(11.5\times10^6)(0.9208)} + \frac{(850)(20)}{(11.5\times10^6)(.727)} = 0.00160 + 0.00203$
$\theta = \underline{0.00364\ RAD} = 0.209\ DEG.$

5-50 $h = 1.25/2 = 0.625\ \text{IN}$: $h/h = 0.625/0.875 = 0.714$: $C_4 = 0.839$ (FIG. 5-25)

$Q = C_4 h^3 = 0.839(0.875)^3 = 0.562\ \text{IN}^3$

$\tau = \dfrac{T}{Q} = \dfrac{850\ \text{LB·IN}}{0.562\ \text{IN}^3} = \underline{1512\ \text{PSI}}$ <u>IN SHAFT WITH FLATS</u>

5-51 $C_3 = 0.966$ FOR $h/h = 0.714$ IN FIG 5-25 BY INTERPOLATION

$K = C_3 h^4 = 0.966(0.875)^4 = 0.566\ \text{IN}^4$

$\theta = \dfrac{TL}{GK} = \dfrac{(850)(20)}{(11.5\times10^6)(0.566)} = 0.0026\ \text{RAD}$ IN SHAFT WITH FLATS

$\theta_{TOT} = \underset{(\text{ROUND})}{0.0016} + \underset{(\text{FLATS})}{0.0026} = \underline{0.0042\ \text{RAD}}$ (0.241 DEG.)

5-52 $\left.\begin{aligned} Q &= 0.208\,a^3 = 0.208(8)^3 = 106.5\ \text{mm}^3 \\ K &= 0.141\,a^4 = 0.141(8)^4 = 577.5\ \text{mm}^4 \end{aligned}\right\}$ (FIG. 5-25)

$\tau = S_{ys} = 0.5\,S_y = 0.5(1070) = 535\ \text{MPa}$

$T = \tau Q = (535\ \text{N/mm}^2)(106.5\ \text{mm}^3) = 57.0\times10^3\ \text{N·mm} = 57.0\ \text{N·m}$

$\theta = \dfrac{TL}{GK} = \dfrac{(57.0\times10^3\ \text{N·mm})(200)\ \text{mm}}{(43000\ \text{N/mm}^2)(577.5\ \text{mm}^4)} = \underline{0.459\ \text{RAD}}$ (26.3 DEG)

5-53 $\theta = 3.0\ \text{DEG}(\pi\ \text{RAD}/180\ \text{DEG}) = 0.0524\ \text{RAD}.$

$K = \dfrac{2t(a-t)^2(b-t)^2}{(a+b-2t)} = \dfrac{2(0.25)(3.75)^2(3.75)^2}{(4+4-0.5)} = 13.18\ \text{IN}^4$ (FIG. 5-25)

$T = \dfrac{\theta G K}{L} = \dfrac{(0.0524)(11.5\times10^6\ \text{LB/IN}^2)(13.18\ \text{IN}^4)}{(8\cdot12)\ \text{IN}} = \underline{82\,750\ \text{LB·IN}}$

5-54 $Q = 2t(a-t)(b-t) = 2(0.25)(3.75)(3.75) = 7.03\ \text{IN}^3$ (FIG. 5-25)

$\tau = \dfrac{T}{Q} = \dfrac{82\,750\ \text{LB·IN}}{7.03\ \text{IN}^3} = \underline{11\,770\ \text{PSI}}$

FOR ASTM A501 STEEL, $S_y = 36\,000\ \text{PSI}$

$\tau_d = \dfrac{S_y}{2(2)} = \dfrac{36\,000\ \text{PSI}}{4} = \underline{9000\ \text{PSI}}$ <u>NOT SAFE</u>

5-55 $K = \dfrac{2(.25)(3.75)^2(5.75)^2}{(4+6-2(0.25))} = 24.47\ \text{IN}^4$ (FIG. 5-25)

$T = \dfrac{\theta G K}{L} = \dfrac{(0.0524)(11.5\times10^6)(24.47)}{96} = \underline{153\,600\ \text{LB·IN}}$

5-56 $Q = 2(.25)(3.75)(5.75) = 10.78\ \text{IN}^3$ (FIG 5-25)

$\tau = T/Q = 153\,600/10.78 = \underline{14\,250\ \text{PSI}}$ <u>NOT SAFE</u>

5-57 <u>TUBE</u>: $Q = 2(.25)(5.75)(5.75) = 16.53\ \text{IN}^3$

$K = \dfrac{2(.25)(5.75)^2(5.75)^2}{(6+6-0.5)} = 42.53\ \text{IN}^4$ (FIG. 5-25)

$\tau_T = T/Q = 0.0605\,T$

$\theta_T = \dfrac{TL}{GK} = 0.0210(TL/G)$

<u>PIPE</u>: $Z_p = 16.99\ \text{IN}^3$

$J = 2I = 2(28.14) = 56.28\ \text{IN}^4$

$\tau_P = T/Z_p = 0.0589\,T$

$\theta_P = \dfrac{TL}{GJ} = 0.0177(TL/G)$

$\dfrac{\tau_T}{\tau_P} = \dfrac{0.0605}{0.0589} = \underline{1.028}$ $\qquad$ $\dfrac{\theta_T}{\theta_P} = \dfrac{0.0210}{0.0177} = \underline{1.186}$

5-58 $P = T \cdot n \quad T = P/n \quad D = 25\,mm$

$$n = 1150\,\frac{rev}{min} \cdot \frac{2\pi\,rad}{rev} \cdot \frac{1\,min}{60\,s} = 120.4\,rad/s$$

$$T = \frac{P}{n} = \frac{125 \times 10^3\,N \cdot m/s}{120.4\,rad/s} = 1038\,N \cdot m$$

$$\tau_{MAX} = \frac{Tc}{J} = \frac{T}{Z_P} = \frac{1038\,N \cdot m}{\pi (35\,mm)^3/16} \cdot \frac{10^3\,mm}{m} = 123\,\frac{N}{mm^2} = 123\,MPa$$

5-59 SMOOTH POWER. $\tau_d = S_{YS}/N = S_Y/2N$; LET $N = 2$

$\tau_d = S_Y/4$. LET $\tau_{MAX} = \tau_d = S_Y/4$

REQ'D $S_Y = 4(\tau_d) = 4(123\,MPa) = 493\,MPa$

POSSIBLE STEEL: AISI 1040 CD, $S_Y = 565\,MPa$

5-60 REPEATED POWER LET $N = 4$. $\tau_d = S_Y/2N = S_Y/8 = \tau_{MAX}$

REQ'D $S_Y = 8\,\tau_{MAX} = 8(123\,MPa) = 986\,MPa$

POSSIBLE STEEL: AISI 4140 OQT 900. $S_Y = 1193\,MPa$

15% ELONGATION. GOOD DUCTILITY.

5-61 SHOCK LOADING LET $N = 6$, $\tau_d = S_Y/2N = S_Y/12 = \tau_{MAX}$

REQ'D $S_Y = 12\,\tau_{MAX} = 12(123\,MPa) = 1480\,MPa$

POSSIBLE STEEL: AISI 4140 OQT 700, $S_Y = 1462\,MPa$

12% ELONGATION.

MARGINAL STRENGTH, SATISFACTORY DUCTILITY.

5-62 $P = 12.0\,HP$ STEADY. $n = 1150\,rpm$. $S_Y = 80\,ksi$

$T = 63\,000(P)/n = 63\,000(12)/1150 = 657\,IN \cdot LB$

$\tau_{MAX} = T/Z_P$. REQ'D $Z_P = \dfrac{T}{\tau_{MAX}} = \dfrac{657\,IN \cdot LB}{20\,000\,LB/IN^2} = 0.0329\,IN^3 = \dfrac{\pi D^3}{16}$

LET $\tau_{MAX} = \tau_d = S_Y/4 = 80\,ksi/4 = 20\,ksi = 20\,000\,lb/in^2$

$$D_{MIN} = \sqrt[3]{\frac{16 Z_P}{\pi}} = \sqrt[3]{\frac{16(0.0329\,IN^3)}{\pi}} = 0.551\,IN;\ \text{SPECIFY } D = 0.60\,IN$$

5-63 $P = 20.0\,HP$ STEADY. $n = 3450\,rpm$, $S_Y = 101\,KSI$

$T = 63000(P)/n = 63000(20)/3450 = 365\,IN \cdot LB$

$\tau_{MAX} = T/Z_P$. REQ'D $Z_P = \dfrac{T}{\tau_{MAX}} = \dfrac{365\,IN \cdot LB}{25250\,LB/IN^2} = 0.0145\,IN^3 = \dfrac{\pi D^3}{16}$

LET $\tau_{MAX} = \tau_d = S_Y/4 = 101\,ksi/4 = 25.25\,ksi = 25\,250\,psi$

$$D_{MIN} = \sqrt[3]{\frac{16 Z_P}{\pi}} = \sqrt[3]{\frac{16(0.0145)\,IN^3}{\pi}} = 0.419\,IN;\ \text{SPECIFY } D = 0.50\,IN$$

5-64 $D_o = 100\ mm$; $D_i = 60\ mm$; ALLOY STEEL

$\tau_{SOLID} = 200\ MPa = \frac{T}{Z_{P_s}} \cdot T = \tau \cdot Z_{P_s}$

$Z_{P_s} = \pi D_o^3/16 = \pi(100mm)^3/16 = 196350\ mm^3$

$T = \tau \cdot Z_P = (200\ N/mm^2)(196350mm^3) = 3.927\times10^7 N\cdot mm$

$\tau_{HOLLOW} = \frac{T}{Z_P} = \frac{3.927\times10^7\ N\cdot mm}{1.709\times10^5\ mm^3} = 230\ N/mm^2 = \underline{230\ MPa = \tau_H}$

$Z_P = \frac{\pi}{16}\ \frac{D_o^4 - D_i^4}{D_o} = \frac{\pi[100^4 - 60^4]}{16(100)} mm^3 = 170\ 900\ mm^3$

5-65 FIND ANGLE OF TWIST FOR SHAFT OF PROB. 5-64.

$T = 3.927\times10^7\ N\cdot mm$

SOLID SEGMENT: $J = \frac{\pi D_o^4}{32} = \frac{\pi(100mm)^4}{32} = 9.817\times10^6\ mm^4$

$\theta_S = \frac{TL}{GJ} = \frac{(3.927\times10^7 N\cdot mm)(300mm)}{(80\times10^3 N/mm^2)(9.817\times10^6 mm^4)} = 0.0150\ rad.$

$G = 80\ GPa = \frac{80\times10^9 N}{m^2}\times\frac{1m^2}{(10^3 mm)^2} = 80\times10^3\ N/mm^2$

HOLLOW SEGMENT: $J = \frac{\pi}{32}(100^4 - 60^4)\ mm^4 = 8.545\times10^6\ mm^4$

$\theta_H = \frac{TL}{GJ} = \frac{(3.927\times10^7)(300)}{(80\times10^3)(8.545\times10^6)} = 0.0172\ rad$

TOTAL $\theta_T = \theta_S + \theta_H = 0.0150 + 0.0172 = \underline{0.0322\ rad}$ (1.85 DEG)

5-66 FIND τ IN EACH PART OF SHAFT.

$n = \frac{1750\ rev}{min}\times\frac{2\pi\ RAD}{rev}\times\frac{1\ min}{60\ s.} = 183\ rad/s$

$T_A = \frac{P_A}{n} = \frac{15\times10^3\ N\cdot m/s}{183\ rad/s} = \underset{OUT}{81.85\ N\cdot m}$

$T_C = \frac{P_C}{n} = \frac{20\times10^3}{183} = \underset{OUT}{109.1\ N\cdot m}$; $T_B = \frac{P_B}{n} = \frac{35\times10^3}{183} = \underset{IN}{191.0\ N\cdot m}$

$T_{AB} = T_A = 81.85\ N\cdot m$ $\quad T_{BC} = T_C = 109.1\ N\cdot m$

τ AT A: $\underset{GROOVE\ DIA}{D = 9.50\ mm}$, RETAINING RING TO RIGHT OF PULLEY; $K_t = 3.0$

$\tau_A = T_{AB}\cdot K_t/Z_P = \frac{(81.85\ N\cdot m)(3.0)}{\pi(9.5mm)^3/16}\cdot\frac{10^3 mm}{m} = 1251\ N/mm^2 = \underline{1459\ MPa}$ $\underline{\text{VERY HIGH}}$

τ AT BEARING TO RIGHT OF PULLEY: STEPPED SHAFT. $D = 10\ mm$

$D/d = 15/10 = 1.50$; $r/d = 0.5mm/10mm = 0.05$; $K_t = 1.60$

$\tau = T_{AB}\cdot K_t/Z_P = \frac{(81.85)(1.60)(10^3)}{\pi(10)^3/16} = \underline{667\ MPa}$

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5-66 (CONTINUED)

τ TO LEFT OF B: RETAINING RING GROOVE; D=14 mm AT GROOVE

$$\tau = \frac{T_{AB} \cdot K_t}{Z_P} = \frac{(81.85)(3.0)(1000)}{\pi(14)^3/16} = \underline{456\,MPa}$$

τ AT KEYSEAT AT B: D = 15 mm, K_t = 2.0 FOR KEYSEAT

$T = T_{BC} = 109.1$ N·m

$$\tau = \frac{T_{BC} \cdot K_t}{Z_P} = \frac{(109.1)(2.0)(1000)}{\pi(15)^3/16} = \underline{329\,MPa}$$

τ TO RIGHT OF B AT SHOULDER FILLET: D_1=15 mm, D_2=20 mm

$D_2/D_1 = 20/15 = 1.33$; $r/D_1 = 0.50\,mm/15\,mm = 0.033$; $K_t = 1.75$

$$\tau = \frac{T_{BC} \cdot K_t}{Z_P} = \frac{(109.1)(1.75)(1000)}{\pi(15)^3/16} = \underline{288\,MPa}$$

τ AT RIGHT BEARING AT SHOULDER FILLET: D_1=15 mm, D_2=20 mm

SAME CONDITIONS AS AT PULLEY: $\underline{\tau = 288\,MPa}$

τ AT C AT RETAINING RING GROOVE: K_t=3.0; D_g=14.0 mm

$$\tau = \frac{T_{BC} \cdot K_t}{Z_P} = \frac{(109.1)(3.0)(1000)}{\pi(14.0)^3/16} = \underline{607\,MPa}$$

τ AT C AT SLED RUNNER KEY SEAT: D=15 mm; K_t=1.60

$$\tau = \frac{T_{BC} \cdot K_t}{Z_P} = \frac{(109.1)(1.60)(1000)}{\pi(15)^3/16} = \underline{263\,MPa}$$

SUMMARY SEVERAL STRESSES ARE QUITE HIGH. LARGER SHAFT DIAMETERS RECOMMENDED.

5-67 FIND τ IN EACH PART OF SHAFT.

$$n = \frac{1150\,rev}{min} \cdot \frac{2\pi\,rad}{rev} \cdot \frac{1\,min}{60\,s} = 120.4\,rad/s$$

$$T_A = \frac{P_A}{n} = \frac{20\times10^3\,N\cdot m/s}{120.4\,rad/s} = 166\,N\cdot m \times \frac{10^3\,mm}{m} = 1.66\times10^5\,N\cdot mm$$

$$T_C = \frac{P_C}{n} = \frac{12\times10^3\,N\cdot m/s}{120.4\,rad/s} = 99.6\,N\cdot m \times \frac{10^3\,mm}{m} = 9.96\times10^4\,N\cdot mm$$

$$T_B = \frac{P_B}{n} = \frac{32\times10^3\,N\cdot m/s}{120.4\,rad/s} = 266\,N\cdot m \times \frac{10^3\,mm}{m} = 2.66\times10^5\,N\cdot mm$$

$T_{AB} = T_A = 1.66\times10^5$ N·mm $\quad T_{BC} = T_C = 9.96\times10^4$ N·mm

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5-67 (CONTINUED)

τ_1 AT A AT KEYSEAT: $D = 20.0\,mm$; $K_t = 2.0$ - PROFILE KEYSEAT

$$\tau_1 = \frac{T_{AB}\,K_t}{Z_P} = \frac{1.66 \times 10^5\,N\cdot mm\,(2.0)}{\pi\,(20\,mm)^3/16} = 211\,N/mm^2 = \boxed{211\,MPa = \tau_{MAX}}$$

τ_2 AT SHOULDER TO RIGHT OF A: $D = 20.0\,mm$; $D/d = \frac{30}{20} = 1.50$

$r/d = 1.0/20 = 0.05$; $K_t = 1.62$

$$\tau_2 = \frac{T_{AB}\,K_t}{Z_P} = \frac{(1.66 \times 10^5)(1.62)}{\pi(20)^3/16} = \underline{171\,MPa}$$

τ_3 AT RIGHT OF BEARING SEAT: $D = 30\,mm$; $D/d = \frac{40}{30} = 1.33$

$r/d = 1.0/30 = 0.033$; $K_t = 1.78$

$$\tau_3 = \frac{T_{AB}\,K_t}{Z_P} = \frac{(1.66 \times 10^5)(1.78)}{\pi(30)^3/16} = \underline{55.7\,MPa}$$

τ_4 AT RETAINING RING TO LEFT OF B: $D = 40.0\,mm$; $K_t = 3.0$

$$\tau_4 = \frac{T_{AB}\,K_t}{Z_P} = \frac{(1.66 \times 10^5)(3.0)}{\pi(40)^3/16} = \underline{39.6\,MPa}$$

τ_5 AT KEYSEAT AT B: $K_t = 2.0$, $D = 40\,mm$

$$\tau_5 = \frac{T_{AB}\,K_t}{Z_P} = \tau_4 \cdot \frac{K_{t5}}{K_{t4}} = 39.6\,MPa \cdot \frac{2.0}{3.0} = \underline{26.4\,MPa}$$

τ_6 AT STEP TO RIGHT OF B: $D = 40\,mm$, $D/d = \frac{50.0}{40.0} = 1.25$

$r/d = 1.0/40 = 0.025$; $K_t = 1.85$

$$\tau_6 = \frac{T_{BC}\,K_t}{Z_P} = \frac{(9.96 \times 10^4)(1.85)}{\pi(40)^3/16} = \underline{14.7\,MPa}$$

τ_7 AT STEP FROM 50 TO 30 mm DIA.: $D = 30.0\,mm$, $D/d = 50/30 = 1.67$

$r/d = 1.0/30 = 0.033$; $K_t = 1.82$

$$\tau_7 = \frac{T_{BC}\,K_t}{Z_P} = \frac{(9.96 \times 10^4)(1.82)}{\pi(30)^3/16} = \underline{10.7\,MPa}$$

τ_8 AT LEFT OF BEARING: $D = 20.0\,mm$; $D/d = 30/20 = 1.50$

$r/d = 1.0/20 = 0.05$; $K_t = 1.62$

$$\tau_8 = \frac{T_{BC}\,K_t}{Z_P} = \frac{(9.96 \times 10^4)(1.62)}{\pi(20)^3/16} = \underline{10.3\,MPa}$$

τ_9 AT STEP TO LEFT OF C: $D = 15.0\,mm$; $D/d = 20/15 = 1.33$; $r/d = 1/15 = 0.067$

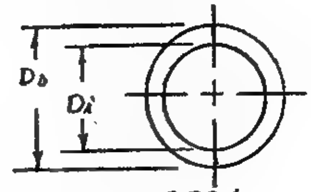
$K_t = 1.50$: $\tau_9 = \frac{T_{BC}\,K_t}{Z_P} = \frac{(9.96 \times 10^4)(1.50)}{\pi(15)^3/16} = \underline{22.5\,MPa}$

τ_{10} AT KEYSEAT AT C: $\tau_{10} = \frac{T_{BC}\,K_t}{Z_P} = \tau_9\,\frac{K_{t10}}{K_{t9}} = (22.5)\,\frac{2.0}{1.5} = \underline{30.1\,MPa}$

$K_t = 2.0$.

5-68 DESIGN SHAFT $P = 225\ kW;\ n = 80\ rpm;\ \tau_d = 60\ MPa;\ K_t = 1.0$

$$T = \frac{P}{n} = \frac{225 \times 10^3\ N\cdot m/s}{8.38\ rad/s} = 26\,857\ N\cdot m$$



$$n = 80\ \frac{REV}{MIN} \cdot \frac{2\pi\ RAD}{REV} \cdot \frac{1\ MIN}{60\ S} = 8.38\ rad/s$$

$$\tau = \frac{T}{Z_P};\quad REQ'D\ Z_P = \frac{T}{\tau_d} = \frac{26857\ N\cdot m}{60\ N/mm^2} \cdot \frac{10^3\ mm}{m}$$

$$Z_P = 4.476 \times 10^5\ mm^3 = \frac{\pi}{16}\ \frac{D_o^4 - D_i^4}{D_o}$$

$$BUT\ Z_P = \frac{\pi[(1.25 D_i)^4 - D_i^4]}{(16)\ 1.25 D_i} = \frac{\pi(1.44 D_i^4)}{(16)(1.25 D_i)} = 0.226\ D_i^3$$

$$THEN\ REQ'D\ D_i = \sqrt[3]{\frac{Z_P}{0.226}} = \sqrt[3]{\frac{4.476 \times 10^5\ mm^3}{0.226}} = 125.5\ mm$$

$$D_o \approx 1.25 D_i = 1.25(125.5) = 157\ mm$$

LET $\underline{D_o = 160\ mm}$; $D_i \approx \frac{D_o}{1.25} = \frac{160}{1.25} = 128\ mm$; USE $\underline{D_i = 125\ mm}$

$$CHECK\ Z_P = \frac{\pi}{16}\ \frac{D_o^4 - D_i^4}{D_o} = \frac{\pi[(160^4) - (125^4)]}{(16)(160)} = 5.05 \times 10^5\ mm^3$$

OK BUT HIGH

TRY $\underline{D_o = 160\ mm,\ D_i = 130\ mm}$

$$Z_P = \frac{\pi[(160)^4 - (130)^4]}{16\ (160)} = 4.54 \times 10^5\ mm^3\ \ OK$$

OR: LET $D_o = 160\ mm$, SOLVE FOR REQ'D. $D_{i\,MAX}$ FOR $Z_P = 4.476 \times 10^5\ mm^3$

$$Z_P = \frac{\pi[(160)^4 - D_i^4]}{16\ (160)};\quad (16)(160)\ Z_P = \pi\ [160^4 - D_i^4]$$

$$\frac{16(160)\ Z_P}{\pi} = 160^4 - D_i^4;\quad D_i^4 = 160^4 - \frac{16(160)(4.476 \times 10^5)}{\pi}$$

$D_{i\,MAX} = 130.6\ mm$; $\underline{USE\ D_o = 160\ mm;\ D_i = 130\ mm}$

$$CHECK\ FOR\ WALL\ THICKNESS:\ t = \frac{D_o - D_i}{2} = \frac{160 - 130}{2} = 15\ mm$$

$$MEAN\ RADIUS = \frac{(D_o + D_i)/2}{2} = 72.5\ mm$$

$r_{MEAN}/t = 72.5/15 = 4.83 < 10$. SHAFT IS NOT THIN-WALLED

BUCKLING NOT LIKELY.

5-69 $D = 4.0\ mm$; $\theta = 180\ DEG \times \frac{\pi\ RAD}{180°} = \pi\ RAD$; $\tau_{MAX} = 150\ MPa = \frac{Tc}{J}$

$$J = \frac{\pi D^4}{32} = \frac{\pi(4.0\ mm)^4}{32} = 25.1\ mm^4$$

$$T_{MAX} = \frac{\tau_{MAX} J}{c} = \frac{(150\ N/mm^2)(25.1\ mm^4)}{2.0\ mm} = 1885\ N{\cdot}mm$$

$$\theta = \frac{TL}{GJ} ; \quad L_{MIN} = \frac{\theta G J}{T_{MAX}}$$

$$G = 26\ GPa = \frac{26 \times 10^9\ N}{m^2} \times \frac{1 m^2}{(10^3\ mm)^2} = 26.0 \times 10^3\ N/mm^2$$

$$\underline{L_{MIN}} = \frac{(\pi\ RAD)(26 \times 10^3\ N/mm^2)(25.1\ mm^4)}{1885\ N{\cdot}mm} = 1088\ mm = \underline{1.088\ m}$$

5-70 TORSION BAR: $L = 200\ mm = 0.200\ m$. $D_o/D_i = 1.50$

$$\text{TORSIONAL STIFFNESS} = \frac{\theta}{T} = \frac{0.015\ DEG}{1.0\ N{\cdot}m} \times \frac{\pi\ RAD}{180\ DEG} = \frac{0.2618 \times 10^{-3}\ RAD}{1.0\ N{\cdot}m}$$

$$\theta = \frac{TL}{GJ} ; \quad \text{REQ'D } J = \frac{TL}{\theta G}$$

$$G = 43\ GPa = (43 \times 10^9\ N/m^2)(1.0 m^2/10^6 mm^2) = 43 \times 10^3\ N/mm^2$$

$$J = \frac{(1.0\ N{\cdot}m)(200\ mm)}{(0.2618 \times 10^{-3}\ RAD)(43 \times 10^3\ N/mm^2)} \times \frac{10^3\ mm}{1\ m} = 17766\ mm^4.$$

$$J = \frac{\pi(D_o^4 - D_i^4)}{32} = \frac{\pi[(1.5 D_i)^4 - D_i^4]}{32} = 0.3988\ D_i^4$$

$$D_i = \frac{J}{0.3988} = \sqrt[4]{\frac{17766\ mm^4}{0.3988}} = \underline{14.53\ mm} = D_i$$

$$D_o = 1.50(D_i) = 1.50(14.53\ mm) = \underline{21.79\ mm} = D_o$$

ALTERNATE DESIGN: PREFERRED SIZE FOR $D_o = 22.0\ mm$
FIND REQ'D D_i FOR $J = 17766\ mm^4$

$$J = \frac{\pi(D_o^4 - D_i^4)}{32} ; \quad D_o^4 - D_i^4 = \frac{32 J}{\pi} ; \quad D_i^4 = D_o^4 - \frac{32 J}{\pi}$$

$$D_i = \sqrt[4]{D_o^4 - \frac{32 J}{\pi}} = \sqrt[4]{22.0^4 - \frac{32(17766)}{\pi}} = \underline{15.19\ mm = D_i}$$

$\underline{\text{FOR } D_o = 22.0\ mm}$

5-71 USE FIRST DESIGN FROM 5-70. $D_o = 21.79\ mm$, $D_i = 14.53\ mm$

$$\tau = \frac{Tc}{J} = \frac{T D_o}{J\,2} ; \quad \theta = \frac{TL}{GJ} \text{ OR } T = \frac{\theta G J}{L}$$

$$\text{THEN } \tau = \frac{\theta G J}{L} \cdot \frac{D_o}{2J} = \frac{\theta G D_o}{2L} = \frac{(0.1745\ RAD)(43 \times 10^3\ N/mm^2)(21.79\ mm)}{2(200\ mm)}$$

$$\theta = 10° \times \frac{\pi\ RAD}{180°} = 0.1745\ RAD$$

$$\tau = 408.8\ N/mm^2 = \underline{408.8\ MPa}$$

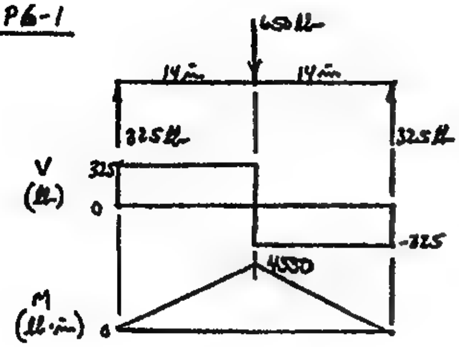
5-72 FIND θ WHEN $\tau_d = \frac{S_{ys}}{3} = \frac{S_y}{2(3)} = \frac{S_y}{6} = \frac{903\,MPa}{6} = 150.5\,MPa$

AISI 4140 OQT 1100; $S_y = 903\,MPa$, 18% ELONGATION

$$\tau = \frac{TC}{J} \;;\; T = \frac{\tau_d J}{C} = \frac{(150.5\,N/mm^2)(17766\,mm^4)}{21.79\,mm/2} \times \frac{1\,m}{1000\,mm} = 245.4\,N{\cdot}m$$

$$\theta = \frac{TL}{GJ} = \frac{(245.4\,N{\cdot}m)(200\,mm)}{(43\,000\,N/mm^2)(17766\,mm^4)} \times \frac{10^3\,mm}{m} = 0.064\,RAD \times \frac{180°}{\pi\,RAD} = \underline{3.68\,DEG}$$

P6-1



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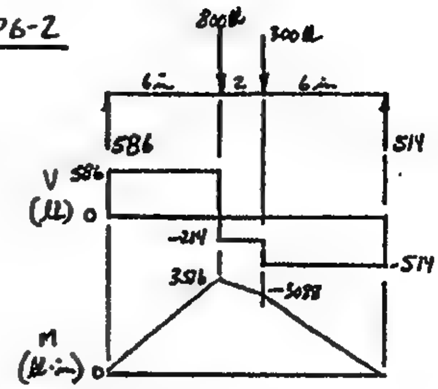

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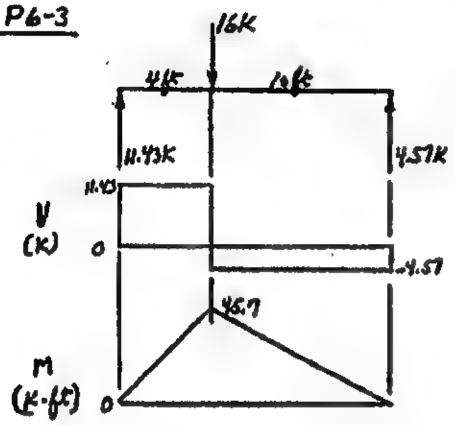

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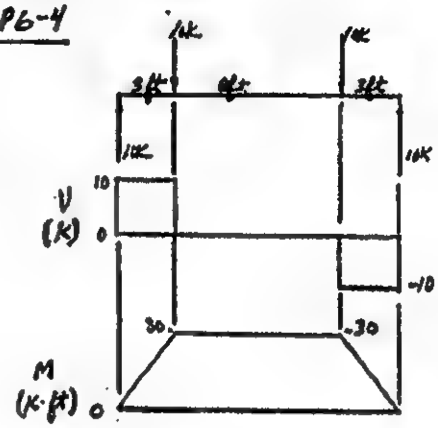

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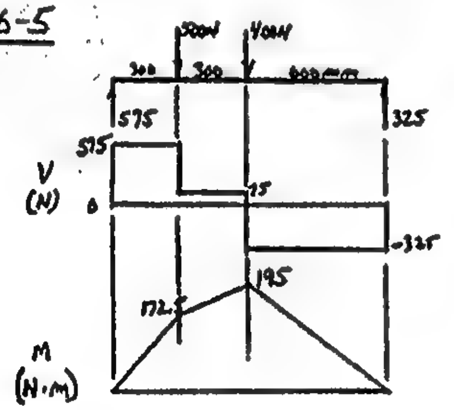

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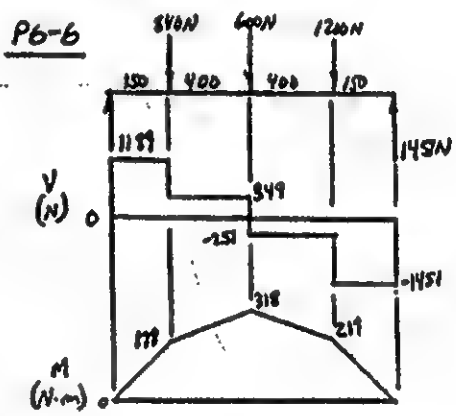

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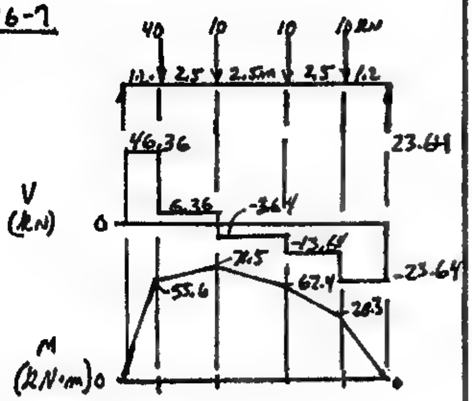

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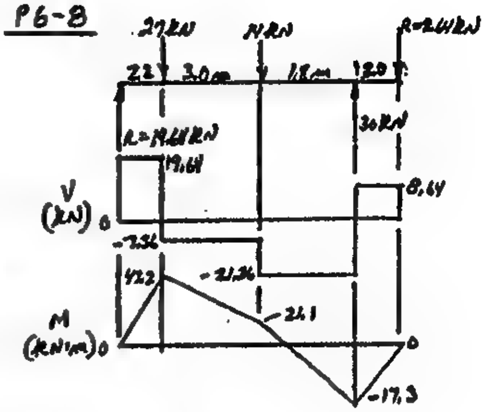

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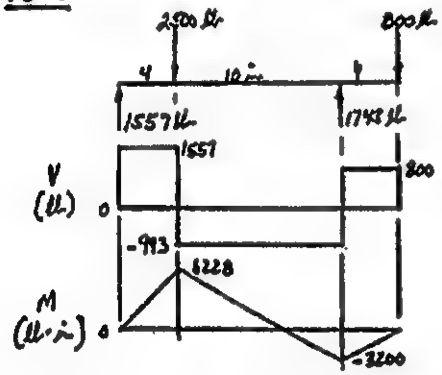

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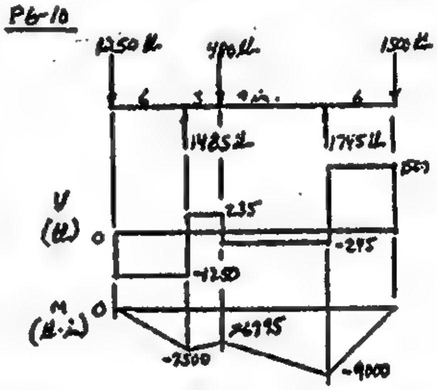

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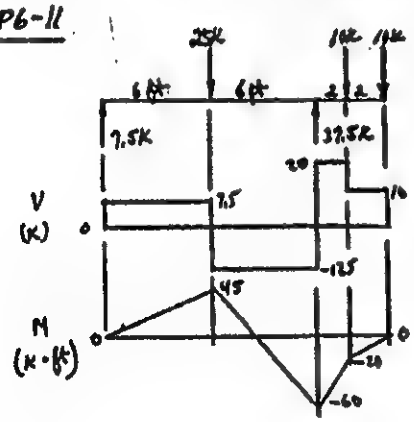

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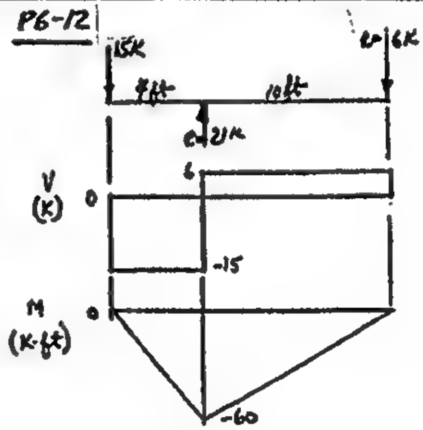

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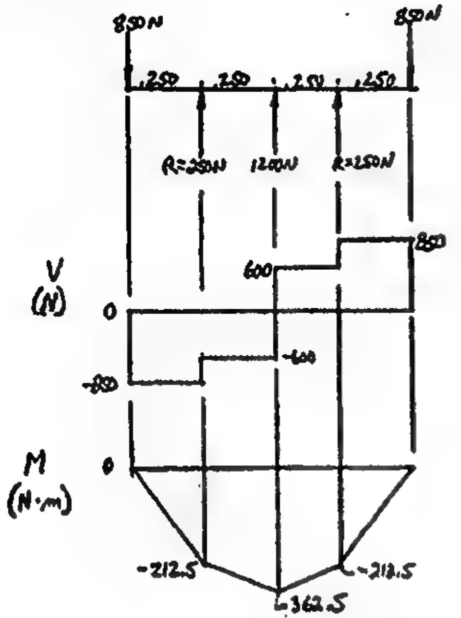

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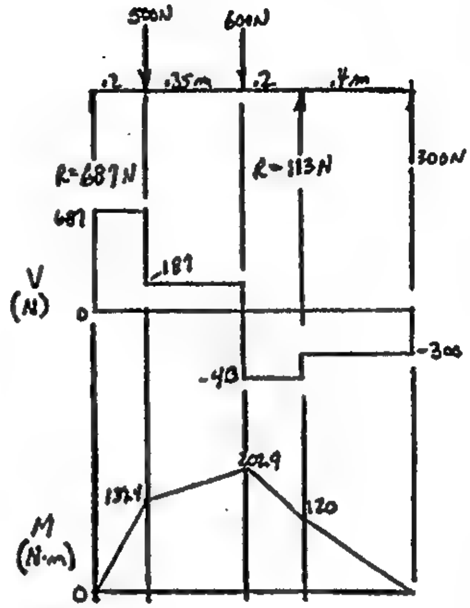

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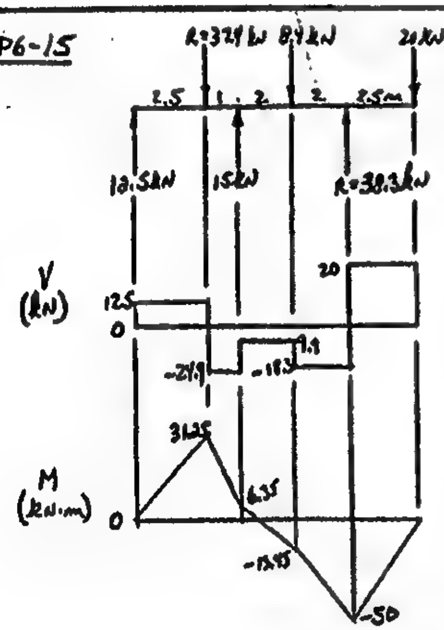

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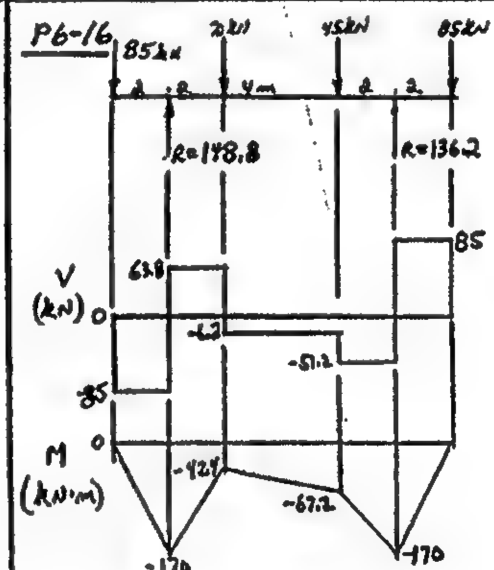

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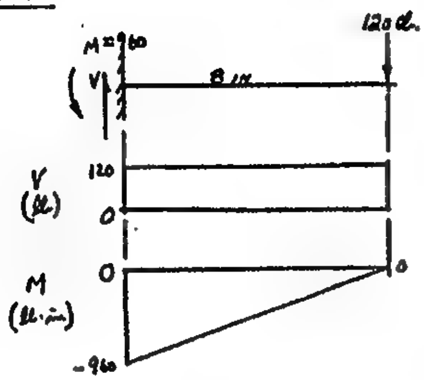

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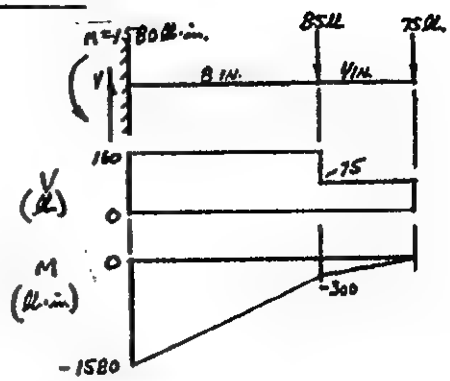

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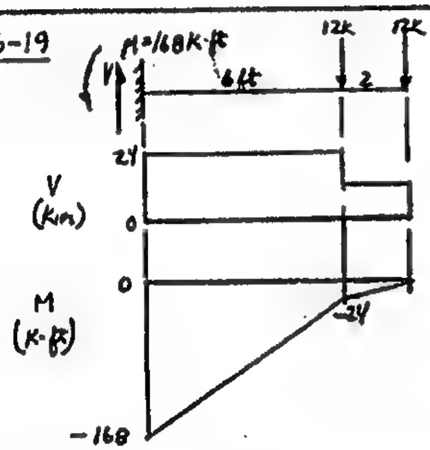

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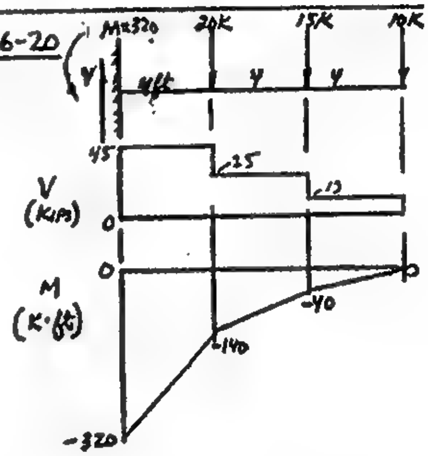

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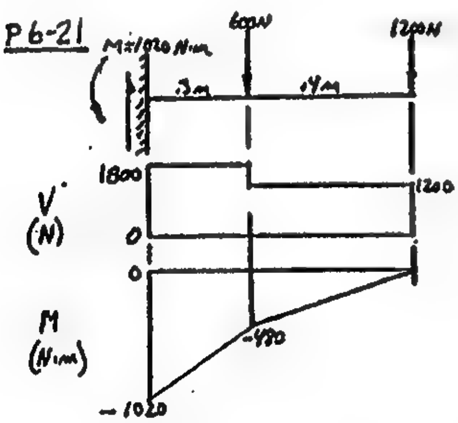

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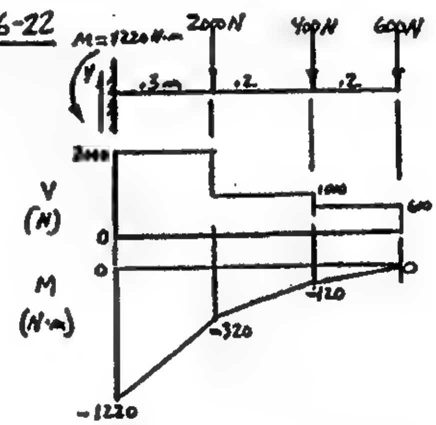

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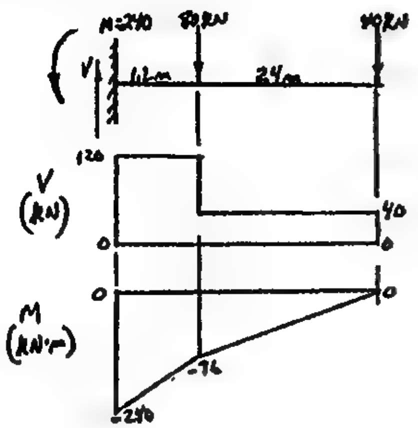

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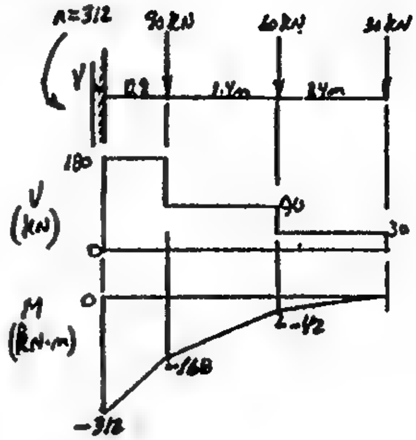

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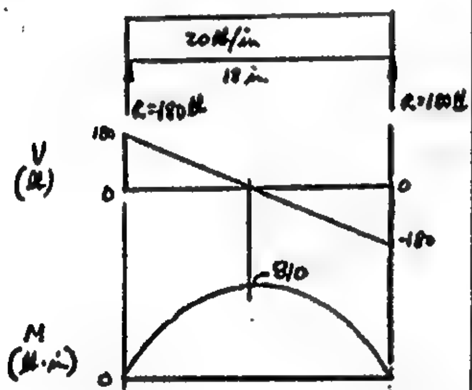

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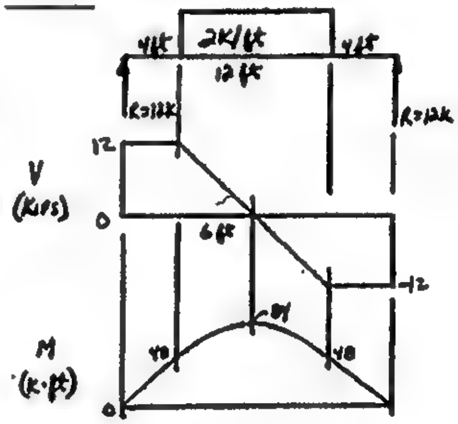

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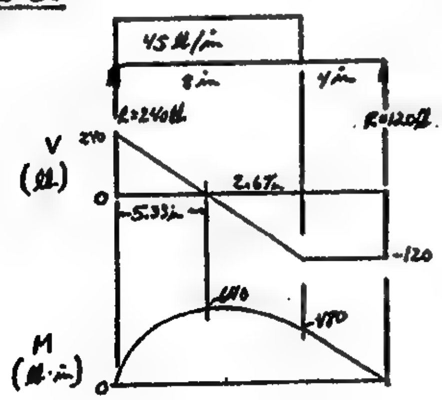

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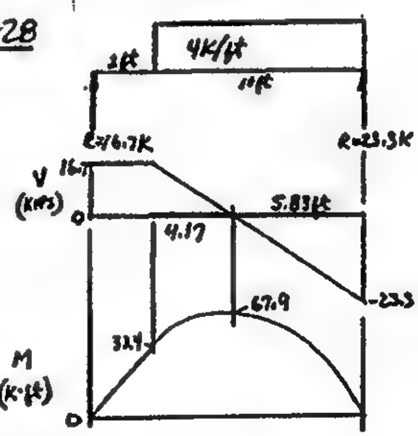

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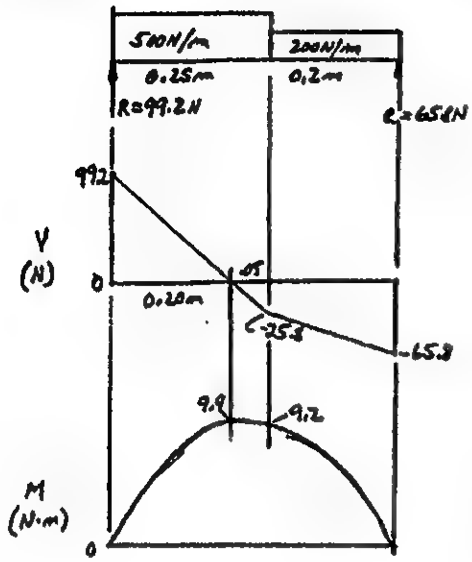

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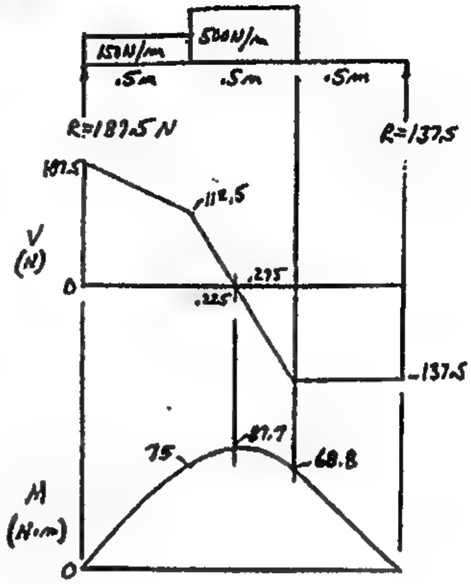

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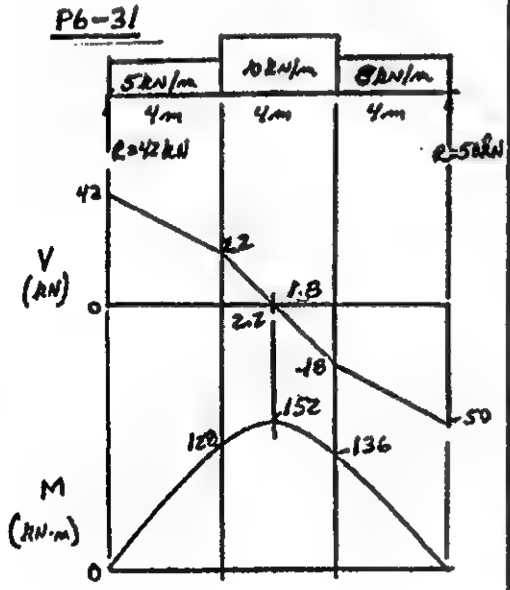

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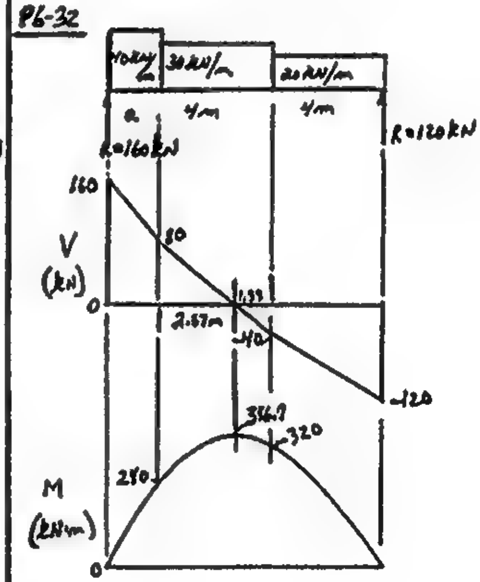

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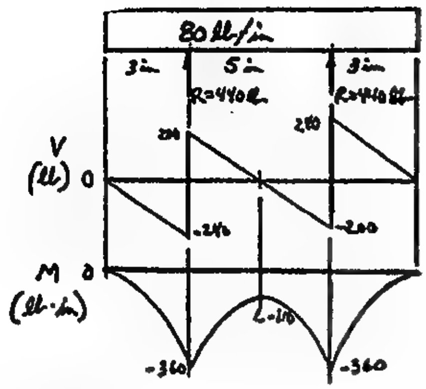

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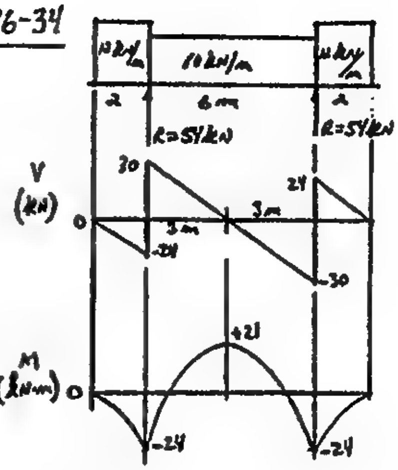

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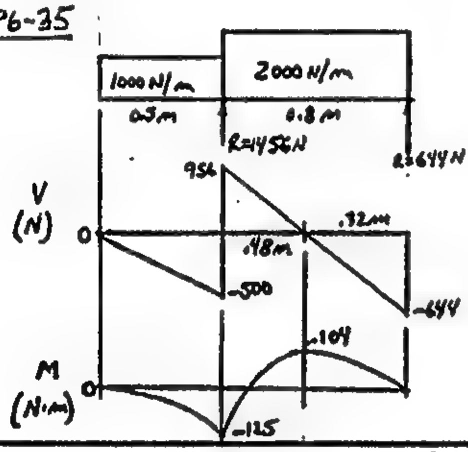

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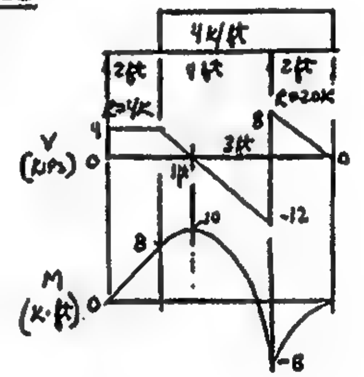

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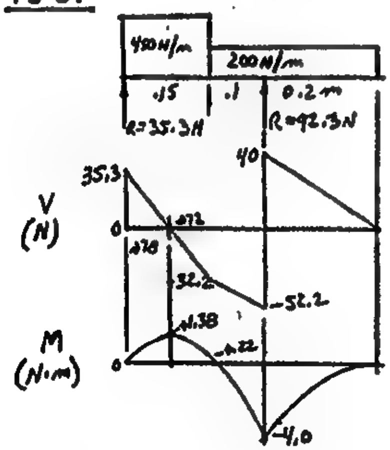

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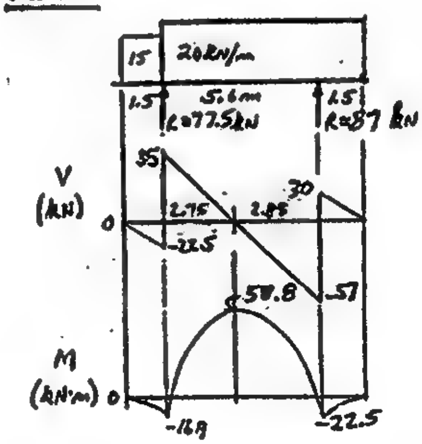

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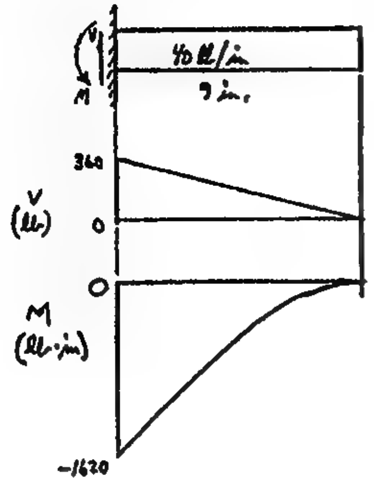

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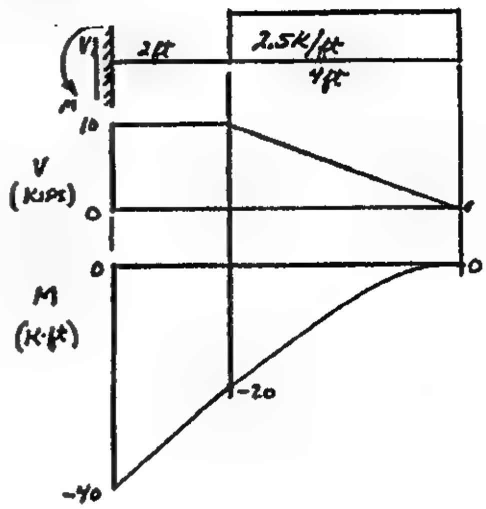

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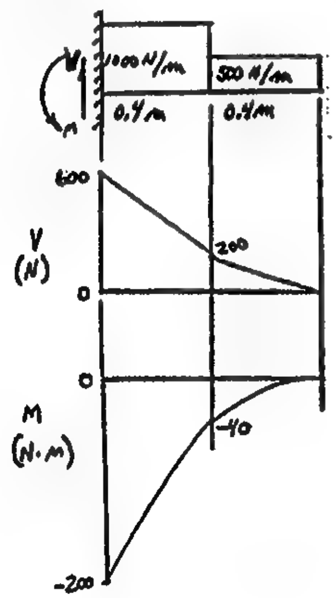

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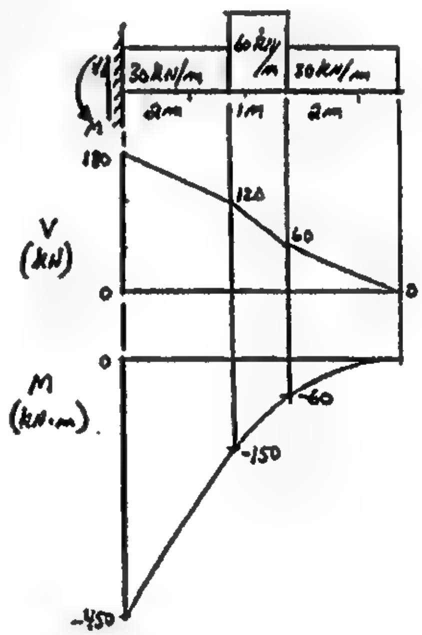

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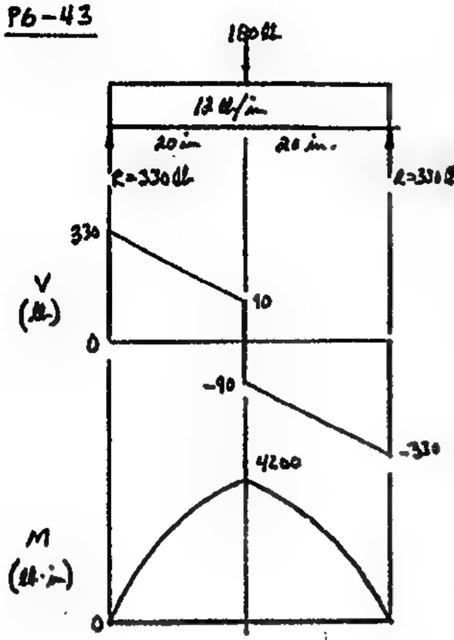

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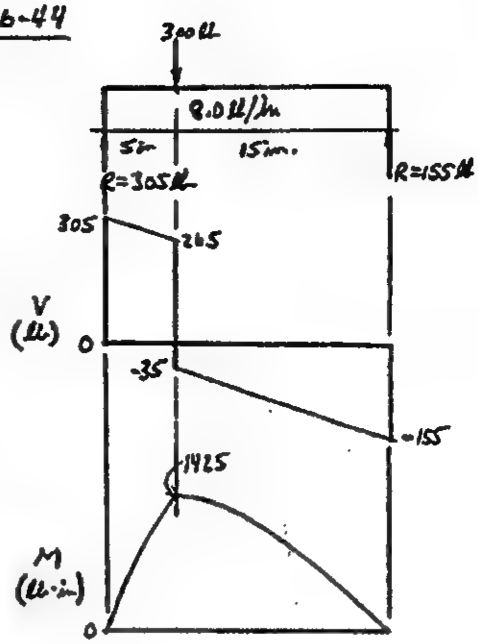

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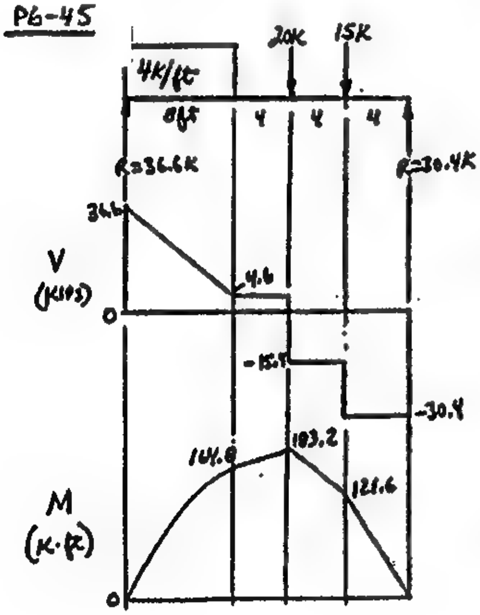

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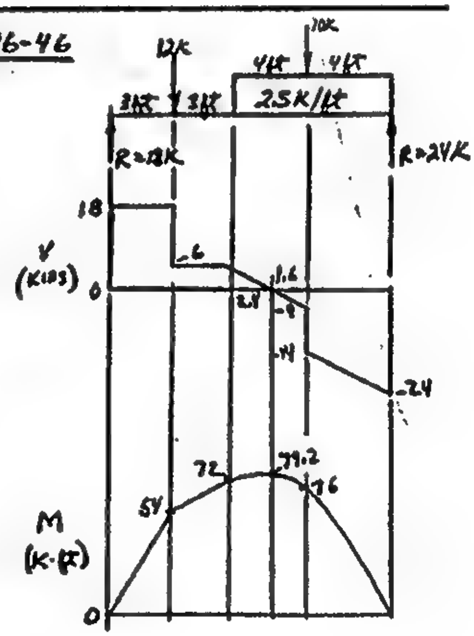

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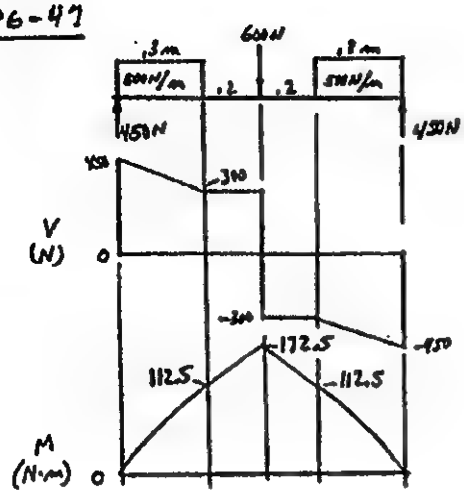

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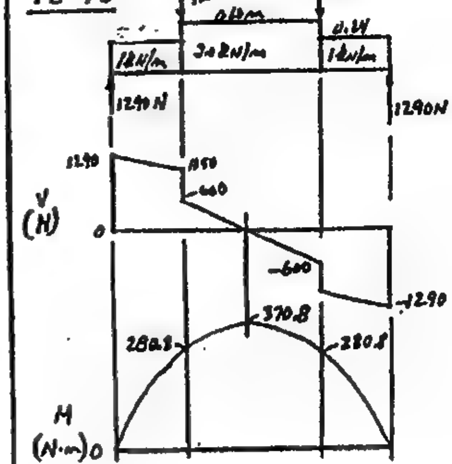

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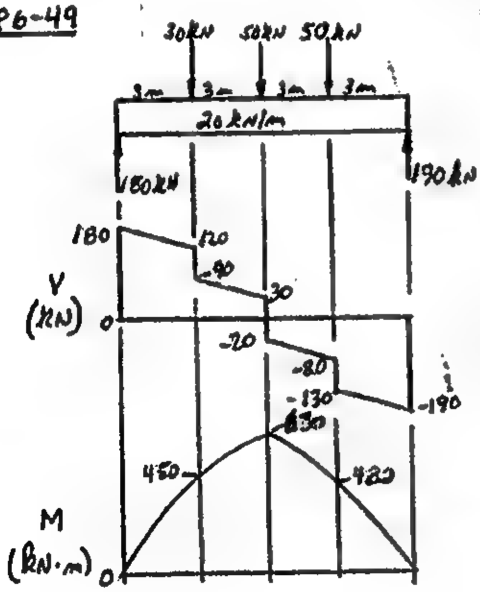

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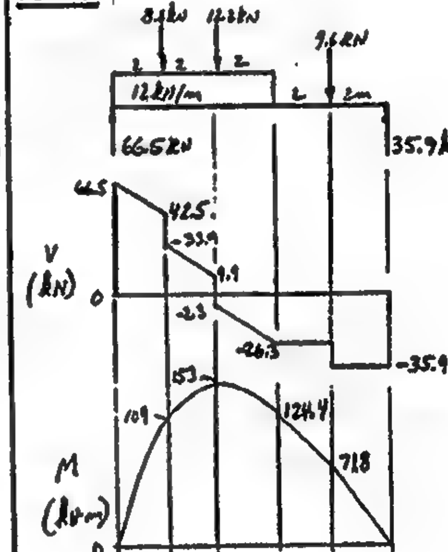

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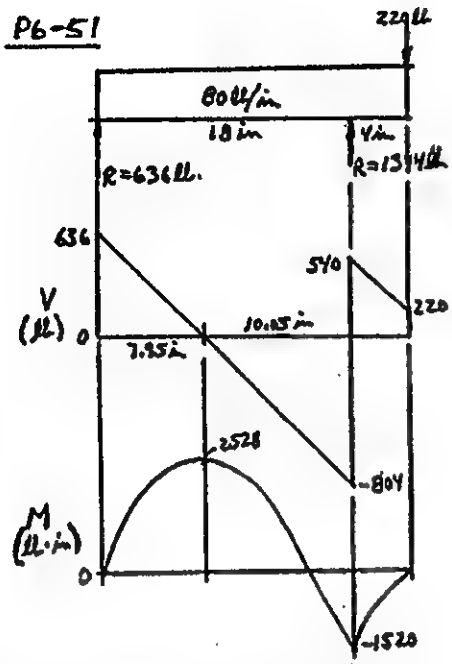

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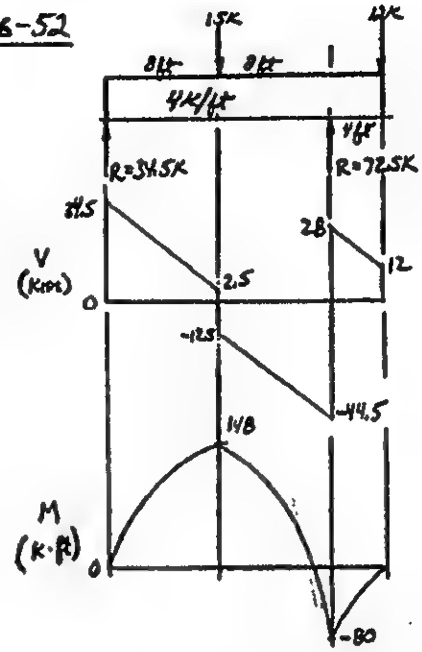

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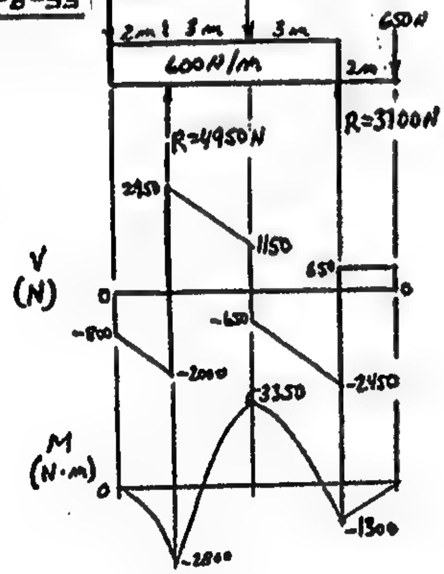

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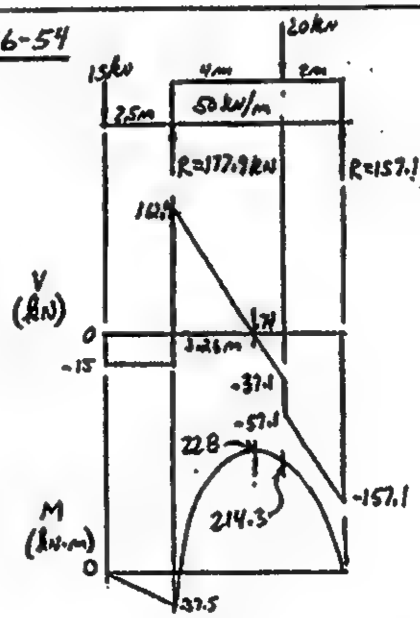
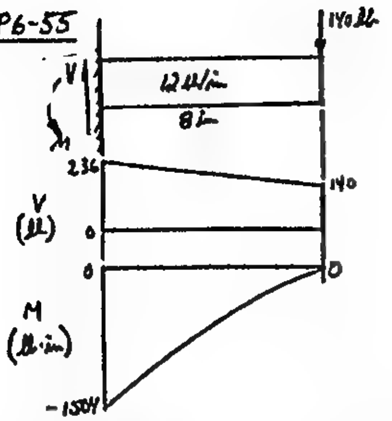
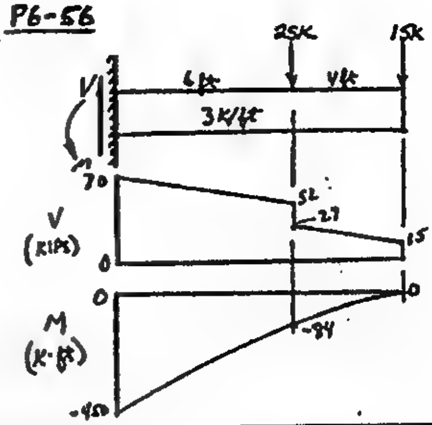


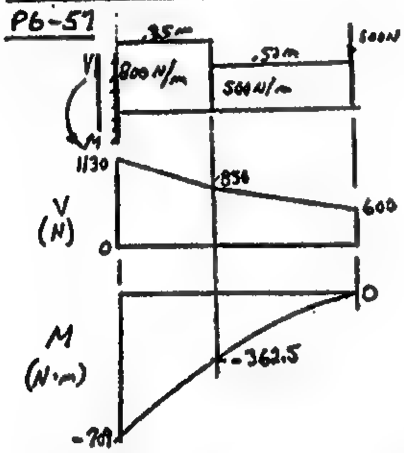
P6-55



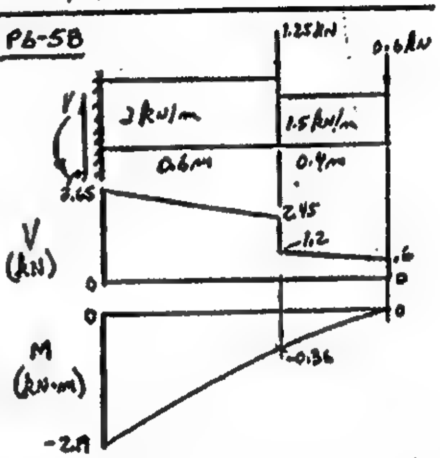
P6-56



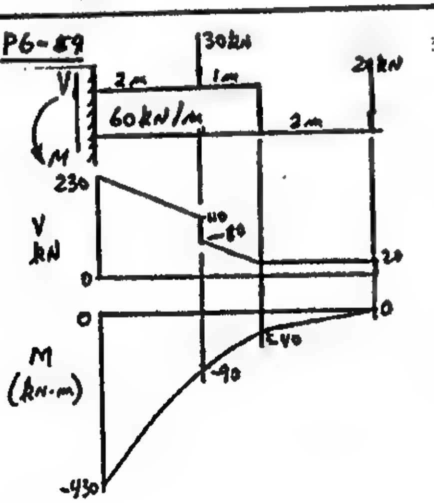
P6-57



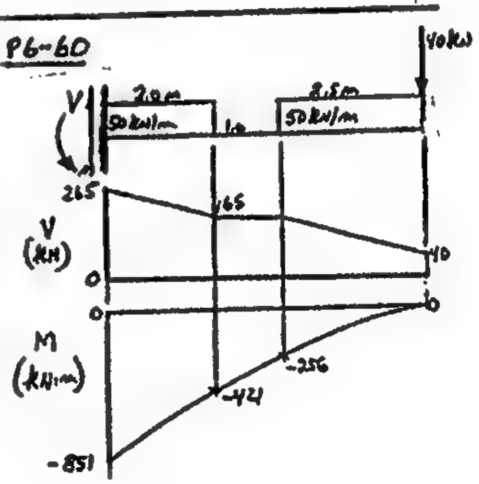
P6-58

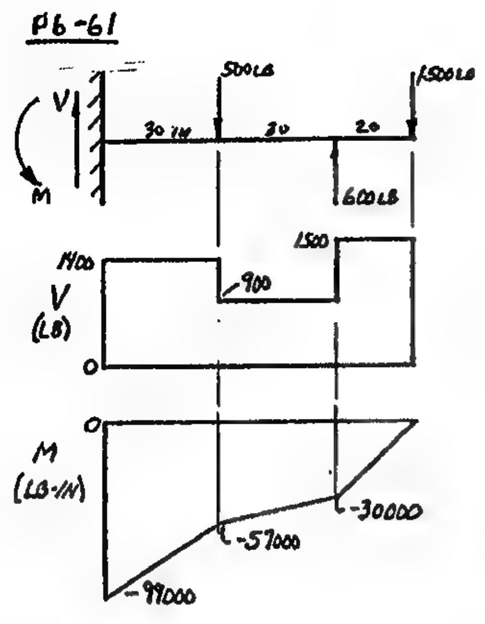


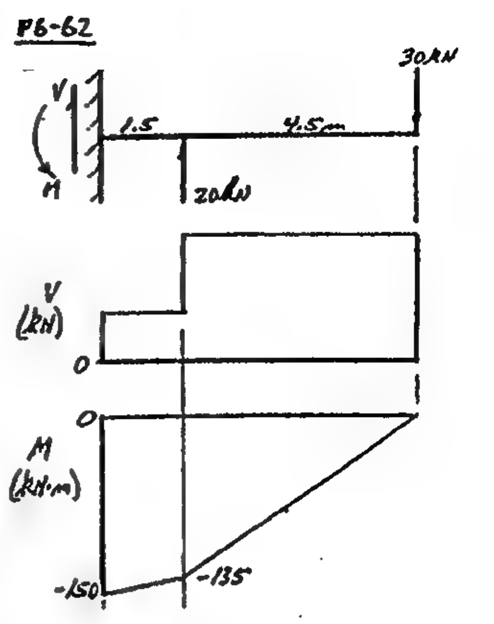
P6-59

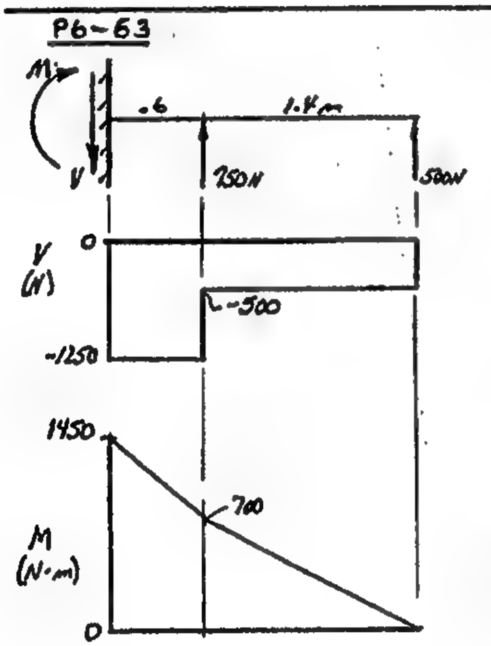


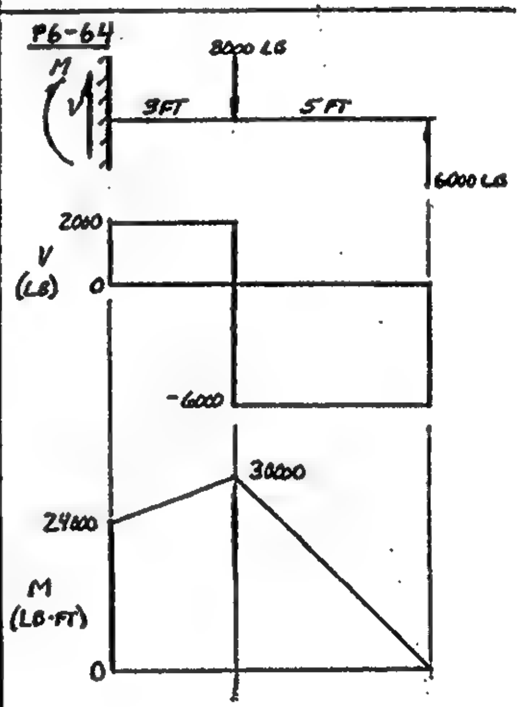
P6-60



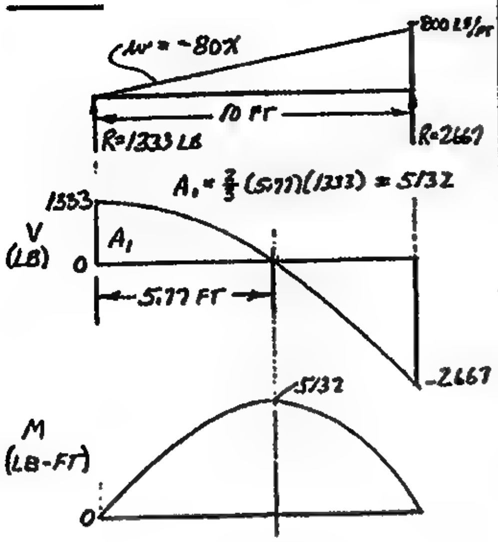

P6-61
500 LB
1500 LB
30 IN
30
20
V
M
600 LB
1400
900
1500
V (LB)
0
0
M (LB·IN)
-57000
-30000
-99000


P6-62
30 kN
1.5
4.5 m
V
M
20 kN
V (kN)
0
0
M (kN·m)
-150
-135

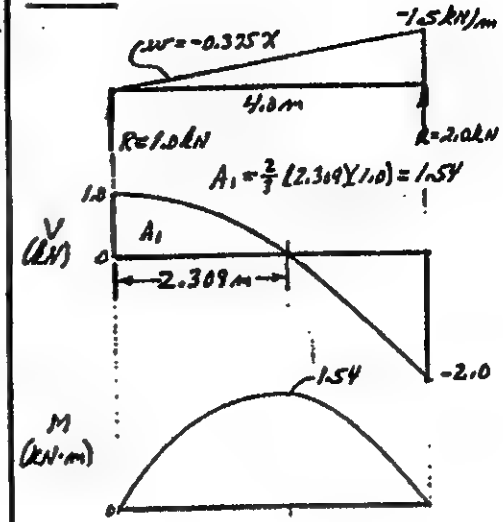

P6-63
M
.6
1.4 m
V
750 N
500 N
V (N)
0
-500
-1250
1450
700
M (N·m)
0


P6-64
8000 LB
M
V
3 FT
5 FT
6000 LB
2000
V (LB)
0
-6000
30000
24000
M (LB·FT)
0

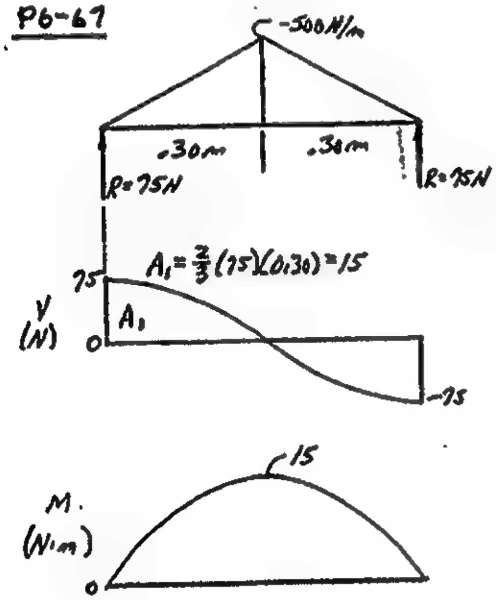
P6-65



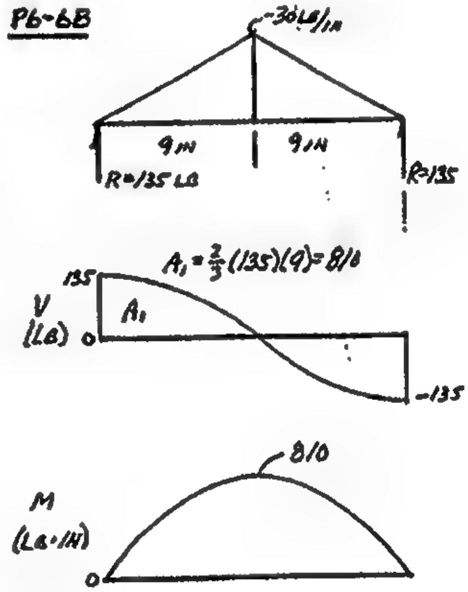
P6-66



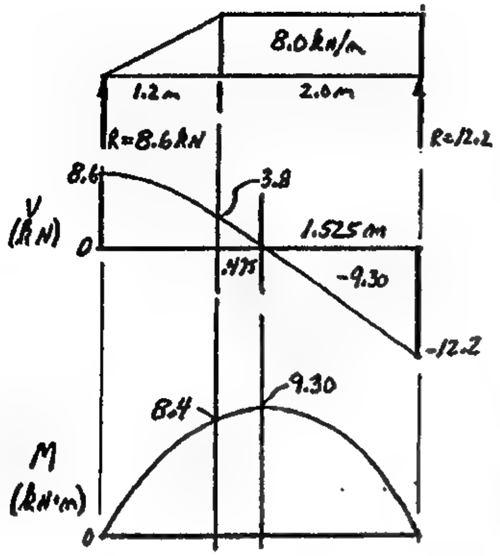
P6-67



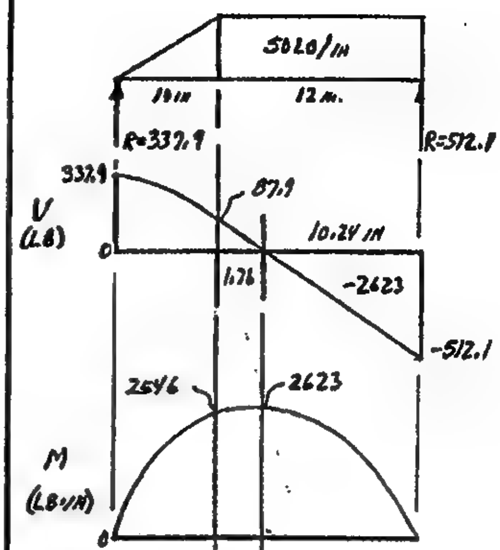
P6-68



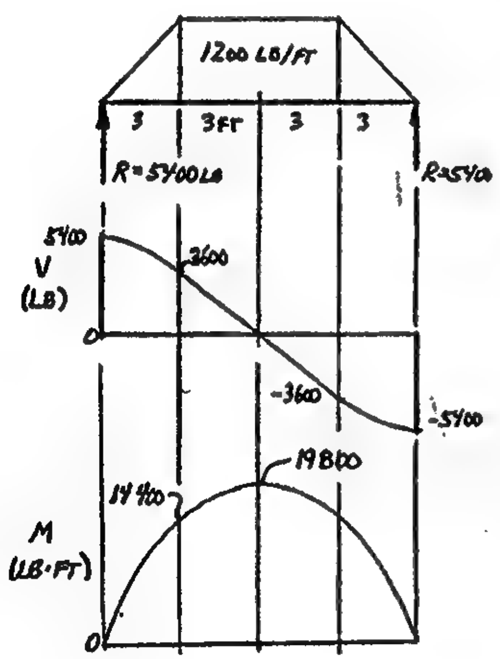
P6-69



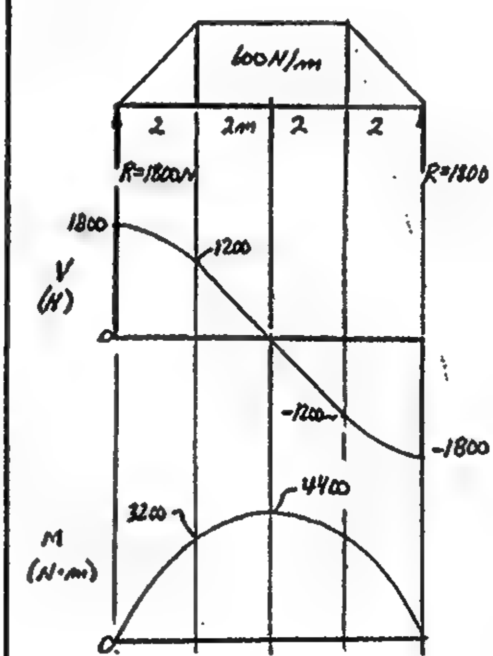
P6-70



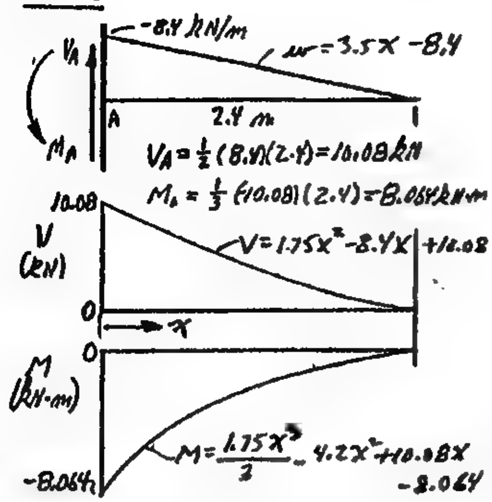
P6-71



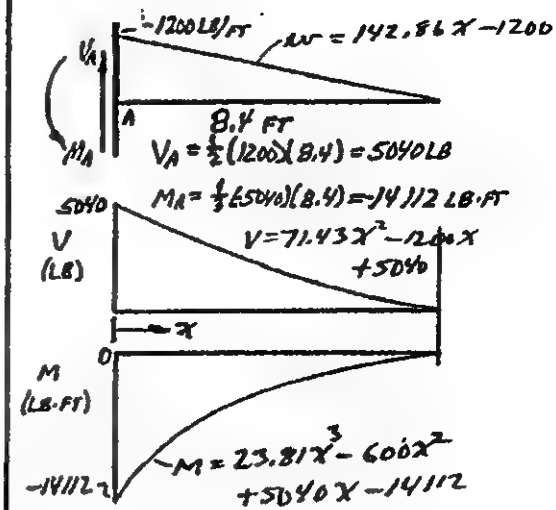
P6-72



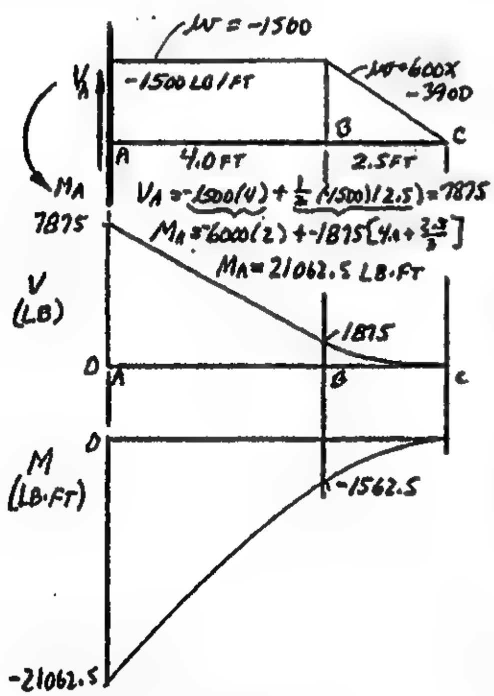
P6-73



P6-74



P6-75



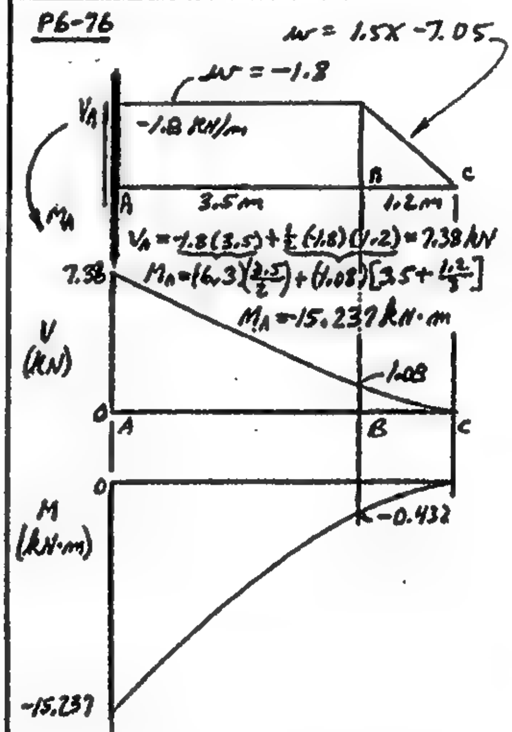
$V_{AB} = -1500x + 7875$

$M_{AB} = -750x^2 + 7875x - 21062$

$V_{BC} = 300x^2 - 3900x + 12675$

$M_{BC} = 100x^3 - 1950x^2 + 12675x - 27462.5$

P6-76



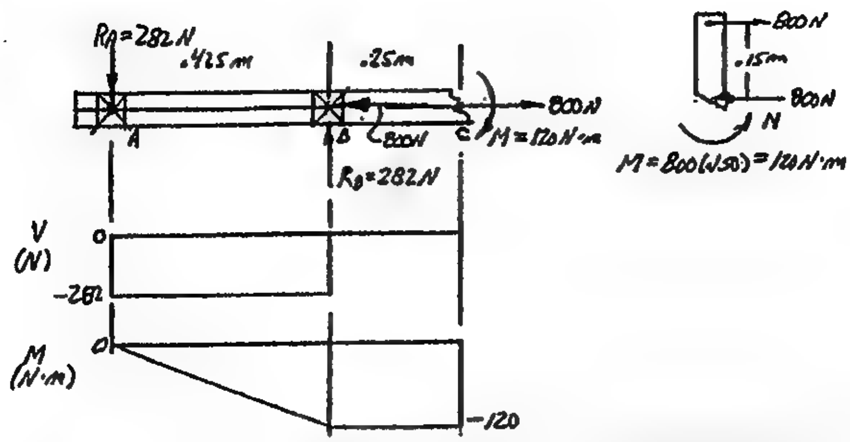
$V_{AB} = -1.8x + 7.38$

$M_{AB} = -0.9x^2 + 7.38x - 15.237$

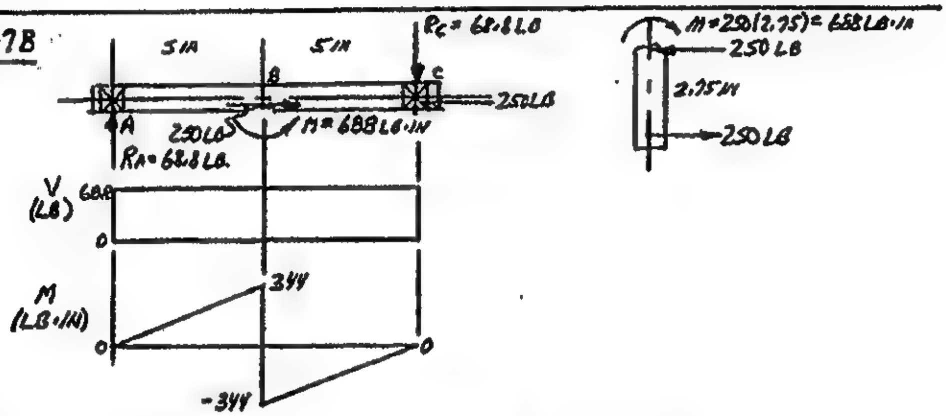
$V_{BC} = 0.75x^2 - 7.05x + 16.57$

$M_{BC} = 0.25x^3 - 3.525x^2 + 16.57x - 25.96$

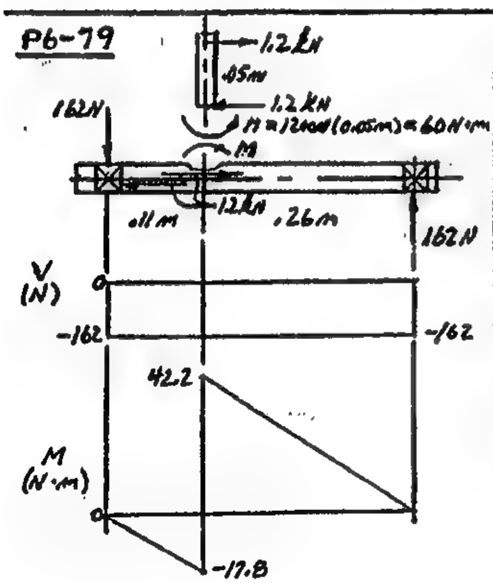
P6-77



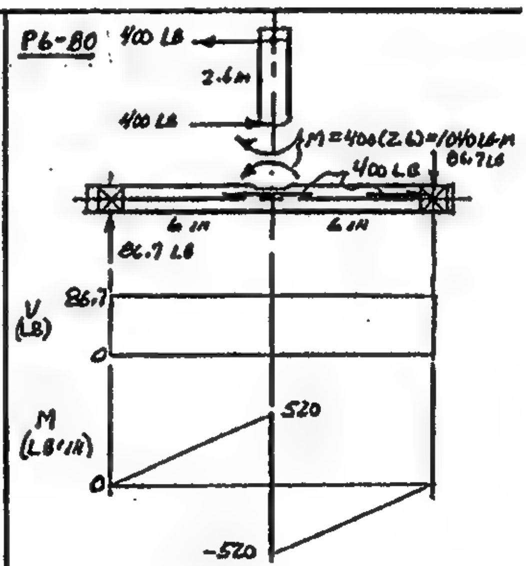
P6-78



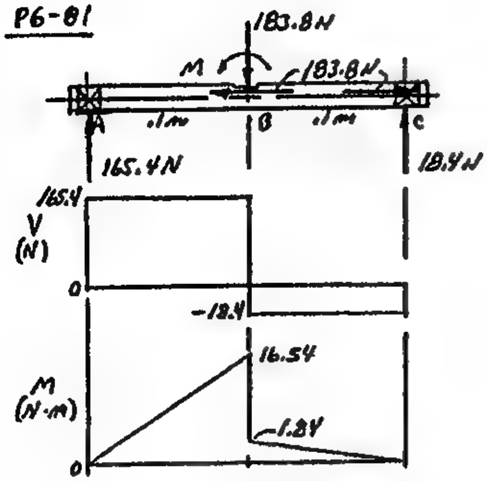
P6-79

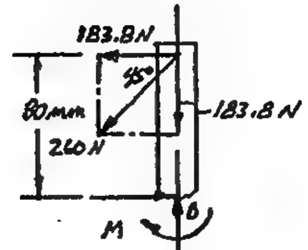


P6-80



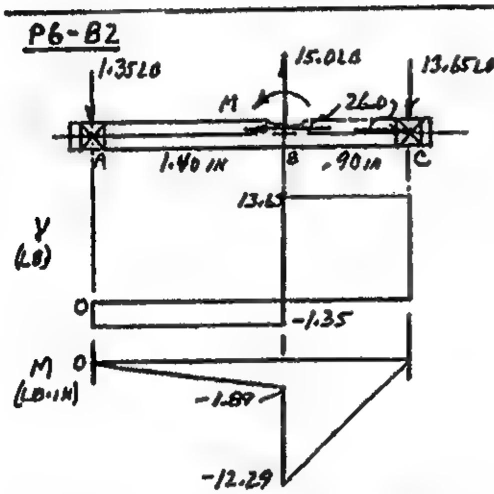
P6-81

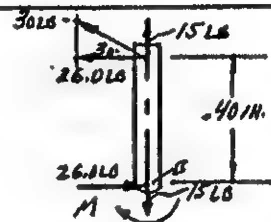




$M = 183.8N(0.08m) = 14.7 N\cdot m$

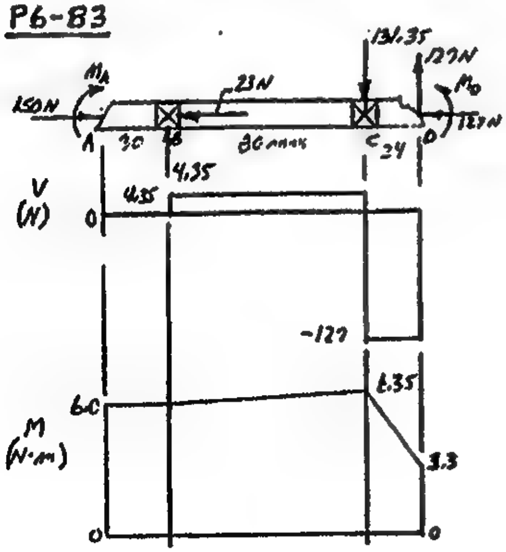
P6-82

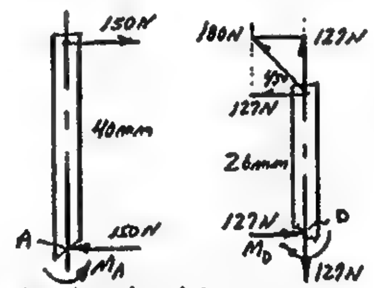




$M = (26.0)(0.40) = 10.4 LB\cdot IN$

P6-83

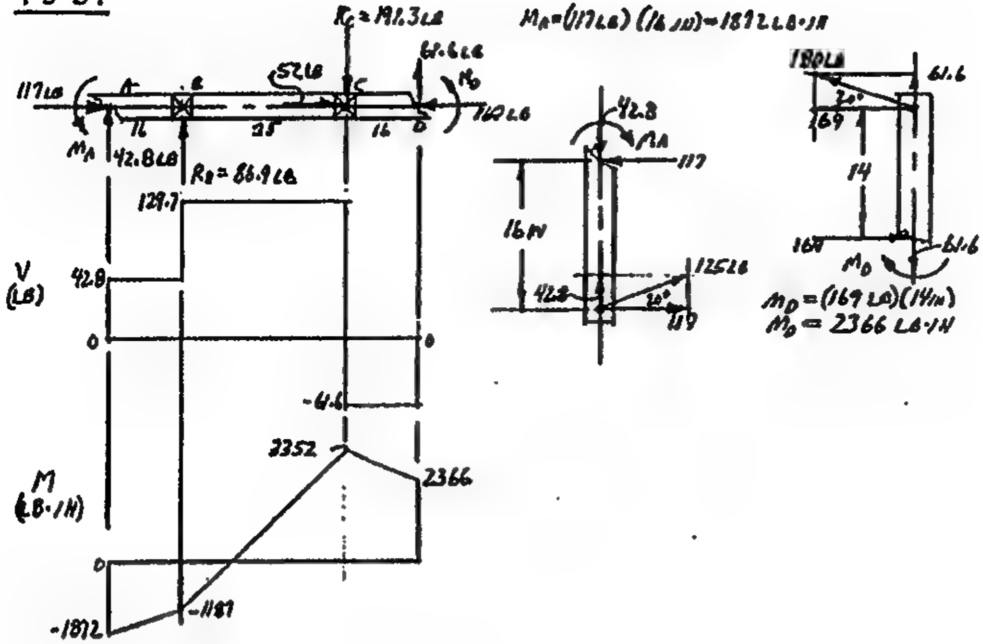




$M_A = 150N(.04m) = 6.0 N\cdot m$

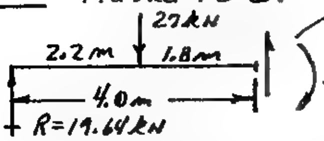
$M_D = 129N(.026m) = 3.3 N\cdot m$

P6-84



$R_C = 191.3$ LB
$M_A = (117\text{ LB})(16\text{ IN}) = 1872$ LB·IN
117 LB
52 LB
61.6 LB
M_D
160 LB
M_A
42.8 LB
$R_B = 86.9$ LB
16
25
16
A
B
C
D
V (LB)
M (LB·IN)
129.7
42.8
0
-61.6
2352
2366
-1872
-1187
42.8
117
16 IN
125 LB
20°
180 LB
61.6
169
14
160
M_D
$M_D = (169\text{ LB})(14\text{ IN})$
$M_D = 2366$ LB·IN

P6-85 FIGURE P6-8.

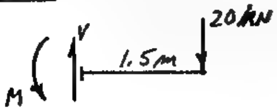


$\underline{V} = 19.64 - 27.0 = \underline{-7.36\,kN}$

$M = (19.64\,kN)(4.0\,m) - (27.0\,kN)(1.8\,m)$

$\underline{M = 29.96\,kN\cdot m}$

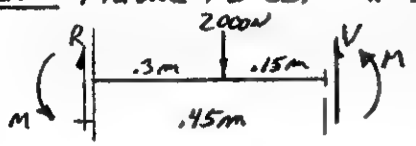
P6-86 FIGURE P6-15. USE PART OF BEAM TO RIGHT OF CUT SECTION.



$\underline{V = 20\,kN}$

$\underline{M} = (-20\,kN)(1.50\,m) = \underline{-30\,kN\cdot m}$

P6-87 FIGURE P6-22. $R = 3000\,N$; $M = -1220\,N\cdot m$

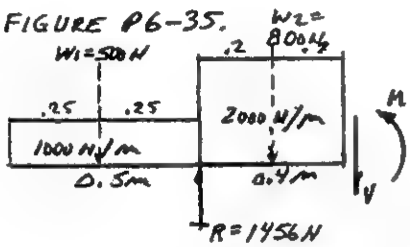


$\underline{V} = 3000N - 2000N = \underline{1000N}$

$M = -1220\,N\cdot m + 3000N(0.45m) - 2000N(0.15m)$

$\underline{M = -170\,N\cdot m}$

P6-88 FIGURE P6-35.

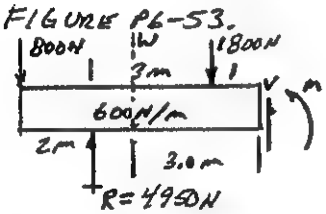


$\underline{V} = 1456N - 500N - 800N = \underline{56N}$

$M = (1456N)(0.4m) - (500N)(.65m) - (800N)(0.2m)$

$\underline{M = 97.4\,N\cdot m}$

P6-89 FIGURE P6-53.



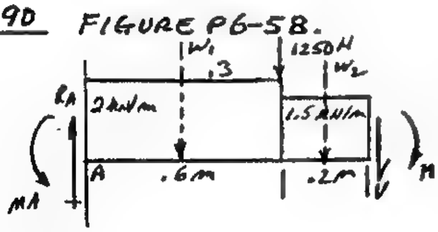
$w = (600\,N/m)(6.0\,m) = 3600N$

$\underline{V} = 4950 - 800 - 3600 - 1800 = \underline{-1250\,N}$

$M = (4950)(4) - 800(6) - 1800(1) - 3600(3)$

$\underline{M = 2400\,N\cdot m}$

P6-90 FIGURE P6-58.



$W_1 = 2\,kN/m(.6m) = 1.2\,kN$

$W_2 = 1.5\,kN/m(0.2m) = 0.30\,kN$

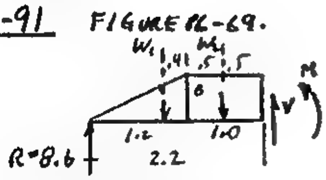
$R_A = 3.65\,kN$, $M_A = -2.19\,kN\cdot m$

$\underline{V} = 3.65 - 1.2 - 1.25 - 0.30 = \underline{0.90\,kN}$

$M = -2.19 + 3.65(0.8) - 1.2(0.5) - 0.3(0.1) - 1.25(0.2)$

$\underline{M = -0.15\,kN\cdot m}$

P6-91 FIGURE P6-69.

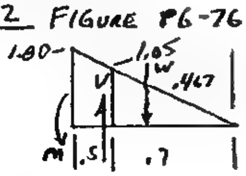


$W_1 = (.5)(8\,kN/m)(1.2m) = 4.8\,kN$; $W_2 = (8)(1) = 8\,kN$

$\underline{V} = 8.6 - 4.8 - 8.0 = \underline{-4.2\,kN}$

$\underline{M} = 8.6(2.2) - 4.8(1.4) - 8.0(0.5) = \underline{8.2\,kN\cdot m}$

P6-92 FIGURE P6-76. USE RIGHT PART FOR FBD.

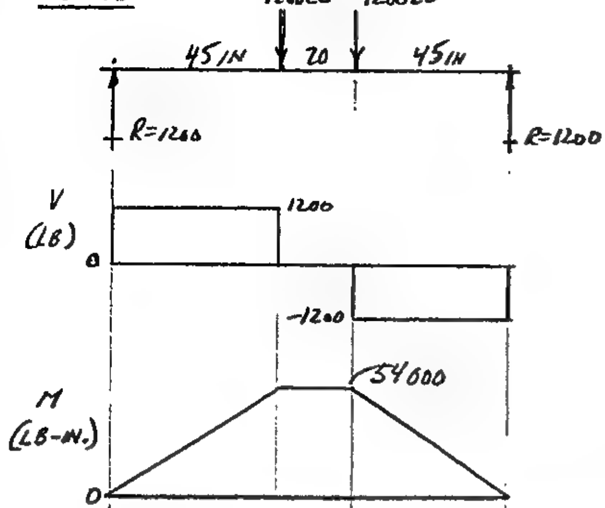


$w = (0.5)(1.05\,kN/m)(0.7m) = 0.377\,kN$

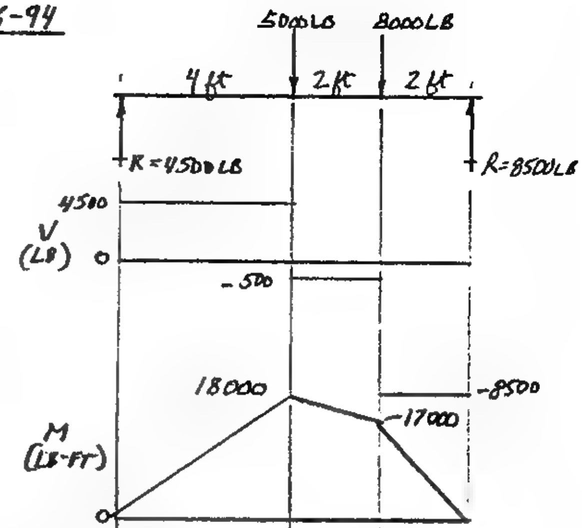
AT CUT: $\underline{V = w = 0.377\,kN}$

$\underline{M} = -w(0.7 - 0.467) = (0.377\,kN)(.233) = \underline{-0.0878\,kN\cdot m}$

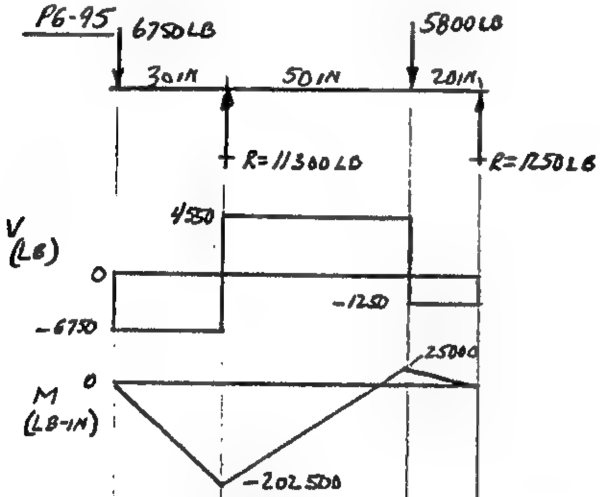
P6-93



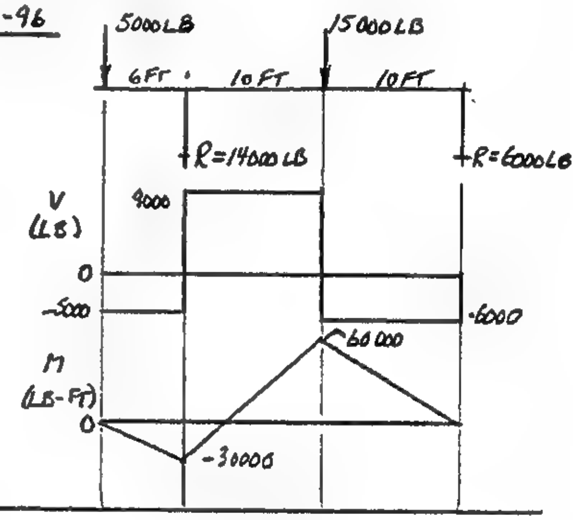
P6-94



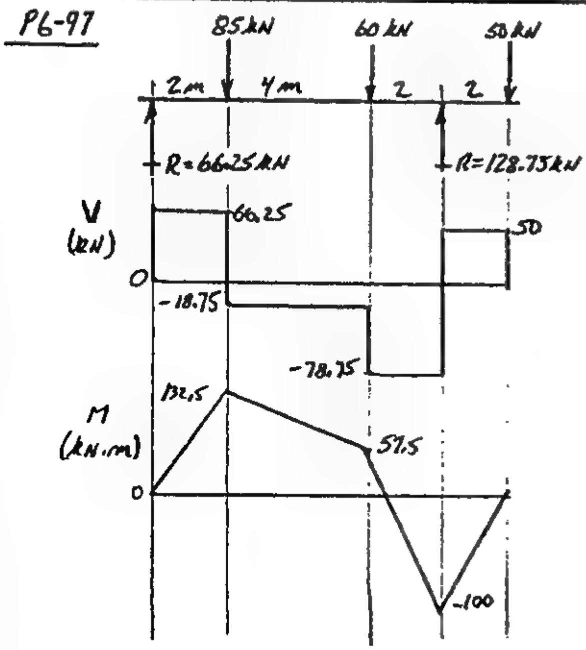
P6-95



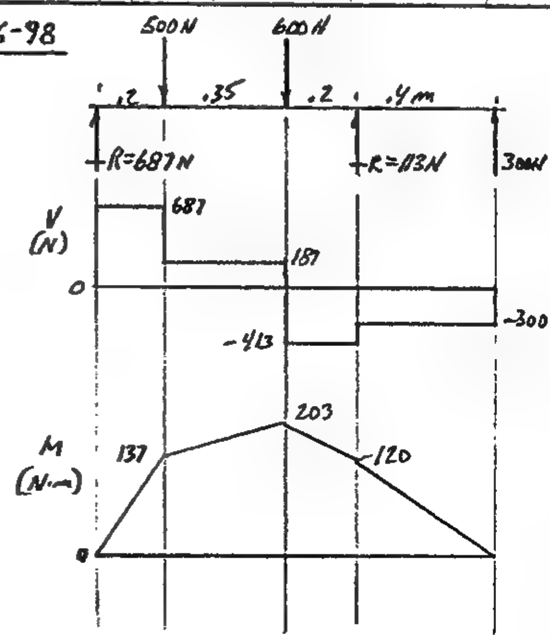
P6-96



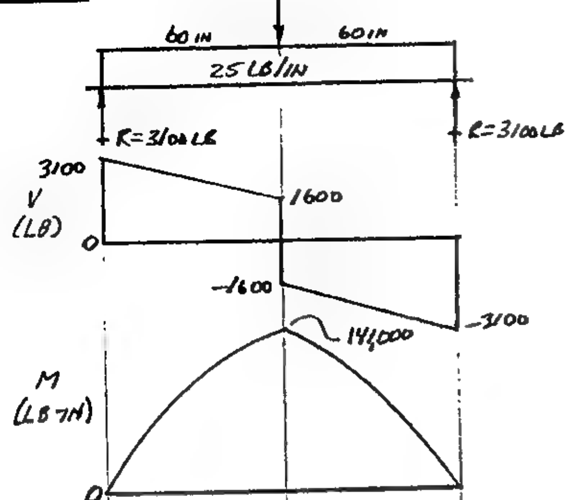
P6-97



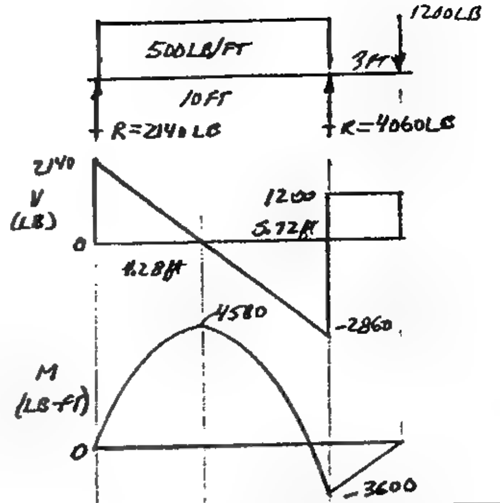
P6-98



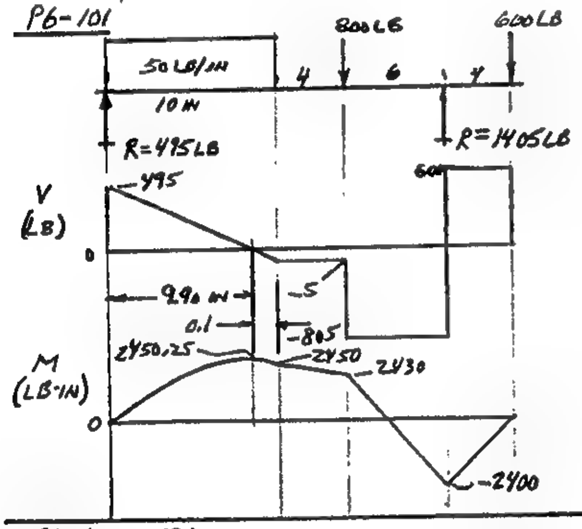
P6-99



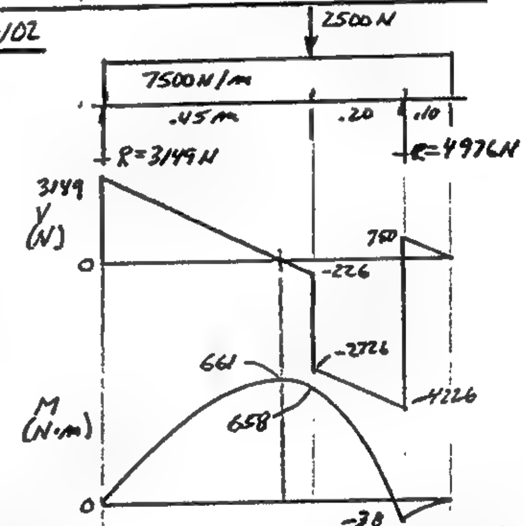
P6-100



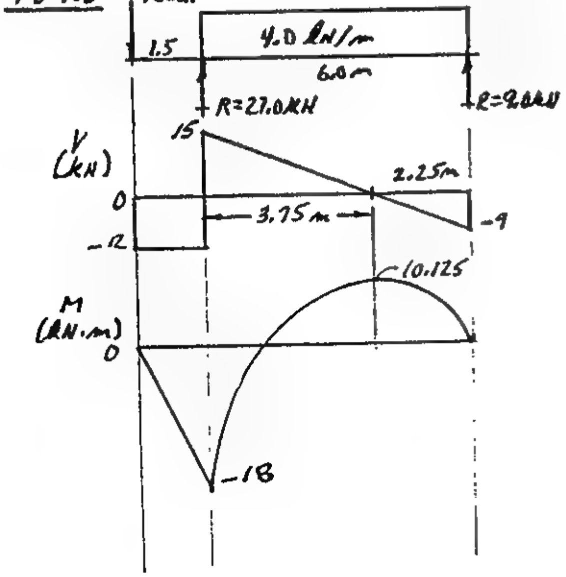
P6-101



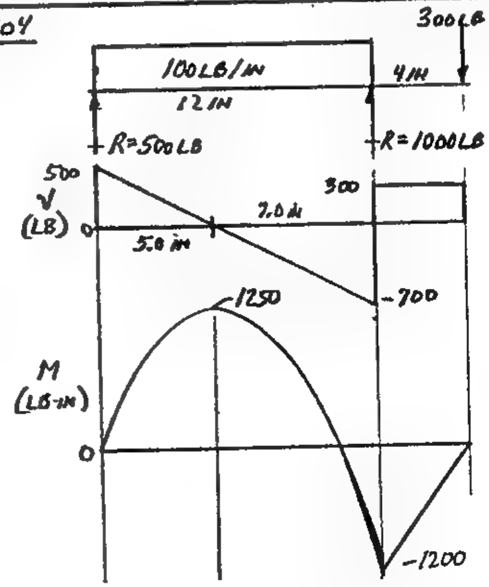
P6-102



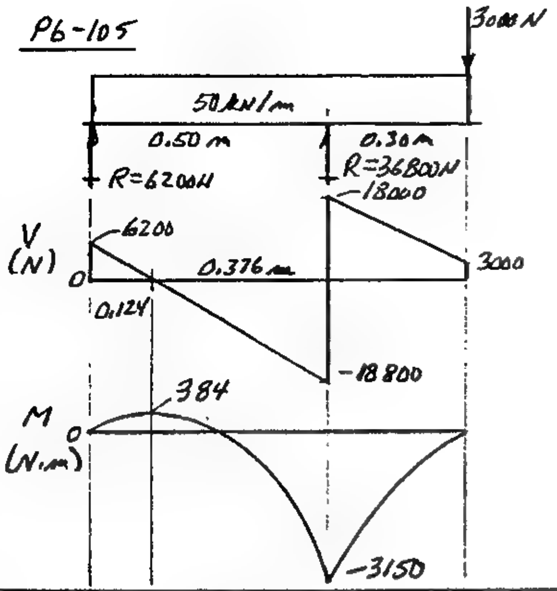
P6-103



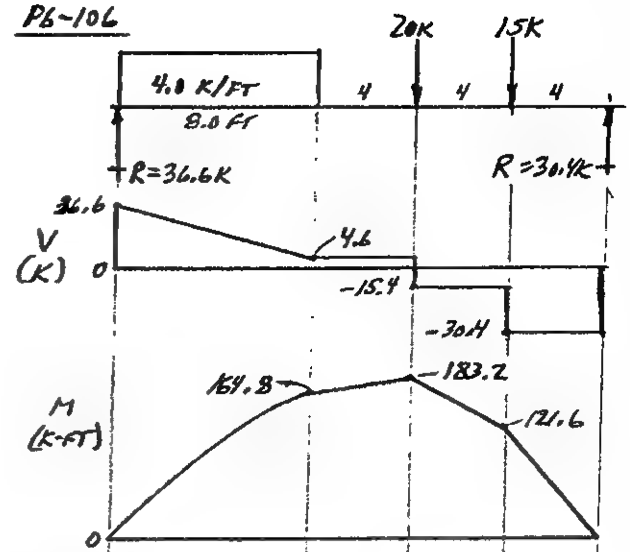
P6-104



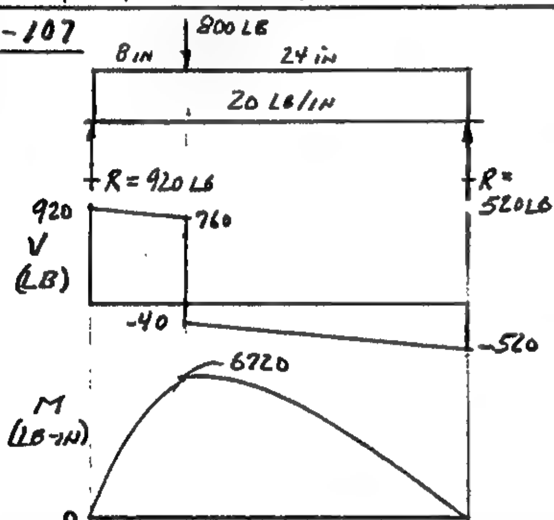
P6-105



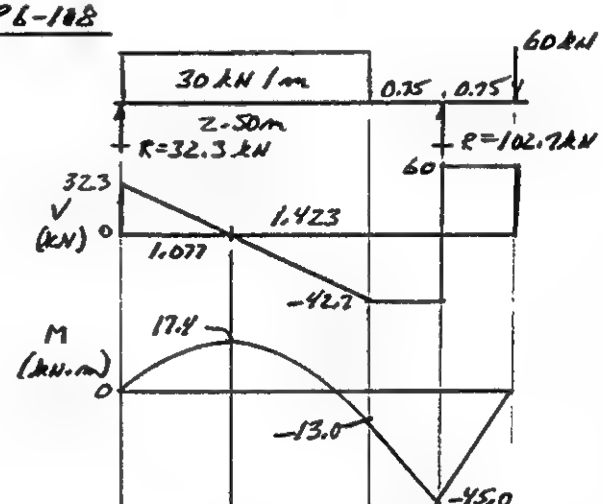
P6-106



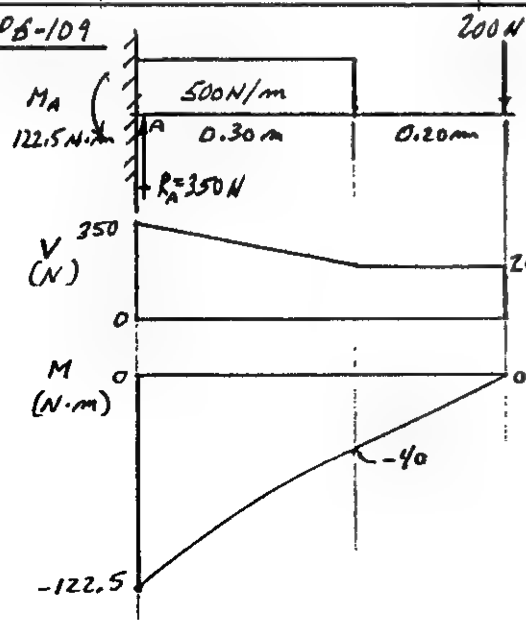
P6-107



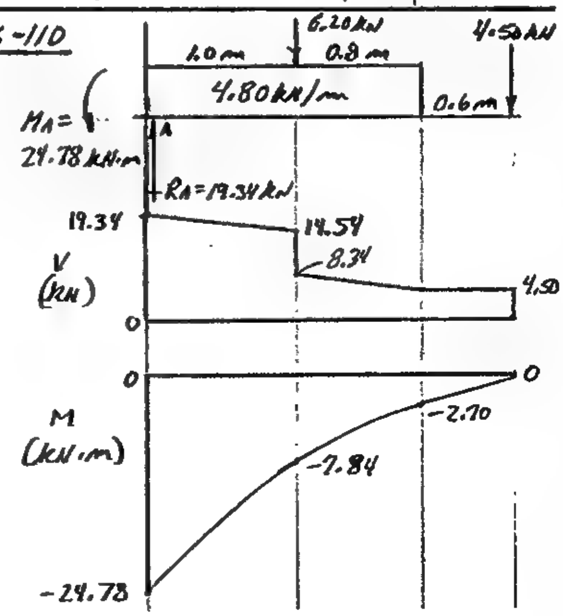
P6-108



P6-109



P6-110



CHAPTER 7 Centroids and Moments of Inertia of Areas

Notes concerning the format of solutions for Chapter 7 problems:

- Problem solutions for the moments of inertia of the shapes shown in Figures P7-1 through P7-40 are shown in the tabular format recommended in Section 7-6.
- Calculations were completed using a spreadsheet.
- The requested result includes the vertical *Y* distance to the centroidal axis from the reference axis and the moment of inertia *I* of the composite shape relative to the horizontal centroidal axis.
- In most problems, the reference axis for computing the location of the horizontal centroidal axis was taken as the base of the section. Exceptions are noted on the top or bottom lines of the solution. For example, in Figure P7-17, the reference axis is at the axis of symmetry at the mid-height of the shape, found by inspection.
- The left-most column of the solution gives a brief description of the part of the composite shape being analyzed.
- For some shapes, internal parts removed from the outer shape are shown to be negative.
- For composite shapes having parts that are commercially available structural shapes, pipes, or tubes, or wood beams, reference should be made to the Appendix tables for pertinent data.

FIGURE P7-1	Units:	Inches	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Vertical	0.5000	1.0000	0.5000	0.16670	0.3365	0.0566	0.2233	
2-Horizontal	0.3125	0.1250	0.0391	0.00163	0.5384	0.0906	0.0922	
Total area =	0.8125	Sum Ay=	0.5391			Total I =	0.3156	in^4
Y=	0.6635	in						

FIGURE P7-2	Units:	Inches	NOTE: 6x8 rectangle with 5x6 rectangle removed					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Total 6x8	48.00	4.00	192.00	256.00	0.00	0.00	256.00	
2-Void 5x6	-30.00	4.00	-120.00	-90.00	0.00	0.00	-90.00	
Total area =	18.00	Sum Ay=	72.00			Total I =	166.00	in^4
Y=	4.00	in	NOTE: Reference axis is base of the shape					

FIGURE P7-3	Units:	Inches	NOTE: 6x8 rectangle with 4x6 rectangle removed					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Total 6x8	48.00	4.00	192.00	256.00	0.00	0.00	256.00	
2-Void 4x6	-24.00	4.00	-96.00	-72.00	0.00	0.00	-72.00	
Total area =	24.00	Sum Ay=	96.00			Total I =	184.00	in^4
Y=	4.00	in	NOTE: Reference axis is base of the shape					

FIGURE P7-4	Units:	mm	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Vertical	5000	100	5.000E+05	1.667E+07	52.50	1.38E+07	3.04E+07	
2-Horizontal	4375	213	9.297E+05	2.279E+05	60.00	1.58E+07	1.60E+07	
Total area =	9375	Sum Ay=	1.430E+06			Total I =	4.64E+07	mm^4
Y=	152.50	mm						

FIGURE P7-5	Units:	mm	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Vertical	250	30.0	7.500E+03	5.208E+04	5.00	6.25E+03	5.83E+04	
2-Horiz-bot	100	2.5	2.500E+02	2.080E+02	32.50	1.06E+05	1.06E+05	
3-Horiz-top	200	57.5	1.150E+04	4.170E+02	22.50	1.01E+05	1.02E+05	
Total area =	550	Sum Ay=	1.925E+04			Total I =	2.66E+05	mm^4
Y=	35.00	mm						

FIGURE P7-6	Units:	mm	NOTE: Both vertical rectangles (10x30) combined					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-20X30	600	15	9.000E+03	4.500E+04	2.50	3.75E+03	4.88E+04	
2-Hor-20X10	200	5	1.000E+03	1.667E+03	7.50	1.12E+04	1.29E+04	
Total area =	800	Sum Ay=	1.000E+04			Total I =	6.17E+04	mm^4
Y=	12.50	mm	NOTE: Reference axis is base of the shape					

FIGURE P7-7	Units:	mm	NOTE: Entire vertical stem; 2 horiz. flanges each 5x15					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-5X40	200	20.0	4.000E+03	2.667E+04	0.00	0E+00	2.67E+04	
2-Horiz-bot	75	2.5	1.875E+02	1.562E+02	17.50	2.30E+04	2.31E+04	
3-Horiz-top	75	37.5	2.812E+03	1.562E+02	17.50	2.30E+04	2.31E+04	
Total area =	350	Sum Ay=	7.000E+03			Total I =	7.29E+04	mm^4
Y=	20.00	mm	NOTE: Reference axis is base of the shape					

FIGURE P7-8	Units:	mm	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-5X40	200	20.0	4.000E+03	2.667E+04	0.00	0E+00	2.67E+04	
2-Ver-5x40	200	20.0	4.000E+03	2.667E+04	0.00	0E+00	2.67E+04	
3-Hor-30x5	150	20.0	3.000E+03	3.125E+02	0.00	0E+00	3.12E+02	
Total area =	550	Sum Ay=	1.100E+04			Total I =	5.36E+04	mm^4
Y=	20.00	mm						

FIGURE P7-9	Units:	mm	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-5X30	150	20.0	3.000E+03	1.125E+04	0.00	0E+00	1.12E+04	
2-Horiz-bot	200	2.5	5.000E+02	4.167E+02	17.50	6.12E+04	6.17E+04	
3-Horiz-top	200	37.5	7.500E+03	4.167E+02	17.50	6.12E+04	6.17E+04	
Total area =	550	Sum Ay=	1.100E+04			Total I =	1.35E+05	mm^4
Y=	20.00	mm						

FIGURE P7-10	Units:	mm	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-5X50	250	30.0	7.500E+03	5.208E+04	0.00	0E+00	5.21E+04	
2-Horiz-bot	140	2.5	3.500E+02	2.917E+02	27.50	1.06E+05	1.06E+05	
3-Horiz-top	140	57.5	8.050E+03	2.917E+02	27.50	1.06E+05	1.06E+05	
Total area =	530	Sum Ay=	1.590E+04			Total I =	2.64E+05	mm^4
Y=	30.00	mm						

FIGURE P7-11 Units: mm NOTE: Both vert. combined 10x45; horiz. flanges combined 5x30

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-10X45	450	22.5	1.012E+04	7.594E+04	0.69	2.14E+02	7.62E+04	
2-Hor-5x30	150	2.5	3.750E+02	3.125E+02	19.31	5.59E+04	5.62E+04	
3-Hor-5x25	125	42.5	5.312E+03	2.604E+02	20.69	5.35E+04	5.38E+04	
Total area =	725	Sum Ay=	1.581E+04			Total I =	1.86E+05	mm^4
Y=	21.81	mm	NOTE: Reference axis is base of the shape					

FIGURE P7-12 Units: mm NOTE: All verticals massed together

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-16X16	256	8	2.048E+03	5.461E+03	4.39	4.92E+03	1.04E+04	
2-Hor-4X50	200	18	3.600E+03	2.667E+02	5.61	6.30E+03	6.57E+03	
Total area =	456	Sum Ay=	5.648E+03			Total I =	1.70E+04	mm^4
Y=	12.39	mm	NOTE: Reference axis is base of the shape					

FIGURE P7-13 Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Hor-5x10	50	2.5	1.250E+02	1.045E+02	20.83	2.17E+04	2.18E+04	
2-Ver-5X55	275	27.5	7.562E+03	6.932E+04	4.17	4.77E+03	7.41E+04	
3-Hor-5x20	100	27.5	2.750E+03	2.083E+02	4.17	1.74E+03	1.94E+03	
4-Ver-5x30	150	15.0	2.250E+03	1.125E+04	8.33	1.04E+04	2.17E+04	
5-Hor-5x5	25	52.5	1.312E+03	5.208E+01	29.17	2.13E+04	2.13E+04	
Total area =	600	Sum Ay=	1.400E+04			Total I =	1.41E+05	mm^4
Y=	23.33	mm						

FIGURE P7-14 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Bot plate	0.5200	0.1000	0.0520	0.00173	0.4330	0.0975	0.0992	
2-Bot flanges	0.1200	0.2500	0.0300	0.0001	0.2830	0.0096	0.0097	
3-2 Vert webs	0.3000	0.9500	0.2850	0.05625	0.4169	0.0522	0.1084	
4-Horiz-top	0.12	1.65	0.1980	0.0001	1.1170	0.1497	0.1498	
Total area =	1.06	Sum Ay=	0.5650			Total I =	0.3672	in^4
Y=	0.5330	in						

FIGURE P7-15 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Bot flange	0.1000	0.0500	0.0050	0.00008	1.0176	0.1036	0.1036	
2-2 Verticals	0.4800	1.2000	0.5760	0.23040	0.1323	0.0084	0.2388	
3-Mid-Horiz.	0.1	1.4500	0.1450	0.0000833	0.3823	0.0146	0.0147	
Total area =	0.68	Sum Ay=	0.7260			Total I =	0.3572	in^4
Y=	1.0676	in						

FIGURE P7-16 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Rectangle	1.1250	0.7500	0.8438	0.21094	0.1491	0.0250	0.2360	
2-Semicircle	0.2209	1.6590	0.3665	0.0022148	0.7598	0.1275	0.1297	
Total area =	1.34589	Sum Ay=	1.2102			Total I =	0.3657	in^4
Y=	0.8992	in						

FIGURE P7-17 Units: mm NOTE: Reference axis taken at y=125

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Rectangle	11400	0.0	0E+00	3.430E+07	0.00	0E+00	3.43E+07	
2-Semic-bot	1414	-107.7	-1.52E+05	9.072E+04	107.72	1.64E+07	1.65E+07	
3-Semic-top	1414	107.7	1.52E+05	9.072E+04	107.72	1.64E+07	1.65E+07	
Total area =	14227	Sum Ay=	0E+00			Total I =	6.73E+07	mm^4
Y=	0.00	mm						

FIGURE P7-18 Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Rectangle	1200	20.0	2.400E+04	1.60E+05	7.27	6.35E+04	2.23E+05	
2-Rect rem.	-400	20.0	-8.00E+03	-1.33E+04	7.27	-2.1E+04	-3.4E+04	
3-Triangle	300	46.7	1.40E+04	6.67E+03	19.39	1.13E+05	1.20E+05	
Total area =	1100	Sum Ay=	3.000E+04			Total I =	3.08E+05	mm^4
Y=	27.27	mm						

FIGURE P7-19 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Hor-.5x1.4	0.700	0.250	0.1750	0.0146	0.681	0.3242	0.3388	
2-Ver-.6x2.5	1.500	1.250	1.8750	0.7813	0.319	0.1531	0.9343	
3-2 Tr-.7x1.3	0.910	0.933	0.8493	0.0854	0.003	0.0000	0.0854	
4-Tri-rem	-0.460	0.807	-0.3711	-0.0216	0.124	-0.0071	-0.0287	
5-Hole-rem	-0.049	2.200	-0.1080	-0.0002	1.269	-0.0791	-0.0793	
Total area =	3	Sum Ay=	2.4202			Total I =	1.2506	in^4
Y=	0.9305	in						

FIGURE P7-20 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-2 Vert rect	1.2000	1.0000	1.2000	0.4000	0.2798	0.0940	0.4940	
2-2 Triangles	0.5100	1.1333	0.5780	0.0819	0.1465	0.0109	0.0928	
3-Top-.3x2.4	0.7200	1.8500	1.3320	0.0054	0.5701	0.2341	0.2395	
Total area =	2.4300	Sum Ay=	3.1100			Total I =	0.8263	in^4
Y=	1.2798	in						

FIGURE P7-21 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Vert rect	8.2500	4.2500	35.0625	20.7969	0	0.0000	20.7969	
2-Bot flange	5.2500	0.7500	3.9375	0.9844	3.5	64.3125	65.2969	
3-Top flange	5.2500	7.7500	40.6875	0.9844	3.5	64.3125	65.2969	
Total area =	18.7500	Sum Ay=	79.6875			Total I =	151.3906	in^4
Y=	4.2500	in						

FIGURE P7-22 Units: Inches NOTE: 7.25x7 rectangle with 4.25x5.5 rectangle removed

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Tot 7.25x7	50.75	3.50	177.63	207.23	0.00	0.00	207.23	
2-4.25x5.5 re	-23.38	3.50	-81.81	-58.92	0.00	0.00	-58.92	
Total area =	27.38	Sum Ay=	95.81			Total I =	148.30	in^4
Y=	3.50	in	NOTE: Reference axis is base of the shape					

FIGURE P7-23 Units: Inches NOTE: 24x4.5 rectangle with 21x3.5 rectangle removed

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Tot 24x4.5	108.00	2.25	243.00	182.25	0.00	0.00	182.25	
2-21x3.5 rem	-73.50	2.25	-165.38	-75.03	0.00	0.00	-75.03	
Total area =	34.50	Sum Ay=	77.63			Total I =	107.22	in^4
Y=	2.25	in	NOTE: Reference axis is base of the shape					

FIGURE P7-24 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-2 Verticals	33.75	5.63	189.84	355.96	2.13	152.40	508.36	
2-Top flange	16.88	12.00	202.50	3.16	4.25	304.80	307.97	
Total area =	50.63	Sum Ay=	392.34			Total I =	816.33	in^4
Y=	7.75	in						

FIGURE P7-25 Units: Inches NOTE: Depth of W14x43 is13.66; Ref. axis at centroid;Y=7.33

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-W14x43	12.60	0.00	0.00	428.00	0.00	0.00	428.00	
2-Bot plate	7.25	-7.08	-51.33	0.15	7.08	363.42	363.57	
3-Top plate	7.25	7.08	51.33	0.15104	7.08	363.42	363.57	
Total area =	19.85	Sum Ay=	0.00			Total I =	1155.13	in^4
Y=	0.00	in						

FIGURE P7-26	Units:	Inches	NOTE: Web for C is .387 thick; Y-Y axis down .674 from top					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-S12x50	14.70	6.00	88.20	305.00	1.90	53.31	358.31	
2-C12x25	7.35	11.71	86.09	4.47	3.81	106.62	111.09	
Total area =	22.05	Sum Ay=	174.29			Total I =	469.40	in^4
Y=	7.90	in	NOTE: For C12x25; y = 12.0+.387-.674 = 11.713					
			NOTE: Reference axis is base of the shape					

FIGURE P7-27	Units:	Inches	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-I12x14.292	12.15	6.00	72.92	317.33	1.40	23.73	341.06	
2-Top plate	3.50	12.25	42.88	0.07	4.85	82.41	82.49	
Total area =	15.65	Sum Ay=	115.79			Total I =	423.55	in^4
Y=	7.40	in						

FIGURE P7-28	Units:	Inches	NOTE: Depth of C12 is12.0; Ref. axis at centroid;y=6.50 from bot					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Two C12	14.07	0.00	0.00	319.520	0.00	0.00	319.52	
2-Bot plate	5.00	-6.25	-31.25	0.104	6.25	195.31	195.42	
3-Top plate	5.00	6.25	31.25	0.104	6.25	195.31	195.42	
Total area =	19.07	Sum Ay=	0.00			Total I =	710.35	in^4
Y=	0.00	in						

FIGURE P7-29	Units:	Inches	NOTE: Overall depth=7.0; Ref axis=centroid, 3.50 in from bottom					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Vert. plate	3.00	0.000	0.0000	9.0000	0.000	0.000	9.000	
2-2 Bot angles	2.72	-2.364	-6.4301	0.9580	2.364	15.201	16.159	
3-2 Top angles	2.72	2.364	6.4301	0.9580	2.364	15.201	16.159	
4-Top plate	2.25	3.250	7.3125	0.0469	3.250	23.766	23.813	
5-Bot plate	2.25	-3.250	-7.3125	0.0469	3.250	23.766	23.813	
Total area =	12.94	Sum Ay=	0.0000			Total I =	88.942	in^4
Y=	0.00	in						

FIGURE P7-30	Units:	Inches	NOTE: Overall depth=6.0; Ref axis=centroid, 3.0 in from bottom					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-2 v. plates	3.000	0.000	0.000	9.000	0.000	0.000	9.000	
2-Bot channel	1.760	-2.545	-4.479	0.305	2.545	11.400	11.705	
3-Top plate	1.760	2.545	4.479	0.305	2.545	11.400	11.705	
Total area =	4.760	Sum Ay=	0.000			Total I =	32.409	in^4
Y=	0.000	in	NOTE: For C3x6; y = 3.0-0.455 = 2.545;See App. A-6;x=0.455					

FIGURE P7-31	Units:	Inches	NOTE: Ref axis=centroid, 3.0 in from center of either pipe					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-vert. plate	2.050	0.000	0.000	2.870	0.000	0.000	2.870	
2-Bot pipe	1.070	-3.000	-3.210	0.391	3.000	9.630	10.021	
3-Top pipe	1.070	3.000	3.210	0.391	3.000	9.630	10.021	
Total area =	3.120	Sum Ay=	0.000			Total I =	22.912	in^4
Y=	0.000	in	NOTE: Length of plate = 6.0 in - pipe dia = 6.0 - 1.90 = 4.10 in					

FIGURE P7-32	Units:	Inches	NOTE: Reference axis=centroid=12 in from CL of pipes					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-2 bot pipes	4.46	-12.00	-53.52	6.04	12.00	642.24	648.28	
2-2 Top pipes	4.46	12.00	53.52	6.04	12.00	642.24	648.28	
Total area =	8.92	Sum Ay=	0.00			Total I =	1296.56	in^4
Y=	0.00	in						

FIGURE P7-33	Units:	Inches	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Base plate	2.5000	0.1250	0.3125	0.0130	2.5980	16.8734	16.8864	
2-2 angles	3.3800	1.4900	5.0362	5.5400	1.2330	5.1382	10.6782	
3-Top plate	6.0000	4.5000	27.0000	0.1250	1.7770	18.9473	19.0723	
Total area =	11.8800	Sum Ay=	32.3487			Total I =	46.6370	in^4
Y=	2.7230	in						

FIGURE P7-34	Units:	Inches	NOTE: Reference axis=centroid=3.00 in from bottom					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-channel	3.83	0.00	0.00	17.40	0.00	0.00	17.40	
2-channel	3.83	0.00	0.00	17.40	0.00	0.00	17.40	
Total area =	7.66	Sum Ay=	0.00			Total I =	34.80	in^4
Y=	0.00	in						

FIGURE P7-35	Units:	Inches	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-2 angles	1.876	0.592	1.111	0.696	2.020	7.658	8.354	
2-Bot channel	2.640	0.478	1.262	0.632	2.134	12.027	12.659	
3-2 Ver webs	4.500	3.000	13.500	13.500	0.388	0.676	14.176	
4-top channel	2.640	5.522	14.578	0.632	2.910	22.349	22.981	
Total area =	11.656	Sum Ay=	30.451			Total I =	58.171	in^4
Y=	2.612	in						

FIGURE P7-36	Units:	Inches	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-W6x15	4.430	2.995	13.268	29.100	0.638	1.801	30.901	
2-2 angles	1.876	0.852	1.598	0.696	1.505	4.252	4.948	
Total area =	6.306	Sum Ay=	14.866			Total I =	35.848	in^4
Y=	2.357	in						

FIGURE P7-37	Units:	Inches	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Bot tube	2.590	2.000	5.180	4.690	1.500	5.828	10.518	
2-Top tube	2.590	5.000	12.950	1.540	1.500	5.828	7.368	
Total area =	5.180	Sum Ay=	18.130			Total I =	17.885	in^4
Y=	3.500	in						

FIGURE P7-38	Units:	Inches	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-6x6x1/2	10.400	3.000	31.200	50.500	0.050	0.026	50.526	
2-4x2x1/4	2.590	1.500	3.885	1.540	1.450	5.447	6.987	
3-3x2x1/4	2.090	4.500	9.405	1.150	1.550	5.020	6.170	
Total area =	15.080	Sum Ay=	44.490			Total I =	63.683	in^4
Y=	2.950	in						

FIGURE P7-39	Units:	Inches	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-bot channel	1.590	0.457	0.727	0.319	2.898	13.358	13.677	
2-6x2x1/4	3.590	3.184	11.431	13.800	0.171	0.106	13.906	
3-Top channel	1.590	6.641	10.559	0.319	3.286	17.164	17.483	
Total area =	6.770	Sum Ay=	22.716			Total I =	45.065	in^4
Y=	3.355	in						

FIGURE P7-40	Units:	Inches	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Bot channel	1.881	0.730	1.373	0.980	0.321	0.193	1.173	
2-I-beam	1.726	1.400	2.416	0.680	0.349	0.211	0.891	
Total area =	3.607	Sum Ay=	3.790			Total I =	2.064	in^4
Y=	1.051	in						

FIGURE P7-41 Units: Inches NOTE: Reference axis is base of the shape

Part	Area, A_i	y_i	A_iy_i	I_c	d_i	$A_id_i^2$	I_c+Ad^2
1-W12x30	8.790	6.170	54.234	238.000	2.029	36.169	274.169
2-C6x13	3.830	12.854	49.231	1.050	4.655	83.010	84.060
Total area =	12.620 in^2	Sum Ay =	103.465 in^3			**Total I_c =**	**358.229 in^4**
		Y =	**8.199 in**				

FIGURE P7-42 Units: Inches NOTE: Reference axis is centroid of W-beam shape

Part	Area, A_i	y_i	A_iy_i	I_c	d_i	$A_id_i^2$	I_c+Ad^2
1-W4x13	3.830	0.000	0.000	11.300	0.000	0.000	11.300
2-4x2x1/4	2.590	3.080	7.977	1.540	3.080	24.570	26.110
3-4x2x1/4	2.590	-3.080	-7.977	1.540	3.080	24.570	26.110
Total area =	9.010 in^2	Sum Ay =	0.000 in^3			**Total I_c =**	**63.520 in^4**
		Y =	**0 in**				

FIGURE P7-43 Units: Inches NOTE: Reference axis is bottom of the shape

Part	Area, A_i	y_i	A_iy_i	I_c	d_i	$A_id_i^2$	I_c+Ad^2
1-W12x30	8.790	6.170	54.234	238.000	2.058	37.229	275.229
2-L4x3x1/4	1.690	13.580	22.950	2.770	5.352	48.408	51.178
3-L4x3x1/4	1.690	13.580	22.950	2.770	5.352	48.408	51.178
Total area =	12.170 in^2	Sum Ay =	100.135 in^3			**Total I_c =**	**377.585 in^4**
		Y =	**8.228 in**				

FIGURE P7-44 Units: Inches NOTE: Reference axis is centroid of 6x2 tube

Part	Area, A_i	y_i	A_iy_i	I_c	d_i	$A_id_i^2$	I_c+Ad^2
1-6x2x1/4	3.590	0.000	0.000	13.800	0.000	0.000	13.800
2-1/2x2 Pl	1.000	3.250	3.250	0.010	3.250	10.563	10.573
3-1/2x2 Pl	1.000	-3.250	-3.250	0.010	3.250	10.563	10.573
Total area =	5.590 in^2	Sum Ay =	0.000 in^3			**Total I_c =**	**34.946 in^4**
		Y =	**0 in**				

FIGURE P7-45 Units: Inches NOTE: Reference axis is bottom of the shape

Part	Area, A_i	y_i	A_iy_i	I_c	d_i	$A_id_i^2$	I_c+Ad^2
1-C8x4.147	3.526	4.000	14.104	37.400	1.023	3.692	41.092
2-C8x4.147	3.526	4.000	14.104	37.400	1.023	3.692	41.092
3-C8x4.147	3.526	7.070	24.929	3.250	2.047	14.770	18.020
Total area =	10.578 in^2	Sum Ay =	53.137 in^3			**Total I_c =**	**100.205 in^4**
		Y =	**5.023 in**				

FIGURE P7-46 Units: Inches NOTE: Reference axis is bottom of the shape

Part	Area, A_i	y_i	A_iy_i	I_c	d_i	$A_id_i^2$	I_c+Ad^2
1-1/2x18 Pl	9.000	0.250	2.250	0.188	1.862	31.194	31.382
2-3/4x10 Pl	7.500	5.500	41.250	62.500	3.388	86.103	148.603
3-L8x4x1/2	5.750	1.359	7.814	6.740	0.753	3.258	9.998
4-L8x4x1/2	5.750	1.359	7.814	6.740	0.753	3.258	9.998
Total area =	28.000 in^2	Sum Ay =	59.129 in^3			Total I_c =	**199.981 in^4**
	Y =	**2.112 in**					

FIGURE P7-47 Units: mm NOTE: Reference axis is bottom of the shape

Part	Area, A_i	y_i	A_iy_i	I_c	d_i	$A_id_i^2$	I_c+Ad^2
1-Flanges(2)	72.000	1.500	108.0	54.0	13.1	12434.2	12488.2
2-Vert. webs(2)	150.000	12.500	1875.0	7812.5	2.1	687.9	8500.4
3-Semicircle(+)	157.080	29.240	4593.0	1120.0	14.6	33476.6	34596.6
4-Semicircle(-)	-76.970	27.968	-2152.7	-268.9	13.3	-13669.7	-13938.6
Total area =	302.110 mm^2	Sum Ay =	4423.3 mm^3			Total I_c =	**41646.6 mm^4**
	Y =	**14.641 mm**					

FIGURE P7-48 Units: Inches NOTE: Reference axis is base of the shape

Part	Area, A_i	y_i	A_iy_i	I_c	d_i	$A_id_i^2$	I_c+Ad^2
1-Rect. (2)	1.250	0.625	0.781	0.163	0.391	0.191	0.353
2-Semicircle	0.884	1.568	1.385	0.035	0.552	0.270	0.305
Total area =	2.134 in^2	Sum Ay =	2.167 in^3			Total I_c =	**0.659 in^4**
	Y =	**1.016 in**					

SOLUTIONS FOR PROBLEMS 7-49 THROUGH 7-66

Each problem requires the computation of the radius of gyration = r_X = *(I_X/A)^1/2 with respect to the horizontal centroidal axis.*
Data for *I* and *A* are taken from the solution for the given figure number.

Prob. No.	Fig. No.	I_X	A	r_X
7-49	P7-2	166.0 in^4	18.00 in^2	3.04 in
7-50	P7-3	184.0 in^4	24.00 in^2	2.77 in
7-51	P7-4	4.64E+07 mm^4	9375 mm^2	70.35 mm
7-52	P7-5	2.66E+05 mm^4	550 mm^2	21.99 mm
7-53	P7-6	6.17E+04 mm^4	800 mm^2	8.78 mm
7-54	P7-8	5.36E+04 mm^4	550 mm^2	9.87 mm
7-55	P7-9	1.35E+05 mm^4	550 mm^2	15.67 mm
7-56	P7-11	1.86E+05 mm^4	725 mm^2	16.02 mm
7-57	P7-12	1.70E+04 mm^4	456 mm^2	6.11 mm
7-58	P7-14	0.3672 in^4	1.06 in^2	0.59 in
7-59	P7-15	0.3572 in^4	0.68 in^2	0.72 in
7-60	P7-16	0.3657 in^4	1.35 in^2	0.52 in
7-61	P7-17	6.73E+07 mm^4	14227 mm^2	68.78 mm
7-62	P7-21	151.4 in^4	18.75 in^2	2.84 in
7-63	P7-22	148.3 in^4	27.38 in^2	2.33 in
7-64	P7-23	107.2 in^4	34.50 in^2	1.76 in
7-65	P7-24	816.3 in^4	50.63 in^2	4.02 in
7-66	P7-25	1155.1 in^4	19.85 in^2	7.63 in

SOLUTIONS FOR PROBLEMS 7-67 THROUGH 7-81

Each problem requires the computation of thr radius of gyration = r_Y = (I_Y/A)^1/2 *with respect to the vertical centroidal axis.*
Data for *A* are taken from the solution for the given figure number.
All sections have a vertical axis of symmetry and they can be broken into parts that all have their centroidal axis on the axis of symmetry.
Therefore the total *I* is the algebraic sum of the *I* for all parts.

Prob. No.	Fig. No.	I_1	I_2	I_3	Total I_Y	A	r_Y
7-67	P7-2	18.00 in^4	18.00 in^4	0.50 in^4	36.50 in^4	18.00 in^2	1.424 in
7-68	P7-3*	144.00 in^4	-32.00 in^4	0.00 in^4	112.00 in^4	24.00 in^2	2.160 in
7-69	P7-4	2.60E+05 mm^4	1.12E+07 mm^4	0.00 mm^4	1.14E+07 mm^4	9375 mm^2	34.91 mm
7-70	P7-5	3333.33 mm^4	520.83 mm^4	26667 mm^4	3.05E+04 mm^4	550 mm^2	7.449 mm
7-71	P7-16	0.0527 in^4	0.0078 in^4	0.00 in^4	0.0605 in^4	1.35 in^2	0.212 in
7-72	P7-17	3.42E+06 mm^4	3.18E+05 mm^4	3E+05 mm^4	4.06E+06 mm^4	14227 mm^2	16.885 mm
7-73	P7-21	5.36 in^4	1.55 in^4	5.36 in^4	12.27 in^4	18.75 in^2	0.809 in
7-74	P7-22*	222.30 in^4	-35.18 in^4	0.00 in^4	187.11 in^4	27.38 in^2	2.614 in
7-75	P7-23*	5184.00 in^4	-2701.13 in^4	0.00 in^4	2482.88 in^4	34.50 in^2	8.483 in
7-76	P7-24	25.31 in^4	177.98 in^4	0.00 in^4	203.29 in^4	50.63 in^2	2.004 in
7-77	P7-25	45.20 in^4	127.03 in^4	127.03 in^4	299.25 in^4	19.85 in^2	3.883 in
7-78	P7-26	15.70 in^4	144.00 in^4	0.00 in^4	159.70 in^4	22.05 in^2	2.691 in
7-79	P7-27	35.48 in^4	14.29 in^4	0.00 in^4	49.77 in^4	15.65 in^2	1.783 in
7-80	P7-42	3.86 in^4	4.69 in^4	4.69 in^4	13.24 in^4	9.01 in^2	1.212 in
7-81	P7-44	2.31 in^4	0.33 in^4	0.33 in^4	2.98 in^4	5.59 in^2	0.730 in

*I_1 Is for the large outside rectangle. I_2 is for the internal rectangle and is negative.

ANALYSIS OF BENDING STRESSES

8-1 $\sigma = Mc/I = \left((425\ N\cdot m)(15\ mm)/67500\ mm^4\right)\dfrac{10^3\ mm}{m} = \underline{94.4\ MPa}$

$I = (30)^4/12 = 67\,500\ mm^4$

8-2 $I = \pi D^4/64 = \pi(20)^4/64 = 7854\ mm^4$

$\sigma = \dfrac{Mc}{I} = \dfrac{(120\ N\cdot m)(10\ mm)}{7854\ mm^4} \times \dfrac{10^3\ mm}{m} = \underline{152.8\ MPa}$

8-3 (a) $I = bh^3/12 = 0.75(1.5)^3/12 = 0.211\ in^4$

$\sigma = \dfrac{Mc}{I} = \dfrac{(5800\ lb\cdot in)(0.75\ in)}{0.211\ in^4} = \underline{20\,620\ psi}$

(b) $I = 1.5(0.75)^3/12 = 0.0527\ in^4$

$\sigma = \dfrac{(5800\ lb\cdot in)(0.375\ in)}{0.0527\ in^4} = \underline{41\,240\ psi}$

8-4 $I = 1.5(7.25)^3/12 = 47.63\ in^4;\ c = 7.25/2 = 3.625\ in.$

$\sigma = \dfrac{Mc}{I} = \dfrac{(15\,500\ lb\cdot in)(3.625)\ in}{47.63\ in^4} = \underline{1180\ psi}$

8-5 $M = 30\ k\cdot ft;\ S = 17.1\ in^3$

$\sigma = \dfrac{M}{S} = \dfrac{30\,000\ lb\cdot ft}{17.1\ in^3} \times \dfrac{12\ in}{ft} = \underline{21\,050\ psi}$

8-6 $M = 60\ k\cdot ft;\ S = 38.2\ in^3$

$\sigma = \dfrac{M}{S} = \dfrac{(60\,000\ lb\cdot ft)}{38.2\ in^3} \times \dfrac{12\ in}{ft} = \underline{18\,850\ psi}$

8-7

ALUM. C4X2.331; $I = 1.02\ in^4;\ c_t = 0.78\ in.;\ c_b = 1.47\ in$

BEAM IS IN NEGATIVE BENDING.

TENSILE – TOP

$\sigma = \dfrac{Mc_t}{I} = \dfrac{(9000\ lb\cdot in)(0.78\ in)}{1.02\ in^4} = \underline{6882\ psi}$

COMP. – BOTTOM

$\sigma = \dfrac{-Mc_b}{I} = \dfrac{-(9000\ lb\cdot in)(1.47\ in)}{1.02\ in^4} = \underline{-12\,970\ psi}$

8-8 $\sigma = \dfrac{M}{S} = \dfrac{4550\ lb\cdot in}{0.326\ in^3} = \underline{13\,960\ psi}$

8-9

$M = 71.5\ kN \cdot m = 71500\ N \cdot m \times 8.851\ lb \cdot in/N \cdot m = 6.33 \times 10^5\ lb \cdot in$

$I = 710.4\ in^4\ ;\ C = \bar{Y} = 6.50\ in$

$\sigma = \frac{MC}{I} = \frac{6.33 \times 10^5\ lb \cdot in\ (6.50\ in)}{710.4\ in^4} = \underline{5794\ psi\ (39.9\ MPa)}$

8-10

$S = 22.67\ in^3 \times 1.639 \times 10^4\ mm^3/in^3 = 3.716 \times 10^5\ mm^3$

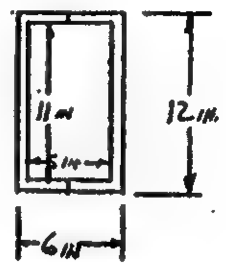
$\sigma = \frac{M}{S} = \frac{(43.2\ kN \cdot m)}{3.716 \times 10^5\ mm^3} \times \frac{10^3\ N}{kN} \times \frac{10^3\ mm}{m} = \underline{116\ MPa}$ $\underline{(16\,800\ psi)}$

8-11

$I = \frac{6(12)^3}{12} - \frac{5(11)^3}{12} = 309.4\ in^4$

$\sigma = \frac{Mc}{I} = \frac{(6000\ lb \cdot ft)(6\ in)}{309.4\ in^4} \times \frac{12\ in}{ft}$

$\sigma = 1396.3\ psi$



DESIGN OF BEAMS

8-12 $\sigma = Mc/I = M/S\ ;\ S = \pi D^3/32\ ;\ D = \sqrt[3]{32S/\pi}$

REQ'D $S = \frac{M}{S_d} = \frac{240\ N \cdot m}{125\ N/mm^2} \times \frac{10^3\ mm}{m} = 1920\ mm^3$

$D = \sqrt[3]{32(1920\ mm^3)/\pi} = \underline{26.9\ mm}$

8-13 $S = bh^2/6 = b(3b)^2/6 = 9b^3/6 = 1.5b^3$

$b = \sqrt[3]{S/1.5} = \sqrt[3]{2636\ mm^3/1.5} = 12.1\ mm$

REQ'D $S = \frac{M}{S_d} = \frac{145\ N \cdot m}{55\ N/mm^2} \times \frac{10^3\ mm}{m} = 2636\ mm^3$

$\underline{b = 12.1\ mm\ ;\ h = 3b = 36.3\ mm}$

8-14 $C_b = \bar{Y} = 152.5\ mm\ ,\ C_t = 225 - 152.5 = 72.5\ mm$

$\sigma_{BOTT} = \frac{MC_b}{I} = \frac{(28\,000\ N \cdot m)(152.5\ mm)}{46.4 \times 10^6\ mm^4} \times \frac{10^3\ mm}{m} = \underline{92.0\ MPa}$

FOR AISI 1020 HR, $S_y = 331\ MPa$

$\sigma_d = S_y/2 = 331\ MPa/2 = 165\ MPa$: SINCE $< S_d$, $\underline{OK}$

8-15 $I = 2.66 \times 10^5\ mm^4\ ;\ C_b = \bar{Y} = 35.0\ mm$

$\sigma = \frac{MC_b}{I} = \frac{(275\ N \cdot m)(35.0\ mm)}{2.66 \times 10^5\ mm} \times \frac{10^3\ mm}{m} = \underline{36.2\ MPa}$

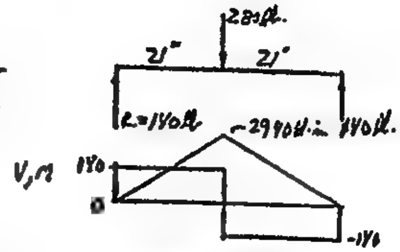
LET $\sigma = \sigma_d = S_u/8$

REQ'D $S_u = 8\sigma = 8(36.2\ MPa) = \underline{290\ MPa}$

COULD USE $\underline{6061\text{-}T6}$, $S_u = 310\ MPa$; 17% ELONGATION

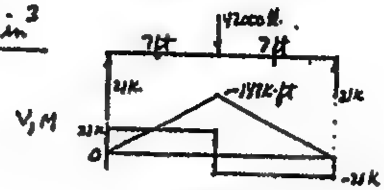
8-16 REQ'D $S = \frac{M}{\sigma_d} = \frac{2940 \text{ lb}\cdot\text{in}}{10\,000 \text{ lb/in}^2} = \underline{0.294 \text{ in}^3}$

USE $\underline{1\tfrac{1}{2} \text{ in SCH 40 PIPE}}$, $= 0.3262 \text{ in}^3$



8-17 $S = \frac{M}{\sigma_d} = \frac{147\,000 \text{ lb}\cdot\text{ft}}{20\,000 \text{ lb/in}^2} \times \frac{12 \text{ in}}{\text{ft}} = \underline{88.2 \text{ in}^3}$

USE A W20X66; $S = 119 \text{ in}^3$

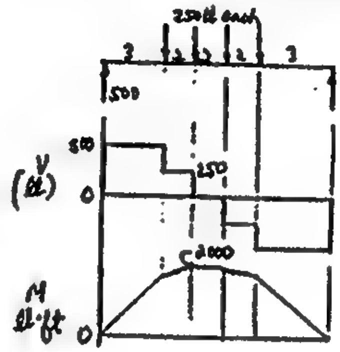


8-18 $I = 107.2 \text{ in}^4$; $C = 4.50 \text{ in}/2 = 2.25 \text{ in}$

$\sigma = \frac{Mc}{I} = \frac{(2000 \text{ lb}\cdot\text{ft})(2.25 \text{ in})(12 \text{ in})}{(107.2 \text{ in}^4)\ \text{ft}}$

$\sigma = \underline{504 \text{ psi}}$

FROM TABLE A18, MINIMUM ALLOWABLE BENDING STRESS = 625 psi - OK



8-19 $I = 30(3)^3/12 - 2\left[14(2)^3/12\right] = 48.83 \text{ in}^4$; $C = 1.50 \text{ in}$

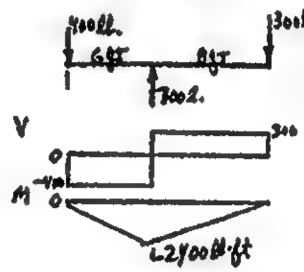
$\sigma = \frac{Mc}{I} = \frac{2400 \text{ lb}\cdot\text{ft}\,(1.50 \text{ in})}{48.83 \text{ in}^4} \times \frac{12 \text{ in}}{\text{ft}}$

$\underline{\sigma = 885 \text{ psi}}$

FOR IMPACT;

REQUIRED $S_u = 12\sigma = 12(885) = 10620$ PSI

FOR 6061-T4 $S_u = 35\,000$ PSI - OK



8-20 $I = 1.86 \times 10^5 \text{ mm}^4$; $C_b = \bar{Y} = 21.8 \text{ mm}$; $C_t = 45 - 21.8 = 23.2 \text{ mm}$

MAX STRESS AT TOP

$\sigma = \frac{M C_t}{I} = \frac{(318 \text{ N}\cdot\text{m})(23.2 \text{ mm})}{1.86 \times 10^5 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = \underline{39.7 \text{ MPa (comp.)}}$

REQ'D $S_y = 2\sigma = 2(39.7 \text{ MPa}) = \underline{79.4 \text{ MPa}}$ OK FOR 6061-T4 $S_y = 145$ MPa

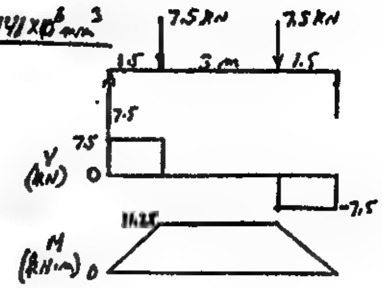
8-21 $I = 1.7 \times 10^4 \text{ mm}^4$; $C_b = \bar{Y} = 12.39 \text{ mm}$

$\sigma = \frac{Mc}{I} = \frac{(195 \text{ N}\cdot\text{m})(12.39 \text{ mm})}{1.7 \times 10^4 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = \underline{142 \text{ MPa}}$

REQ'D $\underline{S_y = 2\sigma = 284 \text{ MPa} - \text{2014-T4 HAS } S_y = 290 \text{ MPa}}$

8-22

$$\text{REQ'D } S = \frac{M}{\sigma_d} = \frac{11250\ N\cdot m}{80\ N/mm^2} \times \frac{10^3\ mm}{m} = \underline{0.141 \times 10^6\ mm^3}$$



(a) ROUND : $S = \pi D^3/32$

$D = \sqrt[3]{32(0.141 \times 10^6)/\pi} = \underline{112.8\ mm}$

$A = \frac{\pi D^2}{4} = \underline{9998\ mm^2}$

(b) SQUARE : $S = b^3/6$

$b = \sqrt[3]{6(0.141 \times 10^6)} = \underline{94.6\ mm}$

$A = b^2 = \underline{8945\ mm^2}$

(c) RECT.; $h = 4b$; $= \frac{bh^2}{6} = \frac{b(4b)^2}{6} = \frac{16b^3}{6} = 8b^3/3$

$b = \sqrt[3]{3S/8} = \sqrt[3]{3(0.141 \times 10^6)/8} = \underline{37.5\ mm}$

$h = 4b = \underline{150\ mm}$

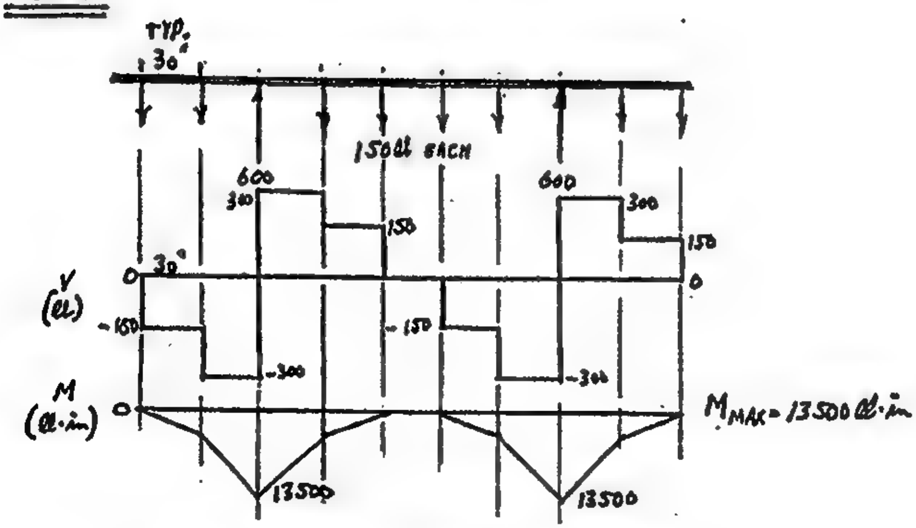
$A = bh = \underline{5625\ mm^2}$

(d) S-BEAM; $S = 0.141 \times 10^6\ mm^3 \times \frac{6.102 \times 10^{-5}\ in^3}{mm^3} = 8.60\ in^3$

$\underline{S8 \times 18.4}$, $S = 14.4\ in^3$

$A = 5.41\ in^2 \times \frac{645.16\ mm^2}{in^2} = \underline{3490\ mm^2}$ LIGHTEST

8-23



$$\text{REQ'D } S = \frac{M}{\sigma_d} = \frac{13500\ lb\cdot in}{10\,000\ lb/in^2} = 1.35\ in^3$$

USE $\underline{\text{3 IN. SCH40 PIPE}}$, $\underline{S = 1.724\ IN^3}$

8-24 $\sigma_d = S_y/4 = 39\,000\ psi/4 = 9750\ psi$

$M = (4800\ LB)(14\ IN) = 67200\ LB\cdot IN$

REQ'D. $S = M/\sigma_d = (67200\ LB\cdot IN)/(9750\ LB/IN^2) = 6.89\ IN^3$

USE EITHER <u>6×4×1/4</u> OR <u>8×2×1/4</u> : EACH WEIGHS 15.6 LB/FT.

8-25 LET $\sigma_d = S_y/4 = 40000\ psi/4 = 10000\ psi$

REQ'D. $S = M/\sigma_d = 67200/10000 = 6.72\ IN^3 \Rightarrow$ <u>6I×4.030</u>

8-26 $\sigma_d = S_y/4 = 36\,000\ psi/4 = 9000\ psi$

REQ'D. $S = M/\sigma_d = 67200/9000 = 7.47\ IN^3 \Rightarrow$ <u>W8×10</u>

8-27 REQ'D $S = 7.47\ IN^3$ FROM 8-26 : <u>NO SUITABLE CHANNEL</u>

8-28 $S_y = 36\ ksi$ - THEN REQ'D. $S = 7.47\ IN^3$ (PROB 8-26) : <u>6-IN SCH 40 PIPE</u>

8-29 DESIGN PROBLEM - MULTIPLE SOLUTIONS POSSIBLE

8-30 FROM FIG P7-15 ; $I = 0.3572\ IN^4$; $C_b = 1.068\ IN.$; $C_t = 1.332\ IN$

$S_{MIN} = \dfrac{I}{C_t} = \dfrac{0.3572\ IN^4}{1.332\ IN} = 0.268\ IN^3$

$\sigma = \dfrac{M}{S} = \dfrac{117\ LB\cdot FT}{0.268\ IN^3} \cdot \dfrac{12\ IN}{FT} = 5239\ psi$

REQ'D $\sigma_d = 4\sigma = 20\,960\ psi$

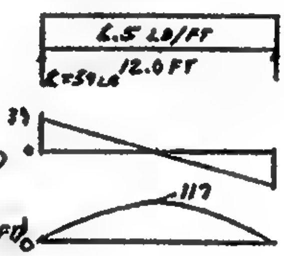
SEVERAL POSSIBLE CHOICES : APP. A-19

EXAMPLE: NYLON ; POLYESTER

HIGH MODULUS OF ELASTICITY

GOOD ELECTRICAL PROPERTIES (TABLE 2-6)

MUST CHECK EXTRUDABILITY.



8-31

$M_{MAX} = 24\ kN\cdot m$ (SEE PROB. P6-34)

$S = \dfrac{M}{\sigma_d} = \dfrac{24000\ N\cdot m}{150\ N/mm^2} \times \dfrac{10^3\ mm}{m} = 160\,000\ mm^3 \times \dfrac{6.102\times10^{-5}\ in^3}{mm^3}$

$S = 9.76\ in^3$; USE <u>W10×12</u>, $S = 10.9\ in^3$

8-32 FROM PROB. P6-35, $M_{MAX} = 125\ N\cdot m$

$\sigma_d = S_u/8 = 648\ MPa/8 = 81\ MPa$

$S = \dfrac{M}{\sigma_d} = \dfrac{125\ N\cdot m}{81\ N/mm^2} \times \dfrac{10^3\ mm}{m} = 1543\ mm^3 = \pi D^3/32$

$D = \sqrt[3]{32S/\pi} =$ <u>$25.1\ mm^3$</u>

8-33 Specify the lightest wide-flange beam. ASTM A36 structural steel. s_y = 36 ksi.

Design stress: $\sigma_d = 0.66\ s_y$ = (0.66)(36 000 psi) = 23 760 psi

From Fig. P6-3: M_{max} = (45.7 K•ft)(1000 lb/K)(12 in/ft) = 548 400 lb-in

Required section modulus: $S = M/\sigma_d$ = (548 400 lb-in)/(23 760 in^2) = 23.1 in^3

Specify: W14x26 steel beam.

8-34 Specify the lightest wide-flange beam. ASTM A36 structural steel. s_y = 248 MPa.

Design stress: $\sigma_d = 0.66\ s_y$ = (0.66)(248 MPa) = 164 MPa

Because beam data are given in U.S. Cust. units, bending moments converted to lb in.

Use $\sigma_d = 0.66\ s_y$ = (0.66)(36 000 psi) = 23 760 psi

From Fig. P6-7: M_{max} = (71.5 kN-m)(8 851 lb-in)/(kN m) = 6.33x10^5 lb-in

Required section modulus: $S = M/\sigma_d$ = (6.33x10^5 lb in)/(23 760 in^2) = 26.6 in^3

Specify: W14x26 steel beam.

Problems 8-33 to 8-42 are similar to 8-33 and 8-34 with the same design stress. Maximum bending moment varies with the beam loading shown in the indicated figure from Chapter 6.

Problems 8-43 to 8-52 use the same set of beam loadings as 8-33 to 8-42 but the objective is to specify the lightest American Standard S-beam. The required section modulus, S, is the same.

Problems 8-53 to 8-62 use the same set of beam loadings as 8-33 to 8-42 but the material is ASTM A572 Grade 60 with s_y = 60 ksi.. Then,

$\sigma_d = 0.66\ s_y$ = (0.66)(60 000 psi) = 39 600 psi (273 MPa)

The objective is to specify the lightest wide flange beam.

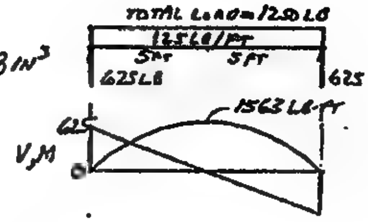
The results of these three sets of problems are summarized below.

Prob. No.	Fig. No.	M max	Reqd. S (in^3)	Lightest W-beam	Prob. No.	Lightest S-beam	Prob. No.	Reqd. S (in^3)	Lightest W-beam
8-33	P6-3	45.7 K-ft	23.1	W14x26	8-43	S10x25.4	8-53	13.9	W12x16
8-34	P6-7	71.5 N-m	26.6	W14x26	8-44	S10x35	8-54	16.0	W12x16
8-35	P6-8	43.2 kN-m	16.1	W12x16	8-45	S8x23	8-55	9.66	W10x12
8-36	P6-11	60.0 K-ft	30.3	W14x26	8-46	S12x35	8-56	18.2	W14x26
8-37	P6-16	170 kN-m	63.3	W18x40	8-47	S15x50	8-57	38.0	W12x30
8-38	P6-36	10.0 K-ft	5.05	W8x10	8-48	S6x12.5	8-58	3.03	W8x10
8-39	P6-40	40.0 K-ft	20.2	W14x26	8-49	S10x25.4	8-59	12.1	W10x15
8-40	P6-52	148 K-ft	74.7	W18x55	8-50	S20x66	8-60	44.8	W18x40
8-41	P6-63	1450 N-m	0.54	W8x10	8-51	S3x5.7	8-61	0.32	W8x10
8-42	P6-64	30.0 K-ft	15.15	W12x16	8-52	S8x23	8-62	9.09	W10x12

8-63 $\sigma_d = 1000\ psi$

REQ'D. $S = \frac{M}{\sigma_d} = \frac{(1563\ LB\cdot FT)(12\ IN/FT)}{850\ LB/IN^2} = 18.8\ IN^3$

USE 2×10 WOOD BEAM



8-64

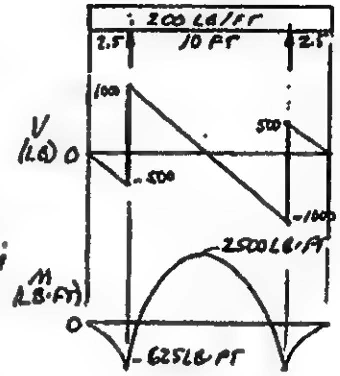
$\sigma_d = 1150\ psi$

PART	A	$\bar{y}$	$A\bar{y}$	I	d	Ad^2	$I+Ad^2$
1	5.25	1.75	9.188	5.36	1.907	19.085	24.445
2	16.87	4.25	71.698	3.164	.593	5.939	9.103

$\Sigma A = 22.12$ $\Sigma A\bar{y} = 80.886$ $I_T = 33.55\ IN^4$

$\bar{Y} = 3.66\ IN = C$

$\sigma = \frac{MC}{I} = \frac{(2500\ LB\cdot FT)(12\ IN/FT)(3.66\ IN)}{33.55\ IN^4} = 3273\ psi$ UNSAFE



8-65

PROCEDURE SAME AS 8-64 ; $\sigma_d = 1150\ psi$

WITH 2×8 VERT. MEMBER, $\sigma = 1064\ psi$

WITH 2×8 VERT. MEMBER:

$\bar{Y} = C = 6.286\ IN$; $I = 177.3\ IN^4$

$\sigma = \frac{(2500)(12)(6.286)}{177.3} = 1064\ psi$ OK

8-66

PROCEDURE SAME AS 8-64 WITH DOUBLE-WIDTH WEB.

WITH 2-2×6 VERTICAL MEMBERS: $\bar{Y} = C = 4.519\ IN$; $I = 146.9\ IN^4$

$\sigma = \frac{(2500)(12)(4.519)}{146.9} = 923\ psi$ OK

8-67

DESIGN PROBLEM — MULTIPLE SOLUTIONS POSSIBLE

8-68.

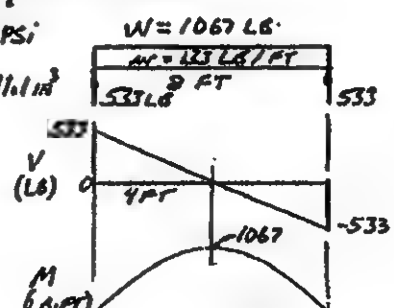
IN THE MIDDLE OF THE DECK, EACH FOOT OF JOIST LENGTH CARRIES A 16 IN WIDE PART OF THE DECK LOAD.

$w = \frac{100\ LB}{FT^2} \times \frac{(16\ IN)(12\ IN)}{FT} \times \frac{1\ FT^2}{144\ IN^2} = 133\ LB/FT$

SPECIFY NO.2 HEMLOCK: $\sigma_d = 1150\ psi$

REQ'D. $S = \frac{M}{\sigma_d} = \frac{(1067\ LB\cdot FT)(12\ IN/FT)}{1150\ LB/IN^2} = 11.1\ IN^3$

USE 2×8; $S = 13.14\ IN^3$



8-69

JOISTS ARE 12 FT LONG: BEAMS

AT ENDS: $w = 133\ LB/FT$:

$V_{MAX} = 800\ LB$ AT SUPPORTS:

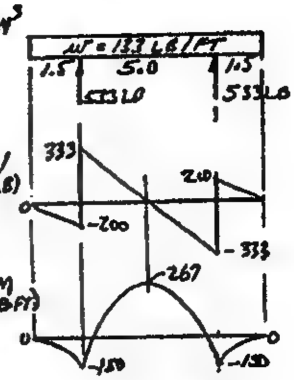
$M_{MAX} = 2400\ LB\cdot FT$ (28800 LB·IN)

REQ'D $S = \frac{M}{\sigma_d} = \frac{28800\ LB\cdot IN.}{1150\ LB/IN^2} = 25.0\ IN^3$ —USE 2×12; $S = 31.6\ IN^3$

8-70

REQ'D $S = \frac{M}{\sigma_d} = \frac{(267\ LB\cdot FT)(12\ IN/FT)}{1150\ LB/IN^2} = 2.79\ IN^3$

USE $\underline{2\times4\ \text{JOISTS}: S = 3.06\ IN^3}$



8-71

JOISTS ARE 12 FT LONG; BEAMS 1.5 FT FROM EACH END: $w = 133\ LB/FT$:

$V_{MAX} = 600\ LB$ AT SUPPORTS:

$M_{MAX} = 1200\ LB\cdot FT$ (14400 LB·IN)

REQ'D. $S = \frac{M}{\sigma_d} = \frac{(14400\ LB\cdot IN)}{1150\ LB/IN^2} = 12.5\ IN^3$

USE $\underline{2\times8 : S = 13.1\ IN^3}$

8-72

$M_{MAX} = 27733\ LB\cdot IN$

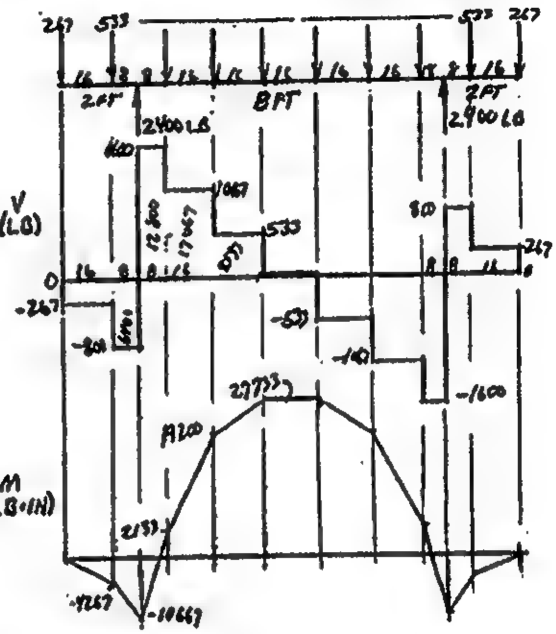
REQ'D $S = \frac{M}{\sigma_d} = \frac{27733\ LB\cdot IN}{1150\ LB/IN^2}$

$S = 24.1\ IN^3$

COULD USE:

$\underline{2\times12 : S = 31.6\ IN^3}$

$4\times8 : S = 30.7\ IN^3$



DESIGN PROBLEM

8-73. TRY $\underline{2\times6}$ $\underline{\text{DECK BOARDS}}$ WITH 3.0 FT SPAN

HEMLOCK: $\sigma_d = 1150\ LB/IN^2$

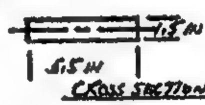
$S = bh^2/6 = (5.5)(1.5)^2/6 = 2.06\ IN^3$

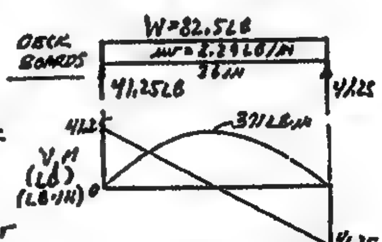
$W = \frac{60\ LB}{FT^2} \times \frac{(5.5)(36)\ IN^2}{BOARD} \times \frac{1\ FT^2}{144\ IN^2} = 82.5\ LB$ TOTAL

$w = 82.5\ LB/36\ IN = 2.29\ LB/IN$

$\sigma = \frac{M}{S} = \frac{371\ LB\cdot IN}{2.06\ IN^3} = 180\ PSI$ $\underline{OK}$

MUST ALSO CHECK DEFLECTION AND SHEAR





$\underline{BEAMS:}$ TOTAL LOAD $= (60\ LB/FT^2)(30\ FT^2)$

$W_T = 1800\ LB$

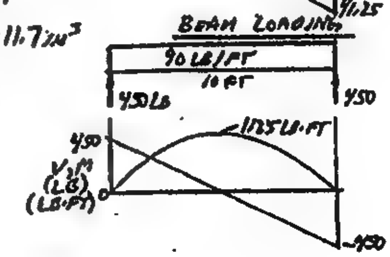
EACH BEAM CARRIES $900\ LB/10\ FT = 90\ LB/FT$

REQ'D. $S = \frac{M}{\sigma_d} = \frac{(1125\ LB\cdot FT)(12\ IN/FT)}{1150\ LB/IN^2} = 11.7\ IN^3$

USE $\underline{4\times6}$ BEAM: $S = 17.65\ IN^3$

$\underline{NOTE}$: MANY OTHER DESIGNS ARE POSSIBLE AND PRACTICAL.

COULD USE $2\times8 : S = 13.14\ IN^3$

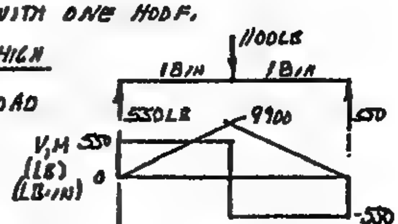


8-74

$\sigma_d = 1150$ PSI FROM PROB. 8-73.

ASSUME DECK BOARDS CARRY 1/2 OF TOTAL LOAD WHEN HORSE STEPS ON IT WITH ONE HOOF.

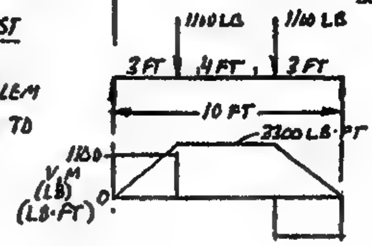
$$\sigma = \frac{M}{S} = \frac{9900\ \text{LB}\cdot\text{IN}}{2.06\ \text{IN}^3} = \underline{4805\ \text{PSI}}\quad \underline{\text{TOO HIGH}}$$



ASSUME SIDE BEAMS CARRY FULL LOAD IF HORSE WALKS TOWARD ONE SIDE. LOAD SPLIT BETWEEN FRONT AND BACK; ASSUME 4FT SPREAD.

$$\sigma = \frac{M}{S} = \frac{(3300\ \text{LB}\cdot\text{FT})(12\ \text{IN/FT})}{17.65\ \text{IN}^3} = \underline{2244\ \text{PSI}}$$

<u>TOO HIGH</u>



<u>CONCLUSION</u>: BRIDGE DESIGNED IN PROBLEM 8-41 WOULD <u>NOT</u> BE SAFE FOR HORSE TO CROSS.

8-75

<u>WOOD BEAM</u>: ASSUME NO. 2 SOUTHERN PINE – $\sigma_d = 1000$ PSI

$$\text{REQD } S = \frac{M}{\sigma_d} = \frac{(15750\ \text{LB}\cdot\text{FT})(12\ \text{IN/FT})}{1000\ \text{LB/IN}^2} = 189\ \text{IN}^3$$

USE <u>10X12 BEAM</u>: <u>$S = 209\ \text{IN}^3$</u>

<u>STEEL W-BEAM</u>: ASTM A36 STEEL, $S_y = 36$ KSI

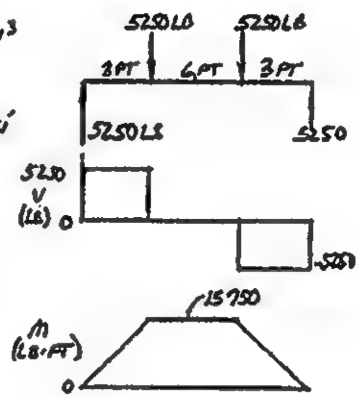
$\sigma_d = 0.66\, S_y = 0.66(36000\ \text{PSI}) = 23760$ PSI

$$\text{REQ'D } S = \frac{M}{\sigma_d} = \frac{(15750\ \text{LB}\cdot\text{FT})(12\ \text{IN/FT})}{23760\ \text{LB/IN}^2}$$

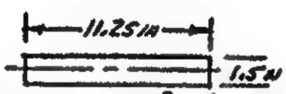
$S = 7.95\ \text{IN}^3$

USE: <u>W6X15</u>; <u>$S = 9.72\ \text{IN}^3$</u>

OR <u>W10X12</u>; <u>$S = 10.9\ \text{IN}^3$</u>



8-76



FOR 2X12: $S = \frac{bh^2}{6} = \frac{(11.25)(1.5)^2}{6}$

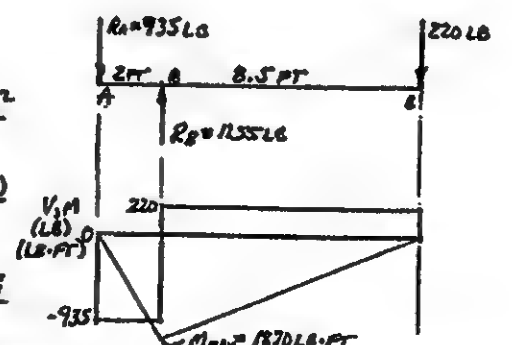
$S = 4.22\ \text{IN}^3$

$$\sigma = \frac{M}{S} = \frac{(1870\ \text{LB}\cdot\text{FT})(12\ \text{IN/FT})}{4.22\ \text{IN}^3}$$

<u>$\sigma = 5319$ PSI</u>

FOR <u>NO. 2 SO. PINE</u>: $\sigma_d = 1000$ PSI

<u>UNSAFE</u>



<u>POSSIBLE REDESIGN APPROACHES</u>:

SHORTER PLANK; STRONGER WOOD; THICKER PLANK; BUILT-UP BEAM.

8-77

$W = m \cdot g = 135\,kg \cdot 9.81\,m/s^2 = 1324\,N$

$W/2 = 662\,N$ ON EACH ROPE

FOR NO. 3 HEMLOCK: $\sigma_d = 4.3\,MPa$

AT A: $S = \pi D^3/32 = \pi(180)^3/32$

$S = 5.73 \times 10^5\,mm^3$

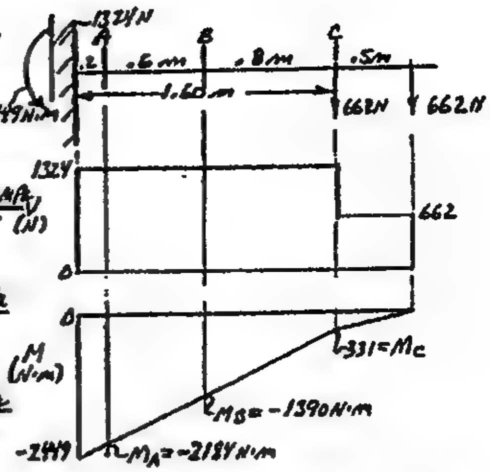
$\sigma = \frac{M}{S} = \frac{2184\,N \cdot m}{5.73 \times 10^5\,mm^3} \cdot \frac{10^3\,mm}{m} = 3.81\,MPa$ OK

AT B: $S = \pi(140)^3/32 = 2.69 \times 10^5\,mm^3$

$\sigma = \frac{M}{S} = \frac{1390 \times 10^3\,N \cdot mm}{2.69 \times 10^5\,mm^3} = 5.16\,MPa$ UNSAFE

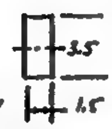
AT C: $S = \pi(90)^3/32 = 7.16 \times 10^4\,mm^3$

$\sigma = \frac{M}{S} = \frac{(331 \times 10^3\,N \cdot mm)}{7.16 \times 10^4\,mm^3} = 4.62\,MPa$ UNSAFE



8-78 $M_{MAX} = 6400\,LB \cdot IN$ AT B

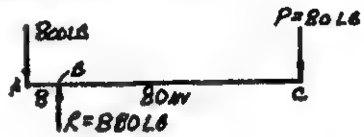
ASSUME 2×4 PLACED AS:



$S = 3.06\,IN^3$

$\sigma = \frac{M}{S} = \frac{6400\,LB \cdot IN}{3.06\,IN^3} = 2092\,PSI$

$\sigma_d = 1000\,PSI$ FOR NO. 2 SO. PINE – UNSAFE



8-79 FROM P7-6: $I = 6.167 \times 10^4\,mm^4$

$C = 17.5\,mm$; $\sigma = \frac{MC}{I}$

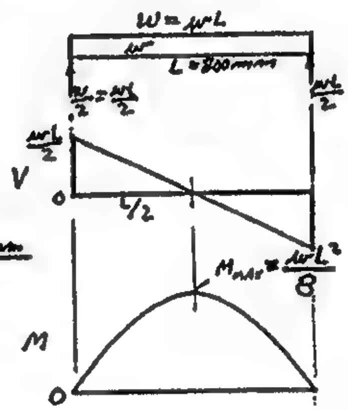
FOR NYLON 6/6: $S_f = 241\,MPa$

THEN $\sigma_d = S_f/2 = 120.5\,MPa$

$M_{MAX} = \frac{\sigma_d \cdot I}{C} = \frac{120.5\,N}{mm^2} \cdot \frac{6.167 \times 10^4\,mm^4}{17.5\,mm}$

$M_{MAX} = 4.25 \times 10^5\,N \cdot mm = wL^2/8$

$w_{MAX} = \frac{8M_{MAX}}{L^2} = \frac{(8)(4.25 \times 10^5\,N \cdot mm)}{(800\,mm)^2} = 5.31\,N/mm$



8-80 FROM P7-9: $I = 1.35 \times 10^5\,mm^4$; $C = 20.0\,mm$

$M_{MAX} = (2.25\,kN)(0.20\,m) = 450\,N \cdot m$

$\sigma = \frac{MC}{I} = \frac{(450\,N \cdot m)(20.0\,mm)}{1.35 \times 10^5\,mm^4} \cdot \frac{10^3\,mm}{m} = 66.7\,MPa$

REQ'D. FLEXURAL STRENGTH $= 3\sigma = 3(66.7) = 200\,MPa$

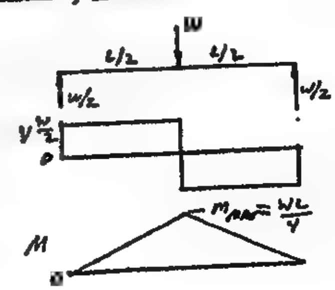
COULD USE: NYLON 6/6, POLYESTER PET, POLYIMIDE, EPOXY

8-81 FROM P7-5: $I = 2.66 \times 10^5\,mm^4$, $C = 35\,mm$

$\sigma_d = \frac{S_f}{2} = \frac{97\,MPa}{2} = 48.5\,MPa = \frac{M_{MAX}\,C}{I}$

$M_{MAX} = \frac{\sigma_d\,I}{C} = \frac{48.5\,N}{mm^2} \cdot \frac{2.66 \times 10^5\,mm^4}{35\,mm} = 3.69 \times 10^5\,N \cdot mm$

$W_{MAX} = \frac{4M_{MAX}}{L} = \frac{(4)(3.69 \times 10^5\,N \cdot mm)}{1250\,mm} = 1180\,N$

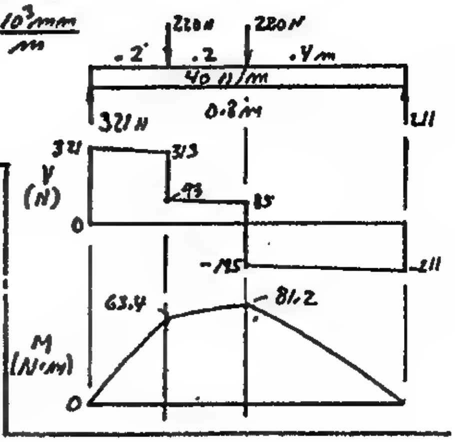


8-82 FROM P7-12: $I = 1.7 \times 10^4\ mm^4$, $C = 12.39\ mm$

$\sigma = \frac{MC}{I} = \frac{(81.2\ N\cdot m)(12.39\ mm)}{1.7 \times 10^4\ mm^4} \cdot \frac{10^3\ mm}{m}$

$\sigma = 59.3\ N/mm^2 = \underline{59.3\ MPa}$

$N = \frac{S_C}{\sigma} = \frac{131\ MPa}{59.3\ MPa} = \underline{2.21}$



8-83 FROM P7-20: $I = 0.8263\ IN^4$

$C = 1.28\ IN$

$\sigma_d = \frac{S_C}{3} = \frac{17000\ PSI}{3} = 5667\ PSI = \frac{M_{MAX} C}{I}$

$M_{MAX} = \frac{\sigma_d I}{C} = \frac{5667\ LB}{IN^2} \cdot \frac{0.8263\ IN^4}{1.28\ IN}$

$M_{MAX} = 3658\ LB\cdot IN = WL/4$ (SEE 8-81)

$W_{MAX} = \frac{4 M_{MAX}}{L} = \frac{4(3658\ LB\cdot IN)}{14.0\ IN} = \underline{1045\ LB}$

8-84 FROM P7-15: $I = 0.3572\ IN^4$; $C = 1.332\ IN$; $L = 8\ FT \times 12\ IN/FT = 96\ IN$

$\sigma_d = S_Y/2 = 21000\ PSI/2 = 10500\ PSI = M_{MAX}\ C/I$

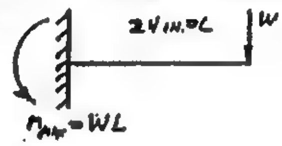
$M_{MAX} = \frac{\sigma_d I}{C} = \frac{10500\ LB}{IN^2} \cdot \frac{0.3572\ IN^4}{1.332\ IN} = 2816\ LB\cdot IN = \frac{wL^2}{8}$ (SEE 8-79)

$w_{MAX} = \frac{8 M_{MAX}}{L^2} = \frac{8(2816\ LB\cdot IN)}{(96\ IN)^2} = 2.44\ \frac{LB}{IN} \times \frac{12\ IN}{FT} = \underline{29.3\ LB/FT}$

8-85 FROM P7-14: $I = 0.3672\ IN^4$; $C = 1.167\ IN$; $\sigma_d = S_u/8 = \frac{62000\ PSI}{8} = 7750\ PSI$

$M_{MAX} = \frac{\sigma_d I}{C} = \frac{7750\ LB}{IN^2} \cdot \frac{0.3672\ IN^4}{1.167\ IN} = 2439\ LB\cdot IN$

$W_{MAX} = \frac{M}{L} = \frac{2439\ LB\cdot IN}{24\ IN} = \underline{102\ LB}$



8-86

HAT SECTION FROM P7-14 WITHOUT LOWER PLATE

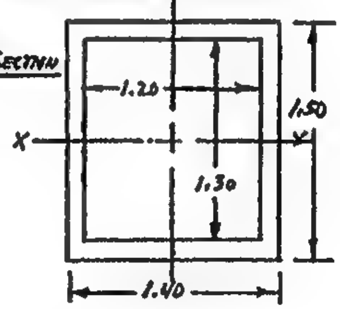
PART	A	y	Ay	I	d	Ad^2	$I+Ad^2$	
1	.12	.05	.006					BOTTOM FLANGE (2 × .6 × .1)
2	.30	.75	.225					2 WEBS 2 × 1.5 × .1
3	.12	1.45	.174					TOP FLANGE (1.2 × 0.1)
$\Sigma A = .54$	$\Sigma Ay = .405$							

$\bar{Y} = .405/.54 = \underline{0.75\ IN}$ SYMMETRICAL → EQUIVALENT SECTION

$I_x = \frac{1.40(1.50)^3}{12} - \frac{1.20(1.30)^3}{12} = 0.174\ IN^4$

$M_{MAX} = \frac{\sigma_d I}{C} = \frac{7750\ LB}{IN^2} \cdot \frac{.174\ IN^4}{0.75\ IN} = 1798\ LB\cdot IN$

$W_{MAX} = \frac{M}{L} = \frac{1798\ LB\cdot IN}{24\ IN} = \underline{75\ LB}$

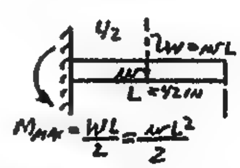


8-87 FROM P7-19: $I = 1.2506\ IN^4$; $C = 1.570\ IN$

$\sigma_d = S_u/6 = 45\ KSI/6 = 7.5\ KSI = 7500\ PSI$

$M_{MAX} = \frac{\sigma_d I}{C} = \frac{7500\ LB}{IN^2} \cdot \frac{1.2506\ IN^4}{1.570\ IN} = 5974\ LB\cdot IN$

$w_{MAX} = \frac{2 M_{MAX}}{L^2} = \frac{2(5974\ LB\cdot IN)}{(42\ IN)^2} = \underline{6.77\ LB/IN}$



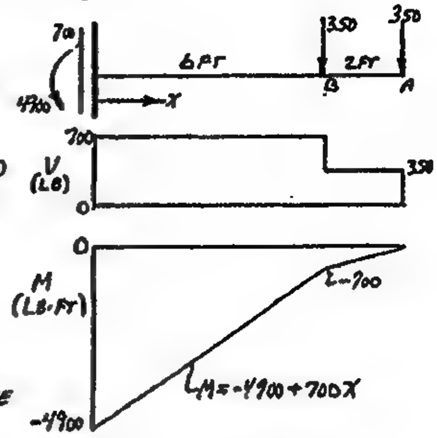
8-88 FROM P7-20: $I = 0.8263\ IN^4$; $C = 1.28\ IN$; $\sigma_d = \frac{S_u}{6} = \frac{33\ KSI}{6} = 5.50\ KSI$

$M_{MAX} = \frac{\sigma_d I}{C} = \frac{5500\ LB}{IN^2} \cdot \frac{0.8263\ IN^4}{1.28\ IN} = 3551\ LB\cdot IN$

$w = \frac{2 M_{MAX}}{L^2} = \frac{2(3551\ LB\cdot IN)}{(42\ IN)^2} = \underline{4.03\ LB/IN}$

BEAMS WITH STRESS CONCENTRATIONS AND VARYING CROSS SECTIONS

8-89



M_x = ALLOWABLE MOMENT AT JOINT

$\sigma = K_t M_x / S$ ($S = 2.394\ in^3$ FOR $3\frac{1}{2}$ in PIPE)

$M_x = \dfrac{\sigma_d S}{K_t} = \dfrac{(20000\ LB/IN^2)(2.394\ IN^3)}{1.78} = 26900$

$\left.\begin{array}{l} r/d = \dfrac{0.25}{4.00} = 0.063 \\ D/d = 4.50/4.00 = 1.125 \end{array}\right\} K_t = 1.78$

$M_x = 26900\ LB \cdot IN (1 FT/12 IN) = -2242\ LB \cdot FT$

LET $M_x = -4900 + 700X = -2242$

$X = 2658/700 = \underline{3.80\ FT}$ AT JOINT

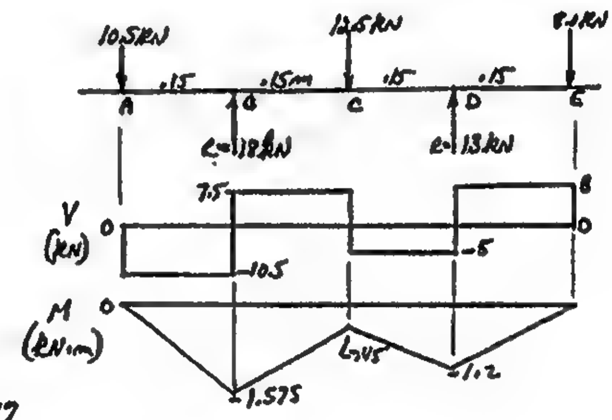
AT D AT WALL: $S = 3.215\ IN^3$ FOR 4 in PIPE

$\sigma = \dfrac{K_t M}{S} = \dfrac{(1.0)(4900\ LB \cdot FT)}{3.215\ IN^3} \times \dfrac{12\ IN}{FT}$

$\underline{\sigma = 18\,300\ PSI}$ IF NO SIGNIFICANT K_t EXISTS AT WALL.

$\underline{OK}$

8-90



$M = 0$ AT A, E

POINT B IS CRITICAL

$\sigma = \dfrac{M K_t}{S}$

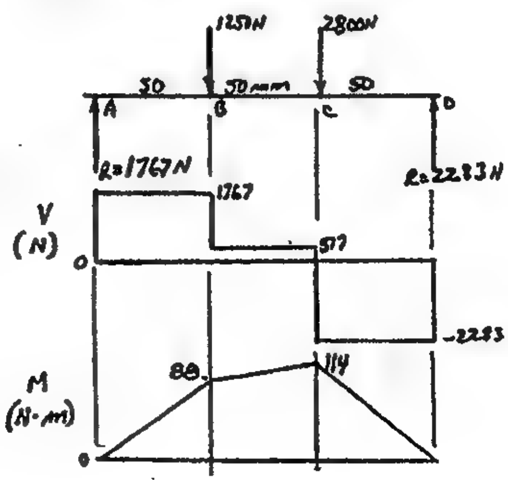
$S = \dfrac{\pi (45)^3}{32} = 8946\ mm^3$

FOR FILLET

$\left.\begin{array}{l} \dfrac{r}{d} = \dfrac{2}{45} = 0.044 \\ \dfrac{D}{d} = \dfrac{55}{45} = 1.22 \end{array}\right\} K_t = 2.17$

$\sigma = \dfrac{M K_t}{S} = \dfrac{1.575 \times 10^3\ N \cdot m\ (2.17)}{8946\ mm^3} \times \dfrac{10^3\ mm}{m} = \underline{382\ MPa}$

8-91



AT C

$S = \dfrac{\pi (18)^3}{32} = 572.6\ mm^3$

$\sigma = \dfrac{M K_t}{S} = \dfrac{(114\ N \cdot m)(2.0)}{572.6\ mm^3} \times \dfrac{10^3\ mm}{m}$

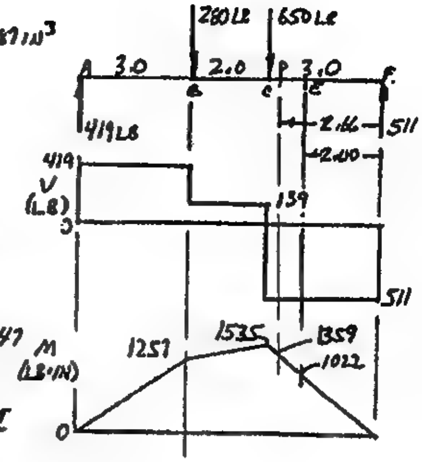
$\underline{\sigma = 398\ MPa}$

8-92

FOR D_1: $S = \pi D_1^3/32 = \pi(0.68)^3/32 = 0.03087\ IN^3$

FOR D_2: $S = \pi(1.00)^3/32 = 0.0982\ IN^3$

FOR D_3: $S = \pi(0.94)^3/32 = 0.08154\ IN^3$



AT C: $K_t = 2.0$ (KEYSEAT)

$\sigma = \frac{K_t \cdot M_C}{S} = \frac{2.0(1535)}{.0982} = 31270\ PSI$

AT D: $r/d = .01/.94 = .0106$; $D/d = 1.00/.94 = 1.06$

$K_t = 2.55$; $M_D = 1359\ LB \cdot IN$ (GROOVE)

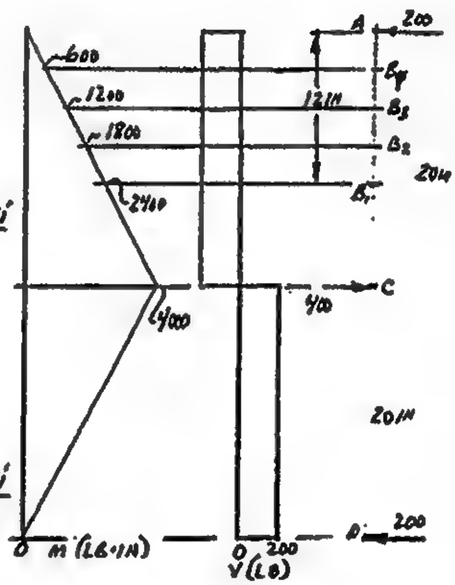
$\sigma = (2.55)(1359)/.08154 = 42500\ PSI$

AT E: $r/d = .06/.68 = 0.088$; $D/d = 1.00/0.68 = 1.47$

$K_t = 1.78$: $M_E = 1022\ LB \cdot IN$ (STEP)

$\sigma = \frac{1.78(1022)}{.03087} = \underline{58930\ PSI}$ HIGHEST

8-93



AT FULCRUM C:

$S = bh^2/6 = .75(2.00)^2/6 = 0.50\ IN^3$

$\sigma = \frac{M}{S} = \frac{4000\ LB \cdot IN}{0.50\ IN^3} = \underline{8000\ PSI}$

AT HOLE B_1: $D_H = 0.75 = d$

$d/w = .75/2.0 = 0.375 \rightarrow K_t = 1.0$

$\sigma = \frac{K_t 6Mw}{(w^3 - d^3)(t)} = \frac{(1.0)(6)(2400)(2.0)}{(2.0^3 - .75^3)(0.25)} = \underline{5067\ PSI}$

AT B_2: $M = 1800$: $\sigma = 3800\ PSI$

AT B_3: $M = 1200$: $\sigma = 2634\ PSI$

AT B_4: $M = 600$: $\sigma = 1267\ PSI$

8-94

AT C: $\sigma = 8000\ PSI$ (FROM 8-93)

AT B_1: $d/w = 1.38/2.0 = 0.69 \rightarrow K_t = 1.40$

$\sigma = \frac{K_t 6Mw}{(w^3 - d^3)t} = \frac{1.40(6)(2400)(2.00)}{(2.0^3 - 1.38^3)(.75)} = \underline{10000\ PSI}$

AT B_2: $\sigma = 7506\ PSI$; AT B_3: $\sigma = 5004\ PSI$

AT B_4: $\sigma = 2500\ PSI$

8-95

$d/w = 1.25/2.00 = 0.625 \rightarrow \underline{K_t = 1.27\ \text{AT HOLE}}$

$\sigma_{B_1} = \frac{K_t 6 M_B w}{(w^3 - d^3)t} = \frac{(1.27)(6)(2.00)}{(2.0^3 - 1.25^3)(.75)} \cdot M_{B_1} = \underline{3.36 M_{B_1}}$

AT FULCRUM, C

$S_c = 0.50\ IN^3$ (SEE 8-93)

$\sigma_c = M_c/S_c = 2.0 M_c$

PIVOT	CD	M_C	σ_C	R_A	AB_1	M_{B_1}	σ_{B_1}
a) END HOLE	20 IN.	4000 LB·IN.	8000 PSI	200 LB	12 IN.	2400 LB·IN.	8064 PSI
b) HOLE B_4	17 IN.	3400	6800	170	9	1530	5141
c) HOLE B_3	14 IN.	2800	5600	140	6	840	2822
d) HOLE B_2	11 IN.	2200	4400	110	3	330	1109
e) HOLE B_1	8 IN.	1600	3200	80	0	0	0

8-96

$M = F(52 + 25/2) = (2500\ N)(64.5\ mm) = 161250\ N \cdot mm$

AT A-A: $S = bh^2/6 = 16(25)^2/6 = 1667\ mm^3$

$\sigma = M/S = (161250\ N \cdot mm)/1667\ mm^3 = \underline{96.8\ MPa}$

8-97

AT B-B: $d/w = 12/25 = 0.48 \rightarrow K_t = 1.0$

$\sigma = \frac{K_t 6Mw}{(w^3 - d^3)(t)} = \frac{(1.0)(6)(161250)(25)}{(25^3 - 12^3)(16)} = \underline{108.8\ MPa}$

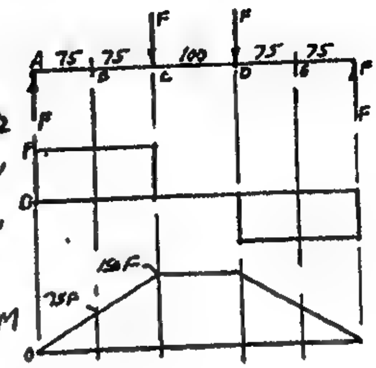
8-98 AT B-B: $d/w = 15/25 = 0.6 \rightarrow K_t = 1.21$

$$\sigma = \frac{(1.21)(6)(16)(250)(25)}{(25^3-15^3)(16)} = \underline{149.3\ MPa}$$

8-99 LET $\sigma = \sigma_d = S_u/8$; REQ'D. $S_u = 8\sigma = 8(149.3) = \underline{1195\ MPa}$

POSSIBLE STEEL: AISI 4140 OQT 900, $S_u = 1289\ MPa$; 15% ELONG.

8-100



AT C: $S = bh^2/6 = (12)(60)^2/6 = 7200\ mm^3$

AT B: $S = 12(40)^2/6 = 3200\ mm^3$

$r/h = 10/40 = 0.25$; $H/h = 60/40 = 1.50 \rightarrow K_t = 1.42$

$\sigma_d = S_u/8 = 669/8 = 83.6\ MPa = \frac{K_t M}{S}$

$M_{MAX} = \frac{\sigma_d S}{K_t} = \frac{83.6N}{mm^2}\frac{3200mm^3}{1.42} = 188450\ N\cdot mm$

$M_B = 75F$: $F = \frac{M}{75} = \frac{188450\ N\cdot mm}{75\ mm} = \underline{2513N}$ MAX

AT C: $M_{MAX} = \frac{(83.6)(7200)}{1.0} = 602100\ N\cdot mm$

$M_C = 150F$: $F = \frac{M}{150} = \frac{602100}{150} = 4014N$

8-101 AT B: $r/h = 2.0/40 = 0.05$; $H/h = 1.50$: $K_t = 2.2$

$M_{MAX} = \frac{\sigma_d S}{K_t} = \frac{83.6N\ (3200\ mm^3)}{mm^2\ (2.20)} = 121636 = 75F$

$F = M_{MAX}/75 = \underline{1622\ N}$

8-102 LET DISTANCE FROM A TO B BE $\underline{a}$. THEN $M_B = Fa$: LET $\sigma_B = \sigma_C$

$\sigma_B = \frac{K_t M_B}{S_B} = \frac{1.42(F\cdot a)}{3200} = \frac{F\cdot a}{2254}$ (DATA FROM PROB. 8-100)

$\sigma_C = \frac{K_t M_C}{S_C} = \frac{(1.0)(150F)}{7200} = \frac{F}{48}$

$\frac{F\cdot a}{2254} = \frac{F}{48}$; $a = \frac{2254}{48} = \underline{47\ mm}$

8-103 $\frac{K_t M_B}{S_B} = \frac{K_t M_C}{S_C}$; $K_{tB} = \frac{M_C}{M_B}\cdot\frac{S_B}{S_C}\cdot K_{tC} = \frac{150F}{75F}\cdot\frac{3200}{7200}\cdot 1.0 = 0.889$ IMPOSSIBLE

8-104 AT B: $S = 3200\ mm^3$; $r/h = 0.25$; $H/h = 75/40 = 1.88$; $K_t = 1.42$ SAME AS PROB 8-100

NO CHANGE IN LIMITING VALUE OF $F = \underline{2513N}$.

8-105 BECAUSE MAXIMUM STRESS OCCURS AT B, A HOLE CAN BE DRILLED AT C.

$\sigma = \sigma_d = 83.6\ MPa = \frac{K_t 6 M w}{(w^3-d^3)(t)}$

$F = 2513\ N$ MAXIMUM FROM PROB 8-100. THEN $M_C = 150F = 3.77\times10^5\ N\cdot mm$

$w = 60$, $t = 12$: BUT IF $d > 0.5w$, $K_t > 1.0$ (APP. A21-4)

SOLVE FOR d:

$$d = \left[w^3 - \frac{K_{t_C} 6 M_C w}{\sigma_d t}\right]^{1/3} = \left[60^3 - \frac{(6)(60)(3.77\times10^5)K_t}{83.6(12)}\right]^{1/3}$$

$$d = \left[2.16\times10^5 - (1.35\times10^5)K_t\right]^{1/3}$$

BY ITERATION: IF $K_t = 1.0$, $d = 43.3$; BUT $d/w = 0.72$ AND $K_t = 1.42$

$K_t = 1.30$, $d = 34.3$; $d/w = 0.57$, $K_t = 1.14$

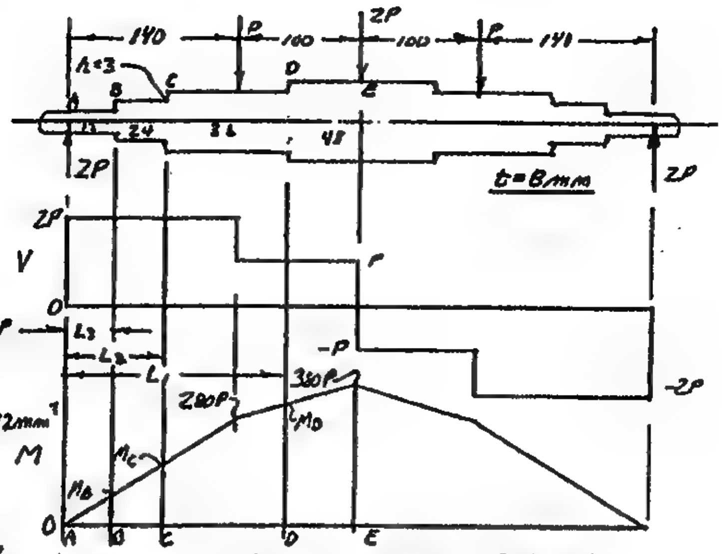
$K_t = 1.20$, $d = 37.8$; $d/w = 0.63$, $K_t = 1.24$

$K_t = 1.22$, $\underline{d = 37.2\ mm}$; $d/w = 0.62$, $K_t = 1.22$ OK

MAXIMUM HOLE SIZE

8-106

$\sigma = \frac{K_t M}{S}$



$\underline{M_B = 2PL_3}$

$\underline{M_C = 2PL_2}$

$M_D = 280P + (L_1 - 140)P$

$\underline{M_D = 140P + L_1 P}$

$S = th^2/6$

$S_B = 8(12)^2/6 = 192 \text{ mm}^3$

$S_C = \frac{8(24)^2}{6} = 768$

$S_D = \frac{8(36)^2}{6} = 1728$

$S_E = \frac{8(48)^2}{6} = 3072$

$L_1 = 160\text{mm};\ L_2 = 80\text{mm};\ L_3 = 40\text{mm};\ P = 200\text{ N}$

POINT	M(N·mm)	S(mm³)	r/h	H/h	K_t	σ (MPa)
B	16000	192	.25	2.0	1.42	118.3 MAXIMUM
C	32000	768	.125	1.50	1.68	70.0
D	64000	1728	.083	1.33	1.82	67.4
E	76000	3072	—	—	1.0	24.7

8-107

LET $\sigma = \sigma_d = S_u/8$

ONE POSSIBLE CHOICE

REQD $S_u = 8\sigma = 8(118.3\text{MPa}) = 946\text{ MPa}$ ∴ AISI 1141 OQT 900, $S_u = 1007$ MPa

8-108

DATA FROM 8-106 EXCEPT:

AT B: $r/h = 1.5/12 = 0.125 \rightarrow K_t = 1.75$: $\sigma_d = \underline{145.8\text{ MPa}}$ MAXIMUM

AT C: $r/h = 1.5/24 = 0.063 \rightarrow K_t = 2.00$: $\sigma_C = 83.3$ MPa

AT D: $r/h = 1.5/36 = 0.042 \rightarrow K_t = 2.27$: $\sigma_D = 84.1$ MPa

8-109

$P = 400\text{ N}$, K_t AND S FROM PROB 8-106

$\sigma_d = S_u/8 = 1170\text{MPa}/8 = 146\text{ MPa} = \frac{K_t M}{S}$; $M_{MAX} = \frac{\sigma_d S}{K_t}$

AT B: $M_{B_{MAX}} = \frac{146\text{N}}{\text{mm}^2}\frac{192\text{ mm}^3}{1.42} = 19740\text{ N·mm} = 2PL_3$

$\underline{L_3} = \frac{19740\text{ N·mm}}{2(400)\text{ N}} = \underline{24.7\text{mm}}$

AT C: $M_{C_{MAX}} = \frac{(146)(768)}{1.68} = 66742\text{ N·mm} = 2PL_2$

$\underline{L_2} = \frac{66742}{2(400)} = \underline{83.4\text{mm}}$

AT D: $M_{D_{MAX}} = \frac{(146)(1728)}{1.82} = 138620\text{ N·mm} = P(140 + L_1)$

$\underline{L_1} = \frac{M_D}{P} - 140 = \frac{138620}{400} - 140 = \underline{206.5\text{mm}}$

B-110

$\sigma_d = S_u/8 = 1014\ MPa/8 = 126.8\ MPa = \frac{K_t M}{S}$; K_t AND S FROM 8-106

AT B: $M_{MAX} = \frac{\sigma_d S_B}{K_{tB}} = \frac{126.8 N}{mm^2} \cdot \frac{192\ mm^3}{1.42} = 17138\ N\cdot mm = 2PL_3$

$P_{MAX} = \frac{M}{2L_3} = \frac{17138\ N\cdot mm}{2(40\ mm)} \doteq 214\ N$ GOVERNING VALUE

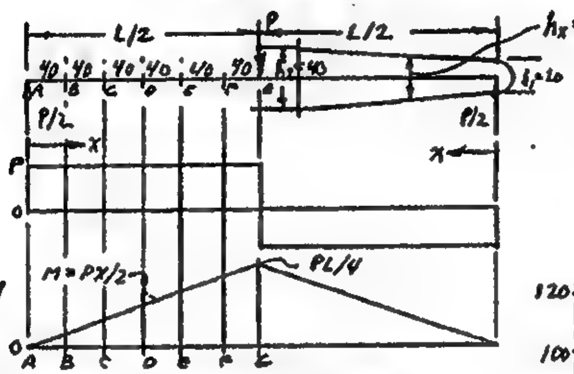
AT C: $M_{MAX} = \frac{(126.8)(768)}{1.68} = 57943\ N\cdot mm = 2PL_2$

$P_{MAX} = \frac{M}{2L_2} = \frac{57943}{2(80)} = 362\ N$

AT D: $M_{MAX} = \frac{(126.8)(1728)}{1.82} = 120\,342\ N\cdot mm = P(L_1+140)$

$P_{MAX} = \frac{M}{L_1+140} = \frac{120\,342}{180+140} = 376\ N$

B-111



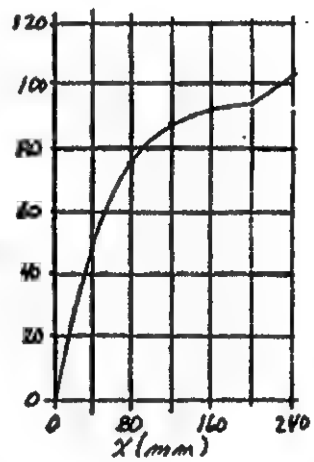
$h_x = 20 + 0.1x$ FOR $x \leq 200$

$\sigma = M/S$

$S = th^2/6$

$P = 5.0\ kN$

x (m)	M (kN·m)	h (mm)	S mm³	σ MPa
0	0	20	1333	0
0.040	.020P = 0.10	24	1920	52.1
0.080	.040P = 0.20	28	2613	76.5
0.120	.060P = 0.30	32	3413	87.9
0.160	.080P = 0.40	36	4320	92.6
0.200	.100P = 0.50	40	5333	93.8
0.240	.120P = 0.60	40	5333	112.5



B-112

$\sigma_d = \sigma_f/4 = 131\ MPa/4 = 32.75\ MPa = M/S$

$M = Px/2$; $h_x = 20 + 0.2x$ FOR $x \leq 200$; $S = th^2/6$

x	h	S	M	$\sigma = M/S$	P FOR $\sigma = 32.75$ MPa
0	20	1333	0	0	–
40	28	2613	20P	P/130.7	4280
80	36	4320	40P	P/108	3537
120	44	6453	60P	P/107.6	3524 — GOVERNING VALUE WITHIN TAPER
160	52	9013	80P	P/112.7	3691
200	60	12000	100P	P/120	3930
240	60	12000	120P	P/100	3275 MAXIMUM PERMISSIBLE

B-113

$\sigma_d = S_u/8 = 793\,MPa/8 = 99.1\,MPa$; $P = 1.20\,kN = 1200\,N$

BASED ON RESULTS OF PROBS. 111 AND 112, h_1 IS CRITICAL AT MIDDLE

$M = PL/4 = (1200\,N)(480\,mm)/4 = 144\,000\,N\cdot mm$

REQD $S = \frac{M}{\sigma} = \frac{144\,000\,N\cdot mm}{99.1\,N/mm^2} = 1453\,mm^3 = th^2/6$

$h_1 = \sqrt{\frac{6S}{t}} = \sqrt{\frac{6(1453\,mm^3)}{20\,mm}} = 20.9\,mm$

AT $x = 120\,mm$: $M = 60\,P = 60(1200\,N) = 72\,000\,N\cdot mm$

REQD $S = \frac{M}{\sigma} = \frac{72000}{99.1} = 727\,mm^3$

$h_{120} = \sqrt{\frac{6(727)}{20}} = 14.8\,mm$

$\Delta h = 6.1\,mm$

FOR LINEAR SIDES; $h_2 = h_{120} - 6.1 = 8.7\,mm$

LET $h_2 = 8\,mm$; $h_1 = 22\,mm$ FOR CONVENIENT DIMENSIONS

A CHECK OF STRESS AS IN B-111 SHOWS $\sigma < \sigma_d$ FOR ALL SECTIONS.

B-114

$\sigma_d = \frac{s_y}{4} = \frac{60\,000}{4} = 15\,000\,psi$

at B, $M = 10\,000\,lb\cdot ft$

REQ'D $S = \frac{M}{\sigma_d} = \frac{10\,000\,lb\cdot ft\,(12)}{15\,000\,lb/in^2\,(ft)} = 8.00\,in^3$

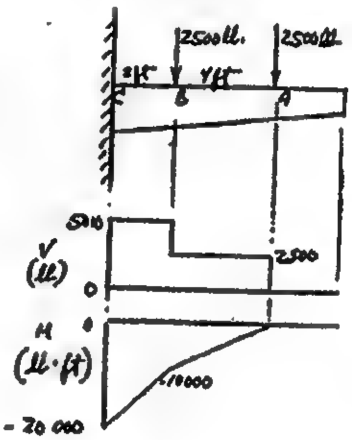
$S = \frac{bh^2}{6} = \frac{(1.5)h^2}{6} = 0.25h^2$

$h = \sqrt{S/0.25} = \sqrt{8.00/0.25} = 5.66\,in$

at C, $M = 20\,000\,lb\cdot ft$

$S = \frac{M}{\sigma_d} = \frac{20\,000(12)}{15000} = 16.00\,in^3$

$h = \sqrt{S/0.25} = 8.00\,in$



B-115 $\sigma_d = 0.66\,S_Y = 0.66(36\,000\,PSI) = 23\,760\,PSI$

$M_{MAX} = wL^2/8 = (2.5\,K/FT)(12\,FT)^2/8 = 45.0\,K\cdot FT$ (540000 LB·IN) (SEE B-79)

REQ'D. $S = \frac{M}{\sigma_d} = \frac{540\,000\,LB\cdot IN}{23\,760\,LB/IN^2} = 22.73\,IN^3$ W14X26 ($S = 35.3\,IN^3$)

B-116 FOR COMPOSITE SECTION: $I = 103 + 2[(3.5)(.25)(6.12)^2] = 168.5\,IN^4$

$C = [11.99 + 2(0.25)]/2 = 6.245\,IN$; $S = I/c = 168.5/6.245 = 26.99\,IN^3$ OK

FOR W12×16 ALONE: $S = 17.1\,IN^3$

$M_{MAX} = \sigma_d \cdot S = (23760\,LB/IN^2)(17.1\,IN^3) = 406\,296\,LB\cdot IN \cdot \frac{1K}{1000\,LB} \cdot \frac{1\,FT}{12\,IN} = 33.9\,K\cdot FT$

FIND x WHERE $M_x = 33.9\,K\cdot FT$

IN GENERAL: $M_x = A_1 + A_2 = Vx + 0.5(15 - V)x$

$M_x = Vx + 7.5x - 0.5Vx = 0.5Vx + 7.5x$

$= 0.5(15 - 2.5x)\,x + 7.5x = 7.5x - 1.25x^2 + 7.5x$

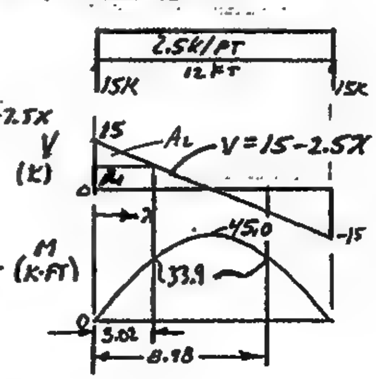
$M_x = -1.25x^2 + 15x$

FOR $M_x = 33.9$;

$-1.25x^2 + 15x - 33.9 = 0$

BY QUADRATIC EQUATION: $x = 3.02\,FT$ OR $8.98\,FT$

EXTEND COVER PLATE OVER MIDDLE 6 FT



ALTERNATE DEVELOPMENT FOR M_x:

$M_x = \int V dx = \int (15 - 2.5x)dx = 15x - 1.25x^2$

8-117 COMPOSITE: FIG P7-26: $I = 469.4\ \text{IN}^4$; $C = 7.90\ \text{IN}$

$$M_{MAX} = \frac{\sigma_d I}{C} = \frac{(23760)(469.4)}{7.90\ \text{IN}} = 1.41 \times 10^6\ \text{LB}\cdot\text{IN}\,(1\text{FT}/12\text{IN}) = 117.6\ \text{K}\cdot\text{FT} = wL^2/8$$

$$w = \frac{8M}{L^2} = \frac{8(117.6)\ \text{K}\cdot\text{FT}}{(15\text{FT})^2} = \underline{4.18\ \text{K/FT}}$$

FOR S12X50: $S = 50.8\ \text{IN}^3$: $M_{MAX} = \sigma_d S = (23760)(50.8) = 1.207 \times 10^6\ \text{LB}\cdot\text{IN} = 100.6\ \text{K}\cdot\text{FT}$

$$w = \frac{8M}{L^2} = \frac{8(100.6)\ \text{K}\cdot\text{FT}}{(15\text{FT})^2} = \underline{3.58\ \text{K/FT}}$$

FLEXURAL CENTER

8-118

$$e = \frac{b^2 h^2 t}{4 I_x} \; ; \; I_x = \frac{38(76)^3}{12} - \frac{34(68)^3}{12} = 4.992 \times 10^5\ \text{mm}^4$$

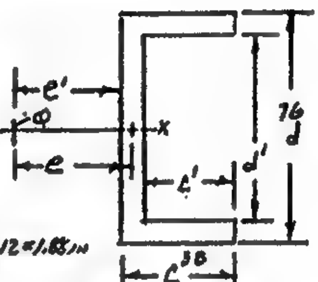
$b = 38 - 2 = 36\ \text{mm}$; $h = 76 - 4 = 72\ \text{mm}$; $t = 4\ \text{mm}$

$$e = \frac{(36^2)(72)^2(4)}{4(4.992 \times 10^5)} = 13.5\ \text{mm FROM MIDDLE OF WEB}$$

Q IS AT $e - 2 = \underline{11.5\ \text{mm}}$ FROM LEFT FACE OF WEB.

8-119

t	c'	d'	I_x	b	h	e (mm)	$e' = e - t/2$
.50	37.5	75	71731	37.75	75.5	14.16	13.91
1.60	36.4	72.8	219745	37.20	74.4	13.94	13.14
3.00	35.0	70.0	389674	36.50	73.0	13.66	12.16



$$I_x = \frac{cd^3}{12} - \frac{c'd'^3}{12}$$

8-120

$h = 2.00 - 0.12 = 1.88\ \text{IN}$; $c = .50 + \frac{.12}{2} = 0.56\ \text{IN}$; $b = 2.00 - .12 = 1.88\ \text{IN}$

$b/h = 1.00$; $c/h = 0.298$; $e/h = 0.46$

$e = 0.46\,h = 0.46(1.88) = 0.865\ \text{IN}$

$e' = e - t/2 = .865 - .06 = \underline{0.805\ \text{IN}}$ FROM LEFT FACE

8-121

t	h	c	b	c/h	b/h	e/h	e	e'
.020	1.980	.5100	1.980	.258	1.00	.48	.950	.940
.063	1.937	.5315	1.937	.274	1.00	.46	.891	.860
.125	1.875	.5625	1.875	.300	1.00	.45	.844	.781

ALL DIMENSIONS IN INCHES

8-122

$h = 80 - 3 = 77\ \text{mm}$; $c = 20 - 1.5 = 18.5\ \text{mm}$; $b = 50 - 3 = 47\ \text{mm}$

$b/h = 47/77 = 0.61$; $c/h = 18.5/77 = 0.24$; THEN $e/h = 0.35$

$e = 0.35\,h = 0.35(77) = 27.0\ \text{mm}$

$e' = e - t/2 = 27.0 - 1.50 = \underline{25.5\ \text{mm}}$ FROM LEFT FACE

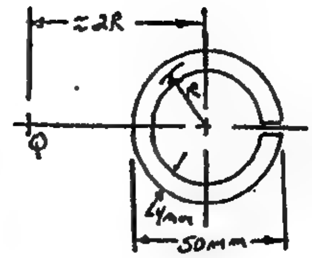
8-123

t	h	c	b	c/h	b/h	e/h	e	e'
0.50	79.5	19.75	49.5	0.248	0.623	.38	30.2	30.0 mm
1.60	78.4	19.20	48.4	0.245	0.617	.37	29.0	28.2 mm
3.00	77.0	18.5	47.0	0.24	0.61	0.35	27.0	25.5 mm

8-124

$R = \frac{D}{2} - \frac{t}{2} = 25 - 2 = 23\ mm$

Q IS AT 2R = 46 mm FROM CENTER



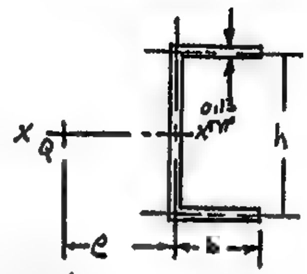
8-125

$I_x = 0.288\ in^4$ (FROM APPENDIX)

OVERALL DIMENSIONS 2.00 IN. DEPTH = D
1.00 IN. WIDTH = B

$h = D - t = 2.00 - 0.13 = 1.87\ in$

$b = B - \frac{t}{2} = 1.00 - \frac{0.13}{2} = 0.935\ in.$



$e = \frac{b^2 h^2 t}{4 I_x} = \frac{(0.935)^2(1.87)^2(0.13)}{4(0.288)}\ in = \underline{0.345\ in}$

8-126

$h = 25 + 5 = 30\ mm$; $C = 20 - 2.5 = 17.5\ mm$; $b = 45 - 5 = 40\ mm$

$c/h = 0.58$; $b/h = 1.33$; THEN $e/h \approx 0.43$ BY EXTRAPOLATION

$e = 0.43 h = 0.43(30) = \underline{12.9\ mm}$

BEAMS MADE FROM ANISOTROPIC MATERIALS

8-127

FROM P7-15: $I = 0.3572\ in^4$; $C_b = 1.068\ in$; $C_t = 1.332\ in$; $L = 6.5\ FT = 78\ in$

FOR TENSION ON BOTTOM: $M_{MAX} = \frac{\sigma_t I}{C_b} = \frac{19000\ LB}{IN^2} \cdot \frac{0.3572\ IN^4}{1.068\ IN} = 6355\ LB \cdot IN$

FOR COMP. ON TOP: $M_{MAX} = \frac{\sigma_c I}{C_t} = \frac{14000\ LB}{IN^2} \cdot \frac{0.3572\ IN^4}{1.332\ IN} = \underline{3754\ LB \cdot IN}$ LIMITING VALUE

FROM 8-47: $M_{MAX} = w L^2 / 8$

$w_{MAX} = \frac{8 M_{MAX}}{L^2} = \frac{8(3754\ LB \cdot IN)}{(78\ IN)^2} = \underline{4.94\ LB/IN}$

8-128

TENSION AT BOTTOM: $M_{MAX} = \frac{\sigma_t I}{C} = \frac{(19000)(0.3572)}{1.332} = \underline{5095\ LB \cdot IN}$ LIMITING VALUE

COMP. ON TOP: $M_{MAX} = \frac{(21000)(0.3572)}{1.068} = 7024\ LB \cdot IN$

$w_{MAX} = \frac{8M}{L^2} = \frac{8(5095)}{(78)^2} = \underline{6.70\ LB/IN}$

8-129

FROM P7-6: $I = 6.167 \times 10^4\ mm^4$; $C_b = 12.5\ mm$; $C_t = 17.5\ mm$

TENSION AT BOTTOM: $M_{MAX} = \frac{\sigma_t I}{C_b} = \frac{(100\ N)(6.167 \times 10^4\ mm^4)}{mm^2 \cdot (12.5\ mm)} = 4.93 \times 10^5\ N \cdot mm$

COMPRESSION AT TOP: $M_{MAX} = \frac{\sigma_c I}{C_t} = \frac{(70)(6.167 \times 10^4)}{17.5} = 2.47 \times 10^5\ N \cdot mm$ LIMITING VALUE

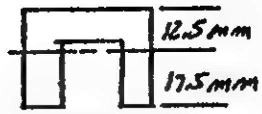
$M_{MAX} = WL/4$ (SEE 8-81)

$W_{MAX} = \frac{4 M_{MAX}}{L} = \frac{4(2.47 \times 10^5\ N \cdot mm)}{1200\ mm} = \underline{822\ N}$

8-130

FROM P7-6 : $I = 6.167 \times 10^4\ mm^4$

$C_t = 12.5\ mm,\ C_b = 17.5\ mm$



COMPRESSION AT TOP: $M_{MAX} = \dfrac{\sigma_{dc} I}{C_t}$

$$M_{MAX} = \frac{70\ N\ (6.167 \times 10^4\ mm^4)}{mm^2\ (12.5\ mm)} = 3.45 \times 10^5\ N \cdot mm \text{ — LIMITING VALUE}$$

TENSION AT BOTTOM: $M_{MAX} = \dfrac{\sigma_{dt} I}{C_b} = \dfrac{(100)(6.167 \times 10^4)}{17.5} = 3.52 \times 10^5\ N \cdot mm$

$$W_{MAX} = \frac{4 M_{MAX}}{L} = \frac{4(3.45 \times 10^5\ N \cdot mm)}{1200\ mm} = \underline{1150\ N}$$

8-131

FROM P7-8 : $I = 5.36 \times 10^4\ mm^4;\ C_b = C_t = 20\ mm$

COMPR. AT TOP: $M_{MAX} = \dfrac{\sigma_{dc} I}{C_t} = \dfrac{(70\ N)(5.36 \times 10^4\ mm^4)}{mm^2 (20.0\ mm)} = 1.876 \times 10^5\ N \cdot mm$ LIMITING VALUE

TENSION AT BOTTOM: $M_{MAX} = \dfrac{\sigma_{dt} I}{C_b} = \dfrac{(100)(5.36 \times 10^4)}{20.0} = 2.68 \times 10^5\ N \cdot mm$

$$W_{MAX} = \frac{4 M_{MAX}}{L} = \frac{4(1.876 \times 10^5\ N \cdot mm)}{1200\ mm} = \underline{625\ N}$$

8-132

FROM P7-9. : $I = 1.35 \times 10^5\ mm^4;\ C_b = C_t = 20\ mm$

COMPRESSION AT TOP IS LIMITING

$$M_{MAX} = \frac{\sigma_{dc} I}{C_t} = \frac{70\ N (1.35 \times 10^5\ mm^4)}{mm^2\ (20\ mm)} = 4.73 \times 10^5\ N \cdot mm$$

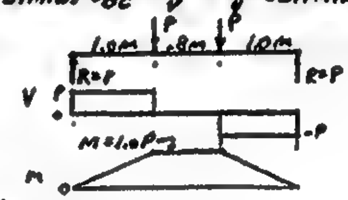
$$W_{MAX} = \frac{4 M_{MAX}}{L} = \frac{4(4.73 \times 10^5\ N \cdot mm)}{1200\ mm} = \underline{1575\ N}$$

8-133

FROM P7-4 : $I = 4.64 \times 10^7\ mm^4;\ C_b = \bar{Y} = 152.5\ mm;\ C_t = 72.5\ mm$

CI - ASTM A48, GR40 : $\sigma_{dt} = \dfrac{S_{ut}}{4} = \dfrac{276\ MPa}{4} = 69\ MPa;\ \sigma_{dc} = \dfrac{S_{uc}}{4} = \dfrac{965}{4} = 241\ MPa$

$\sigma = \dfrac{Mc}{I} : M_{MAX} = \dfrac{\sigma_d I}{C}$



COMP. AT TOP: $M_{MAX} = \dfrac{\sigma_{dc} I}{C_t} = \dfrac{241\ N (4.64 \times 10^7\ mm^4)}{mm^2 (72.5\ mm)}$

$M_{MAX} = 1.54 \times 10^8\ N \cdot mm$

TENS. AT BOTTOM: $M_{MAX} = \dfrac{\sigma_{dt} I}{C_b} = \dfrac{(69)(4.64 \times 10^7)}{152.5}$

$M_{MAX} = 2.10 \times 10^7\ N \cdot mm$ — LIMITING VALUE $= (1.0\ m)(P)$

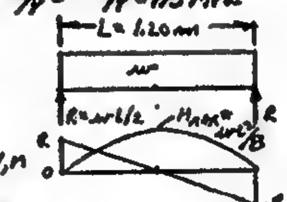
$P_{MAX} = M_{MAX}/1.0\ m = 2.10 \times 10^7\ N \cdot mm / 1000\ mm = 21.0 \times 10^3\ N = \underline{21.0\ kN}$

8-134

FROM P7-5 : $I = 2.66 \times 10^5\ mm^4;\ C_b = \bar{Y} = 35.0\ mm;\ C_t = 25.0\ mm$

ASTM A220, 80002 $\sigma_{dt} = S_{ut}/4 = 655/4 = 164\ MPa;\ \sigma_{dc} = S_{uc}/4 = 1650/4 = 413\ MPa$

$\sigma = \dfrac{Mc}{I} : M_{MAX} = \dfrac{\sigma_d I}{C}$



COMP. AT TOP : $M_{MAX} = \dfrac{\sigma_{dc} I}{C_t} = \dfrac{413\ N (2.66 \times 10^5\ mm^4)}{mm^2 (25.0\ mm)}$

$M_{MAX} = 4.39 \times 10^6\ N \cdot mm$

TENS. AT BOTTOM: $M_{MAX} = \dfrac{\sigma_{dt} I}{C_b} = \dfrac{(164)(2.66 \times 10^5)}{35.0}$

$M_{MAX} = 1.25 \times 10^6\ N \cdot mm$ — LIMITING VALUE $= wL^2/8$

$$w_{MAX} = \frac{8 M_{MAX}}{L^2} = \frac{8(1.25 \times 10^6\ N \cdot mm)}{(1200\ mm)^2} = 6.94 \frac{N}{mm} \cdot \frac{10^3\ mm}{m} = \underline{6.94\ kN/m}$$

TOTAL LOAD $= wL = (6.94\ kN/m)(1.20\ m) = \underline{8.33\ kN}$

8-135

$I = 2.66 \times 10^5\ mm^4$; $C_b = 25\ mm$; $C_t = 35\ mm$ (FROM P7-5 UPSIDE DOWN)

COMPR. AT TOP: $M_{MAX} = \dfrac{\sigma_{d_c} I}{C_t} = \dfrac{(413)(2.66 \times 10^5)}{35} = 3.14 \times 10^6\ N \cdot mm$

TENS. AT BOTTOM: $M_{MAX} = \dfrac{\sigma_{d_t} I}{C_b} = \dfrac{(164)(2.66 \times 10^5)}{25} = 1.74 \times 10^6\ N \cdot mm$

$w = \dfrac{8M}{L^2} = \dfrac{8(1.74 \times 10^6)\ N \cdot mm}{(1200\ mm)^2} = \underline{9.69\ N/mm} = \underline{9.69\ kN/m}$

TOTAL LOAD $= wL = (9.69\ kN/m)(1.20\ m) = \underline{11.63\ kN}$

8-136

$\sigma_{d_t} = S_u/10 = 827\ MPa/10 = 82.7\ MPa$; $\sigma_{d_c} = S_{uc}/10 = 1240\ MPa/10 = 124\ MPa$

PART	A	$\bar{y}$	$A\bar{y}$	I	d	Ad^2	$I + Ad^2$	
1	18750	62.5	1.17×10^6	24.4×10^6	75	105.5×10^6	129.9×10^6	3 RIBS
2	56250	162.5	9.14×10^6	26.4×10^6	25	35.16×10^6	61.56×10^6	TOP
$\Sigma A = 75000$	$\Sigma A\bar{y} = 10.31 \times 10^6$						$I = 191.4 \times 10^6\ mm^4$	

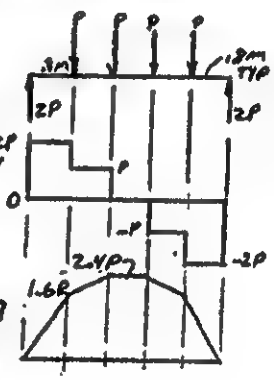
$\bar{Y} = 137.5\ mm = C_b$; $C_t = (200 - \bar{Y}) = 62.5\ mm$

TENSION AT BOTTOM: $M_{MAX} = \dfrac{\sigma_{d_t} I}{C_b} = \dfrac{82.7\ N}{mm^2} \cdot \dfrac{191.4 \times 10^6\ mm^4}{137.5\ mm} = 115.1 \times 10^6\ N \cdot mm$

COMPR. AT TOP: $M_{MAX} = \dfrac{\sigma_{d_c} I}{C_t} = \dfrac{(124)(191.4 \times 10^6)}{62.5} = 379.8 \times 10^6\ N \cdot mm$

$M_{MAX} = 2.4P$

$P_{MAX} = \dfrac{M_{MAX}}{2.4} = \dfrac{115.1 \times 10^6\ N \cdot mm}{2.4\ m\ (10^3\ mm/m)} = \underline{48.0\ kN}$



8-137

INCREASE DEPTH OF RIBS TO 250 mm

THEN $\bar{Y} = 222.5\ mm = C_b$; $C_t = (325 - \bar{Y}) = 102.5\ mm$

$I = 815.8 \times 10^6\ mm^4$

TENSION AT BOTTOM: $M_{MAX} = \dfrac{(82.7)(815.8 \times 10^6)}{222.5}$

$M_{MAX} = 303.2 \times 10^6\ N \cdot mm$

COMPR. AT TOP: $M_{MAX} = \dfrac{(124)(815.8 \times 10^6)}{102.5} = 986 \times 10^6$

$P_{MAX} = \dfrac{M_{MAX}}{2.4\ m} = \dfrac{303 \times 10^6\ N \cdot mm}{2400\ mm} = \underline{126\ kN}$

8-138

DESIGN PROBLEM - MULTIPLE SOLUTIONS POSSIBLE

8-139 ASTM A48 CAST IRON. $S_u = 276\ MPa$ - BRITTLE

$\sigma_d = S_u/6 = 276\ MPa/6 = 46.0\ MPa$

ACTUAL $\sigma_{MAX} = \frac{M}{S} = \frac{(2.4\times10^3 N)(350\ mm)}{\pi(50\ mm)^3/32} = 68.4\ MPa > \sigma_d$ UNSAFE

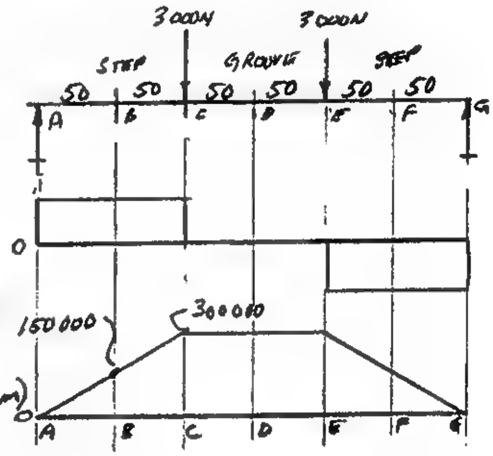
8-140 FROM FIG. 8-15: $M_{MAX} = 45900$ LB·IN. $\sigma_d = S_u/8$ REPEATED LOAD

AL. 6061-T6 $S_u = 45000\ PSI$. $\sigma_d = 45000\ PSI/8 = 5625\ PSI$

$\sigma = M/S$. REQ'D $S = M/\sigma_d = 45900\ LB\text{-}IN / 5625\ LB/IN^2 = 8.16\ IN^3$

SPECIFY 6I × 4.692 ALUMINUM I-BEAM SHAPE. $S = 8.50\ IN^3$

8-141 FIND MAX. STRESS. POINTS B OR D POSSIBLE SECTIONS



AT B: $d = 40\ mm$, $M_B = 150\ 000\ N\cdot mm$

STEP: $d/D = 80/40 = 2.00$, $r/d = 3.0/40 = 0.075$ } $K_t = 1.97$ APP A21-9

$S_B = \frac{\pi d^3}{32} = \frac{\pi(40\ mm)^3}{32} = 6283\ mm^3$

$\sigma_B = \frac{M_B \cdot K_t}{S_B} = \frac{(150\ 000\ N\cdot mm)(1.97)}{6283\ mm^3} = 47.0\ MPa$

AT D: $d_g = 60\ mm$, $M_D = 300000\ N\cdot mm$

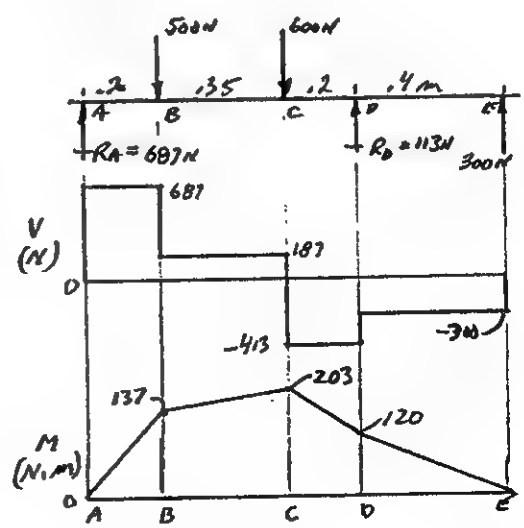
GROOVE: $D/d_g = 80/60 = 1.33$, $r/d_g = 6.0/60 = 0.10$ } $K_t = 2.03$ APP A21-8

$S_D = \pi(d_g)^3/32 = \pi(60\ mm)^3/32 = 21206\ mm^3$

$\sigma_D = \frac{M_D K_t}{S_D} = \frac{(300\ 000\ N\cdot mm)(2.03)}{21\ 206\ mm^3} = 28.7\ MPa$

$\sigma_{MAX} = \sigma_B = 47.0\ MPa$ AT STEP AT B.

8-142 SHAFT $D = 30.0\ mm$. $M_{MAX} = 203\ N\cdot m$



$S = \pi D^3/32 = \pi(30\ mm)^3/32 = 2651\ mm^3$

$\sigma_{MAX} = \frac{M}{S} = \frac{203\ N\cdot m}{2651\ mm^3} \cdot \frac{10^3\ mm}{m} = 76.6\ MPa$

$\sigma_d = S_u/8 = 600\ MPa/8 = 75.0\ MPa$

AISI 1040 WQT 1300. $S_u = 600\ MPa$

BECAUSE $\sigma_{MAX} > \sigma_d$ - UNSAFE

8-143 ASTM A36, $S_y = 36000$ PSI, STATIC LOAD

AISC: $\sigma_d = 0.66\,S_y = 0.66(36000) = 23\,760$ PSI

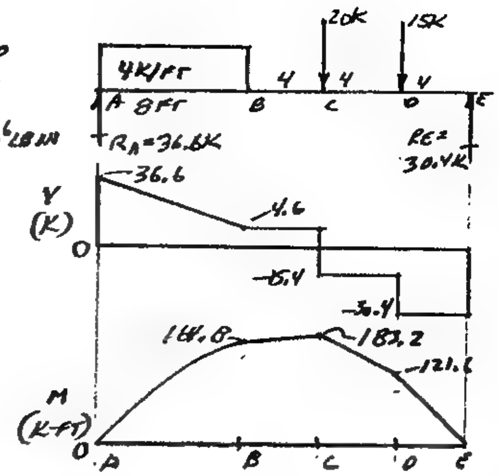
$M_{MAX} = 183.2\text{K-FT} \times 1000\,\text{LB/K} \times 12\,\text{IN/FT} = 2.20 \times 10^6$ LB·IN

$$\text{REQ'D } S = \frac{M_{MAX}}{\sigma_d} = \frac{2.20 \times 10^6 \text{ LB·IN}}{23760 \text{ LB/IN}^2}$$

$S_{MIN} = 92.5$ IN^3

SPECIFY W18×55; $S = 98.3$ IN^3

CHECK SHEAR STRESS IN WEB.
CHECK LATERAL BRACING AND DEFLECTION



8-144 SAME AS 8-143 BUT ASTM A242

$S_y = 50\,000$ PSI; $\sigma_d = 0.66\,S_y = 0.66(50\,000\text{ PSI}) = 33\,000$ PSI

$$\text{REQ'D } S = \frac{M_{MAX}}{\sigma_d} = \frac{2.20 \times 10^6 \text{ LB·IN}}{33000 \text{ LB/IN}^2} = 66.7 \text{ IN}^3$$

SPECIFY W18×40; $S = 68.4$ IN^3. LIGHTER BEAM SAVES 15LB/FT

15LB/FT × 20FT = 300 LB OF STEEL.

BUT COST/LB MAY BE HIGHER. ALSO CHECK WEB SHEAR, DEFLECTION, AND LATERAL BRACING REQUIREMENTS FROM AISC SPECIFICATIONS.

8-145 ASTM A500 GRADE C: $S_Y = 46$ KSI $= 317$ MPa

AISC: $\sigma_d = 0.66\,S_Y = 0.66(317\text{ MPa}) = 209$ MPa

$M_{MAX} = -12.16$ kN·m AT B.

$$\text{REQ'D } S = \frac{M_{MAX}}{\sigma_d} = \frac{12.16\text{ kN·m}}{209\text{ N/mm}^2} \times \frac{10^3\text{ N}}{\text{kN}} \times \frac{10^3\text{ mm}}{\text{m}}$$

$S_{MIN} = 5.82 \times 10^4$ mm^3

CONVERT TO IN^3:

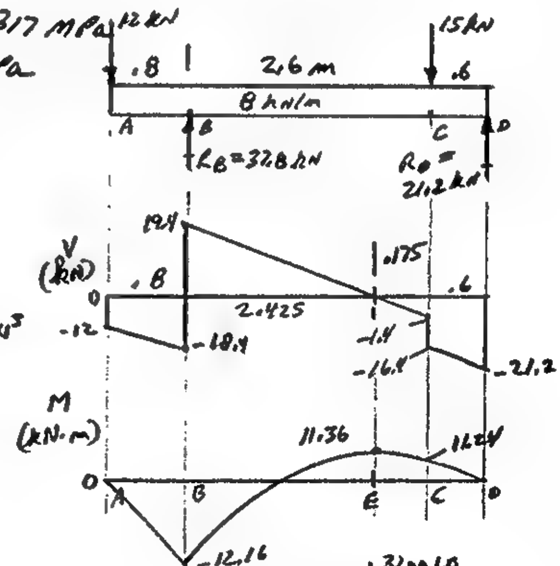
$$S_{MIN} = 5.82 \times 10^4 \text{ mm}^3 \times \frac{(1\text{ IN})^3}{(25.4\text{ mm})^3} = 3.55 \text{ IN}^3$$

SPECIFY EITHER:

4×4×1/4; $S = 4.11$ IN^3

6×2×1/4; $S = 4.60$ IN^3

BOTH WEIGH 12.2 LB/FT



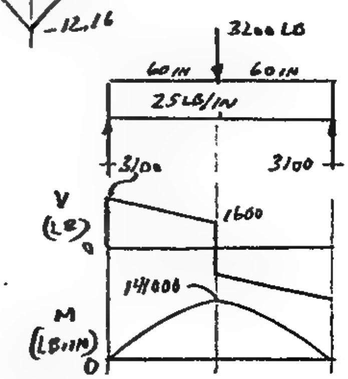
8-146 ASTM A500-C: $S_Y = 46$ KSI

$\sigma_d = S_Y/4 = 46\text{ KSI}/4 = 11\,500$ PSI

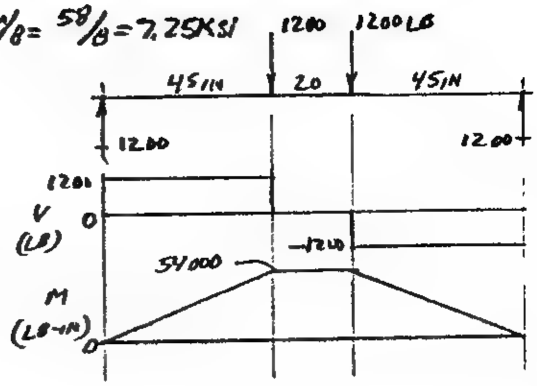
$$\text{REQ'D } S = \frac{M_{MAX}}{\sigma_d} = \frac{141\,000 \text{ LB·IN}}{11\,500 \text{ LB/IN}^2} = 12.26 \text{ IN}^3$$

SPECIFY 8-IN SCHEDULE 40 STEEL PIPE

$S = 16.81$ IN^3



8-147 ASTM A501: $S_u = 58\,KSI$: $\sigma_d = S_u/8 = 58/8 = 7.25\,KSI$



$$REQ'D\ S = \frac{M_{MAX}}{\sigma_d} = \frac{54000\ LB\text{-}IN}{7250\ LB/IN^2}$$

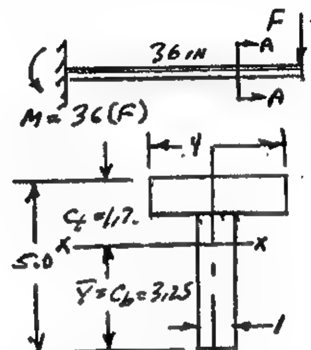
$S_{MIN} = 7.45\ IN^3$

SPECIFY $8 \times 2 \times 1/4$ STEEL TUBE (HSS)

$S = 7.52\ IN^3$

$w = 15.6\ LB/FT$

8-148 $M_{MAX} = 36F$ AT SUPPORT



SECTION A-A

$I_x = 18.16\ IN^4$

ASTM A48 GRADE 60: $S_{ut} = 55\,KSi$

$S_{uc} = 170\,KSi$

$\sigma_{dt} = S_{ut}/4 = 55000\,PSi/4 = 13750\,PSi$

$\sigma_{dc} = S_{uc}/4 = 170000\,PSi/4 = 42500\,PSi$

TOP IS IN TENSION: $\sigma_T = \dfrac{M c_t}{I}$

$$M_{MAX} = \frac{\sigma_{dt} I}{c_t} = \frac{(13750\ LB/IN^2)(18.16\ IN^4)}{1.75\ IN}$$

$M_{MAX} = 142\,685\ LB\cdot IN = (36\ IN)(F)$

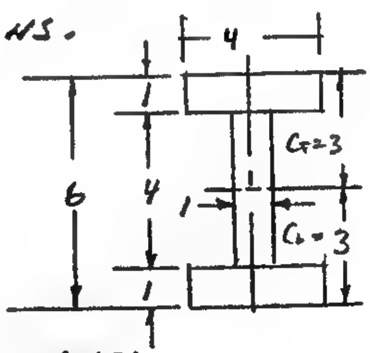
$$F_{ALL} = \frac{M_{MAX}}{36\ IN} = \frac{142685\ LB\cdot IN}{36\ IN} = 3963\ LB.$$

ANSWER

BOTTOM IS IN COMPRESSION: $\sigma_b = \dfrac{M c_b}{I}$

$$M_{MAX} = \frac{\sigma_{dc} I}{c_b} = \frac{(42500)(18.16)}{3.25} = 237{,}477\ LB\cdot IN < M\ AT\ TOP$$

8-149 DESIGN PROBLEM. MANY POSSIBLE SOLUTIONS.



ONE DESIGN: MAKE SHAPE A FULL I-BEAM

THEN $I = 4(6)^3/12 - 3(4)^3/12 = 56.0\ IN^4$

$S = \dfrac{I}{c} = \dfrac{56.0\ IN^4}{3\ IN} = 18.67\ IN^3$ FOR BOTH TENSION AT TOP AND COMPRESSION AT BOTTOM

AT TOP: $\sigma = \dfrac{M}{S} = \dfrac{(6000\ LB)(36\ IN)}{18.67\ IN^3} = 11\,571\ PSi < \sigma_{dt}$

& BOTTOM: OK

SHAPE MAY BE OPTIMIZED BY MAKING TOP FLANGE LARGER THAN BOTTOM FLANGE TO TAKE ADVANTAGE OF $\sigma_{dc} >> \sigma_{dt}$.

SEE EXAMPLE PROBLEM 8-10. TRIAL & ERROR SOLUTION REQ'D.

8-150 ASTM A36 $S_Y = 36000$ psi

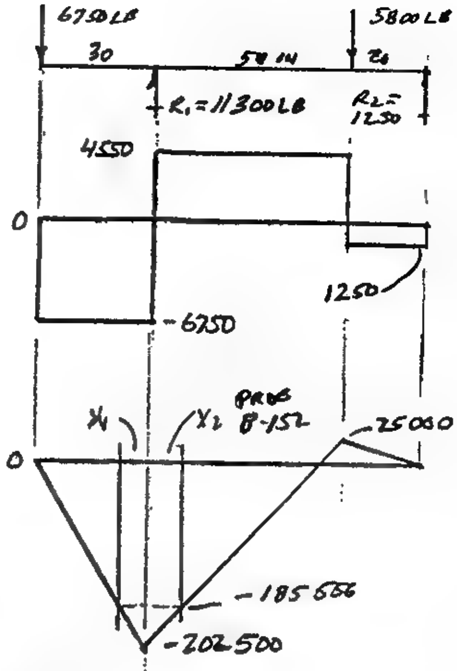
$\sigma_d = 0.66(36000) = 23760$ psi

REQ'D $S > \dfrac{M_{MAX}}{\sigma_d} = \dfrac{202\,500 \text{ LB-IN}}{23760 \text{ LB/IN}^2} = 8.52 \text{ IN}^3$

SPECIFY W10X12 $S = 10.9 \text{ IN}^3$

CHECK BEAM FOR COMPACTNESS, WEB SHEAR, AND LATERAL SUPPORT REQUIRED.

TOTAL WT $= 12 \dfrac{LB}{FT} \times \dfrac{1FT}{12IN} \times 100 IN = 100 LB$



8-151 SHAPE IS IDENTICAL TO THAT IN PROBLEM 7-44.

$I_x = 34.95 \text{ IN}^4$

$C_t = C_b = 3.50$ IN

$S = I/c = 34.95/3.50 = 9.99 \text{ IN}^3 > 8.52 \text{ IN}^3$ SAFE

6x2x1/4 TUBE WEIGHS 12.2 LB/FT $\times$ 100 IN $\times \dfrac{1FT}{12IN} = 101.7$ LB.

PLATES: VOL. IN 1.0 FT

$2[(2(0.5) \text{ IN}^2)12 \text{ IN}] = 24 \text{ IN}^3$

$WT = 24 \text{ IN}^3 \times 0.283 \text{ LB/IN}^3 = 6.79$ LB ; $6.79 \dfrac{LB}{FT} \times 100 IN \times \dfrac{1FT}{12IN} = 56.6$ LB

TOTAL WT $= 101.7 + 56.6 = 158.3$ LB MUCH HEAVIER THAN W10x12.

8-152 DESIGN PROBLEM – MANY POSSIBLE SOLUTIONS.

ONE POSSIBLE DESIGN: NOTE THAT BENDING MOMENT IS HIGH ONLY NEAR R_1. CONSIDER USING W8x10 BEAM WITH $S = 7.81 \text{ IN}^3$ AND ADDING COVER PLATES TO TOP AND/OR BOTTOM TO INCREASE SECTION MODULUS ONLY OVER THAT LENGTH WHERE REQUIRED S IS GREATER THAN 7.81 IN^3.

MAX. ALLOW. $M = \sigma_d \cdot S = (23760 \text{ LB/IN}^2)(7.81 \text{ IN}^3) = 185566$ LB-IN

TO LEFT OF R_1: $X_1 = \dfrac{(202500 - 185556) \text{ LB-IN}}{6750 \text{ LB}} = 2.51$ IN (SEE M-DIAGRAM)

TO RIGHT OF R_1: $X_2 = \dfrac{202500 - 185556}{4550 \text{ LB}} = 3.72$ IN

TOTAL LENGTH OF PLATES IS $2.51 + 3.72 = 6.23$ IN QUITE SMALL

USE 1/4 x 4.00 IN A36 STEEL PLATES TOP AND BOTTOM FOR 6.50 INCHES

I IS INCREASED FROM 30.8 IN^4 TO 63.92 IN^4

S IS INCREASED FROM 7.81 IN^3 TO $15.24 \text{ IN}^3 >> 8.52 \text{ IN}^3$ OK

WT OF BEAM IS 10 LB/FT $\times (100/12)$ FT $= 83.3$ LB

WT OF COVER PLATES IS 3.7 LB

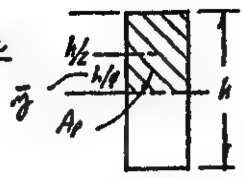
TOTAL WT OF MODIFIED BEAM IS 87.0 LB. THIS IS 13 LB LESS THAN W10X12 BEAM.

GENERAL SHEAR FORMULA

9-1 $\tau = \dfrac{VQ}{It} = \dfrac{(7500\,N)(2.5\times10^5\,mm^3)}{(33.3\times10^6\,mm^4)(50\,mm)} = 1.125\,N/mm^2 = \underline{1.125\,MPa}$

$I = \dfrac{th^3}{12} = \dfrac{50(200)^3}{12} = 33.3\times10^6\,mm^4$

$Q = A_p\bar{y} = (100)(50)(50) = 2.5\times10^5\,mm^3$



PROBLEMS 9-1 THRU 9-4

9-2 $I = (38)(180)^3/12 = 18.47\times10^6\,mm^4$

$Q = (90)(38)(45) = 1.539\times10^5\,mm^3$

$\tau = \dfrac{VQ}{It} = \dfrac{(5000)(1.539\times10^5)}{(18.47\times10^6)(38)} = \underline{1.10\,MPa}$

9-3 $I = (1.5)(7.25)^3/12 = 47.63\,IN^4 : Q = (3.625)(1.5)(1.813) = 9.86\,IN^3$

$\tau = \dfrac{VQ}{It} = \dfrac{(12500\,LB)(9.86\,IN^3)}{(47.63\,IN^4)(1.5\,IN)} = \underline{1724\,PSI}$

9-4 $I = (3.5)(11.25)^3/12 = 415\,IN^4 : Q = (5.625)(3.5)(2.813) = 55.37\,IN^3$

$\tau = \dfrac{VQ}{It} = \dfrac{(20000)(55.37)}{(415)(3.5)} = \underline{762\,PSI}$

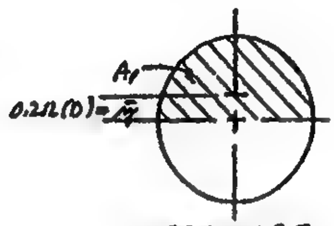
9-5 $I = \pi D^4/64 = \pi(50^4)/64 = 3.07\times10^5\,mm^4$

$A_p = \pi D^2/8 = \pi(50)^2/8 = 982\,mm^2$

$\bar{y} = 0.212\,D = 0.212(50) = 10.6\,mm$

$Q = A_p\bar{y} = (982)(10.6) = 10407\,mm^3$

$\tau = \dfrac{VQ}{It} = \dfrac{(4500)(10407)}{(3.07\times10^5)(50)} = \underline{3.05\,MPa}$



PROBLEMS 9-5 THRU 9-8

9-6 $I = \pi(38)^4/64 = 1.024\times10^5\,mm^4 : Q = \dfrac{\pi(38)^2}{8}\times 0.212(38) = 4568\,mm^3$

$\tau = \dfrac{(2500)(4568)}{(1.024\times10^5)(38)} = \underline{2.94\,MPa}$

9-7 $I = \pi(2.0)^4/64 = 0.785\,IN^4 : Q = [\pi(2.0)^2/8][0.212(2.0)] = 0.666\,IN^3$

$\tau = \dfrac{(7500)(0.666)}{(0.785)(2.0)} = \underline{3180\,PSI}$

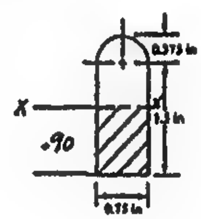
9-8 $I = \pi(0.63)^4/64 = 0.00773\,IN^4 : Q = [\pi(0.63)^2/8][0.212(0.63)] = 0.0208\,IN^3$

$\tau = \dfrac{VQ}{It} = \dfrac{(850)(0.0208)}{(0.00773)(0.63)} = \underline{3632\,PSI}$

9-9 $I_x = 0.366\,IN^4 ;\ \bar{Y} = 0.90\,IN$ FROM P7-16.

$Q = (0.75)(0.90)(0.45) = 0.304\,IN^3$

$\tau = \dfrac{VQ}{It} = \dfrac{(1500)(0.304)}{(0.366)(0.75)} = \underline{1661\,PSI}$



8-150 ASTM A36 $S_y = 36000$ psi

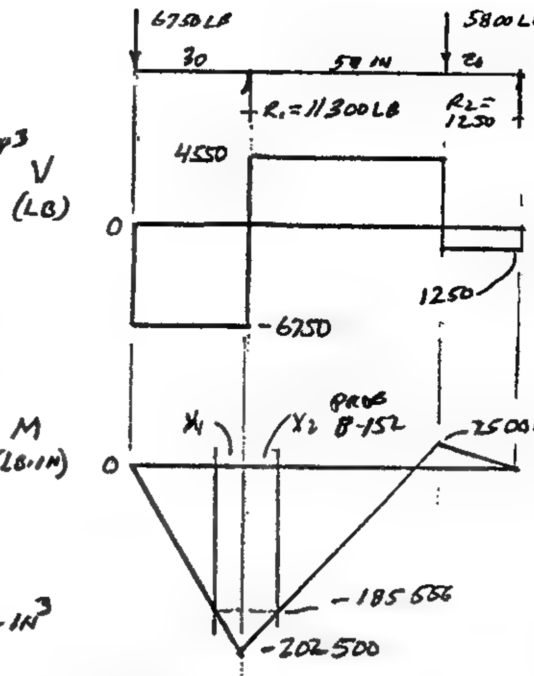
$\sigma_d = 0.66(36000) = 23760$ psi

REQ'D $S = \frac{M_{MAX}}{\sigma_d} = \frac{202\,500 \text{ LB-IN}}{23760 \text{ LB/IN}^2} = 8.52 \text{ IN}^3$

SPECIFY W10×12 $S = 10.9 \text{ IN}^3$

CHECK BEAM FOR COMPACTNESS, WEB SHEAR, AND LATERAL SUPPORT REQUIRED.

TOTAL WT $= 12\frac{\text{LB}}{\text{FT}} \times \frac{1\text{FT}}{12\text{IN}} \times 100\text{IN} = 100\text{LB}$



8-151 SHAPE IS IDENTICAL TO THAT IN PROBLEM 7-44.

$I_x = 34.95 \text{ IN}^4$

$C_t = C_b = 3.50 \text{ IN}$

$S = I/c = 34.95/3.50 = 9.99 \text{ IN}^3 > 8.52 \text{ IN}^3$

SAFE

6×2×1/4 TUBE WEIGHS 12.2 LB/FT $\times$ 100 IN $\times \frac{1\text{FT}}{12\text{IN}} = 101.7$ LB.

PLATES: VOL. IN 1.0 FT

$2[2(0.5) \text{ IN}^2)12\text{IN}] = 24 \text{ IN}^3$

WT $= 24\text{IN}^3 \times 0.283 \text{ LB/IN}^3 = 6.79$ LB ; $6.79\frac{\text{LB}}{\text{FT}} \times 100\text{IN} \times \frac{1\text{FT}}{12\text{IN}} = 56.6$ LB

TOTAL WT $= 101.7 + 56.6 = 158.3$ LB MUCH HEAVIER THAN W10×12.

8-152 DESIGN PROBLEM – MANY POSSIBLE SOLUTIONS.

ONE POSSIBLE DESIGN: NOTE THAT BENDING MOMENT IS HIGH ONLY NEAR R_1. CONSIDER USING W8×10 BEAM WITH $S = 7.81 \text{ IN}^3$ AND ADDING COVER PLATES TO TOP AND/OR BOTTOM TO INCREASE SECTION MODULUS ONLY OVER THAT LENGTH WHERE REQUIRED S IS GREATER THAN 7.81 IN^3.

MAX. ALLOW. $M = \sigma_d \cdot S = (23760 \text{ LB/IN}^2)(7.81 \text{ IN}^3) = 185\,566$ LB-IN

TO LEFT OF R_1: $X_1 = \frac{(202\,500 - 185\,556) \text{ LB·IN}}{6750 \text{ LB}} = 2.51$ IN (SEE M-DIAGRAM)

TO RIGHT OF R_1: $X_2 = \frac{202500 - 185\,556}{4550 \text{ LB}} = 3.72$ IN

TOTAL LENGTH OF PLATES IS $2.51 + 3.72 = 6.23$ IN QUITE SMALL

USE 1/4 × 4.00 IN A36 STEEL PLATES TOP AND BOTTOM FOR 6.50 INCHES

I IS INCREASED FROM 30.8 IN^4 TO 63.92 IN^4

S IS INCREASED FROM 7.81 IN^3 TO 15.24 IN^3 >> 8.52 IN^3 OK

WT OF BEAM IS 10 LB/FT × (100/12) FT = 83.3 LB

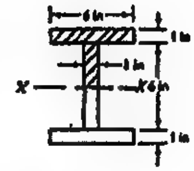
WT OF COVER PLATES IS 3.7 LB

TOTAL WT OF MODIFIED BEAM IS 87.0 LB. THIS IS 13 LB LESS THAN W10×12 BEAM.

9-10 $I_x = 166\ in^4$ FROM P7-2.

$Q = A_1 y_1 + A_2 y_2 = 3(1.5) + 6(3.5) = 25.5\ in^3$

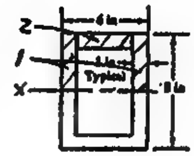
$\tau = \dfrac{VQ}{It} = \dfrac{(850)(25.5)}{(166)(1.0)} = \underline{131\ psi}$



9-11 $I_x = 184\ in^4$ FROM P7-3 $\therefore t = 2(1.0\ in) = 2.0\ in$

$Q = A_1 y_1 + A_2 y_2 = (2)(4)/2) + (4)(3.5) = 30.0\ in^3$

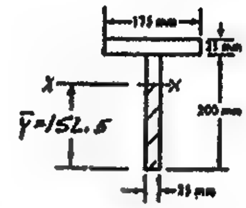
$\tau = \dfrac{(850)(30.0)}{(184)(2.0)} = \underline{69.3\ psi}$



9-12 $I_x = 4.64 \times 10^7\ mm^4$ FROM P7-4

$Q = A_p \bar{y} = (25)(152.5)(76.25) = 2.907 \times 10^5\ mm^3$

$\tau = \dfrac{(112 \times 10^3\ N)(2.907 \times 10^5\ mm^3)}{(4.64 \times 10^7\ mm^4)(25\ mm)} = \underline{28.1\ MPa}$

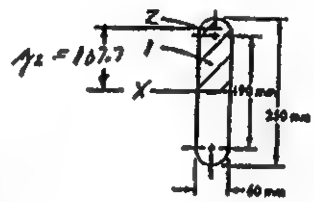


9-13 $I_x = 6.73 \times 10^7\ mm^4$ FROM P7-17

$Q = A_1 y_1 + A_2 y_2 = (60)(95)(47.5) + \dfrac{\pi(60)^2}{8}(107.7)$

$Q = 4.23 \times 10^5\ mm^3$

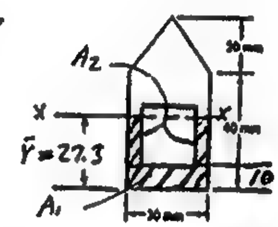
$\tau = \dfrac{(71.2 \times 10^3)(4.23 \times 10^5)}{(6.73 \times 10^7)(60)} = \underline{7.46\ MPa}$



9-14 $I_x = 3.08 \times 10^5\ mm^4$ FROM P7-18. $t = 10\ mm$

$Q = A_p \bar{y} = (300)(22.3) + (173)(8.65) = 8186\ mm^3$

$\tau = \dfrac{(1780)(8186)}{(3.08 \times 10^5)(1.0)} = \underline{4.73\ MPa}$

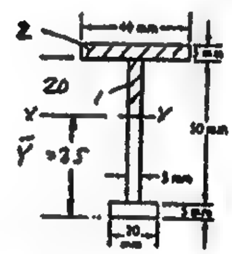


9-15 $I_x = 2.66 \times 10^5\ mm^4$ FROM P7-5

$Q = A_1 y_1 + A_2 y_2 = (20)(5)(10) + (40)(5)(22.5)$

$Q = 5500\ mm^3$

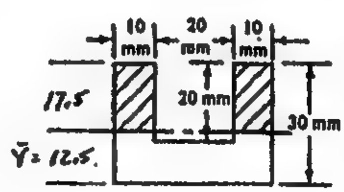
$\tau = \dfrac{(675)(5500)}{(2.66 \times 10^5)(5)} = \underline{2.79\ MPa}$



9-16 $I_x = 6.167 \times 10^4\ mm^4$ FROM P7-6

$Q = A_1 y_1 = (17.5)(20)(17.5/2) = 3063\ mm^3$

$\tau = \dfrac{(2500\ N)(3063\ mm^3)}{(6.167 \times 10^4\ mm^4)(20)\ mm} = \underline{6.21\ MPa}$



9-17 $I_x = 5.36 \times 10^4\ mm^4$ FROM P7-8.

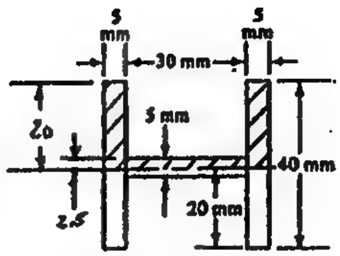
$Q = A_1\bar{y}_1 + A_2\bar{y}_2$

$Q = (2)(5)(20)(20/2) + (30)(2.5)(1.25)$

$Q = 2094\ mm^3$

$\tau = \frac{VQ}{It} = \frac{(10500\ N)(2094\ mm^3)}{(5.36 \times 10^4\ mm^4)(40\ mm)} = \underline{10.3\ MPa}$

NOTE: $\tau_{max} = 30.0$ MPa JUST ABOVE AREA WEB WHERE $Q = 1531\ mm^3$ AND $t = 10\ mm$.



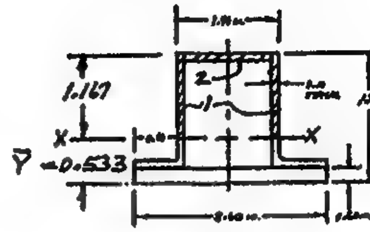
9-18 $I_x = 0.3672\ IN^4$ FROM P7-14

$Q = A_1\bar{y}_1 + A_2\bar{y}_2$

$Q = (2)(.10)(1.167)(1.167/2) + (0.1)(1.2)(1.117)$

$Q = 0.270\ IN^3$

$\tau = \frac{(1200)(0.270)}{(0.3672)(0.20)} = \underline{4416\ psi}$

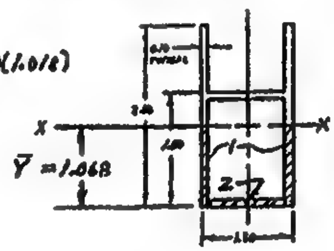


9-19 $I_x = 0.3572\ IN^4$ FROM P7-15

$Q = A_1\bar{y}_1 + A_2\bar{y}_2 = (2)(.10)(1.068)(1.068/2) + (0.1)(1.0)(1.018)$

$Q = 0.2159\ IN^3$

$\tau = \frac{(775)(0.2159)}{(0.3572)(0.20)} = \underline{2342\ psi}$



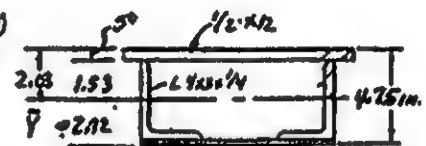
9-20 $I_x = 46.64\ IN^4$ FROM P7-33.

$Q = A_1\bar{y}_1 + A_2\bar{y}_2$

$Q = (2)(0.25)(1.53)(1.53/2) + (0.5)(12)(1.78)$

$Q = 11.27\ IN^3$

$\tau = \frac{(2500)(11.27)}{(46.64)(0.50)} = \underline{1208\ psi}$



PROBLEMS 9-21 THRU 9-30: GIVEN $\tau_{max} = 70\ psi = \frac{VQ}{It}$

$V_{max} = \frac{\tau I t}{Q}$

9-21 $t = 1.50$ in; $h = 3.50$ IN; $I = th^3/12 = 5.36\ IN^4$: $Q = t(h/2)(h/4) = 2.297\ IN^3$

$V = \frac{(70)(5.36)(1.50)}{2.297} = \underline{245\ LB}$

9-22 $t = 3.5$ IN; $h = 1.50$ IN; $I = (3.5)(1.5)^3/12 = 0.984\ IN^4$: $Q = (3.5)(1.50/2)(1.50/4) = 0.984\ IN^3$

$V = (70)(0.984)(3.5)/0.984 = \underline{245\ LB}$

9-23 $t = 1.50$ IN : $h = 11.25$ IN : $I = 178\ IN^4$: $Q = (1.50)(11.25/2)(11.25/4) = 23.73\ IN^3$

$V = (70)(178)(1.50)/23.73 = \underline{788\ LB}$

9-24 $t = 11.25$ IN : $h = 1.50$ IN : $I = 11.25(1.50)^3/12 = 3.16\ IN^4$: $Q = (11.25)(1.50/2)(1.50/4) = 3.16\ IN^3$

$V = (70)(3.16)(11.25)/3.16 = \underline{788\ LB}$

9-25 $t = 9.5\ IN : h = 11.5\ IN : I = 1204\ IN^4 : Q = (9.5)(11.5/2)(11.5/4) = 157\ IN^3$

$V = (70)(1204)(9.5)/157 = \underline{5098\ LB}$

9-26 $t = 11.5\ IN : h = 9.5\ IN : I = 11.5(9.5)^3/12 = 822\ IN^4 :$

$Q = (11.5)(9.5/2)(9.5/4) = 129.7\ IN^3$

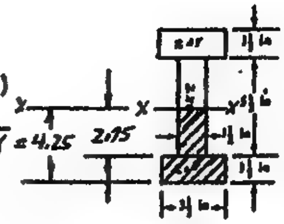
$V = (70)(822)(11.5)/129.7 = \underline{5098\ LB}$

9-27 $I_x = 151.4\ IN^4$ FROM P7-21.

$Q = A_1\bar{y}_1 + A_2\bar{y}_2 = (2.75)(1.5)(2.75/2) + (3.5)(1.5)(3.5)$

$Q = 24.05\ IN^3$

$V = (70)(151.4)(1.50)/24.05 = \underline{661\ LB}$



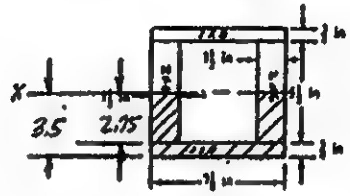
9-28 $I_x = 148.3\ IN^4$ FROM P7-22.

$Q = A_1\bar{y}_1 + A_2\bar{y}_2$

$Q = (2)(1.5)(2.75)(2.75/2) + (.75)(7.25)(3.125)$

$Q = 28.33\ IN^3$

$V = (70)(148.3)(3.0)/28.33 = \underline{1099\ LB}$



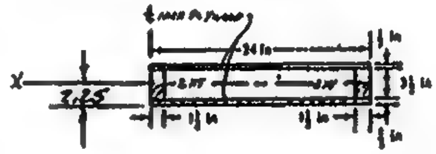
9-29 $I_x = 107.2\ IN^4$ FROM P7-23.

$Q = A_1\bar{y}_1 + A_2\bar{y}_2$

$Q = (2)(1.5)(1.75)(1.75/2) + (.5)(28)(2.00)$

$Q = 28.59\ IN^3$

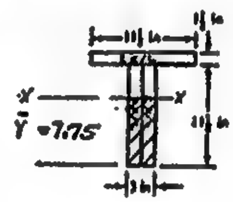
$V = (70)(107.2)(3.0)/28.59 = \underline{787\ LB}$



9-30 $I_x = 816.3\ IN^4$ FROM P7-24.

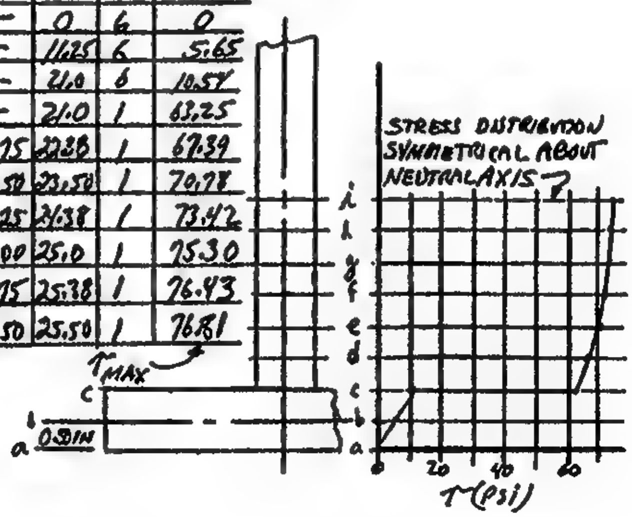
$Q = A_p\bar{y} = (7.75)(3.0)(7.75/2) = 90.09\ IN^3$

$V = (70)(816.3)(3.0)/90.09 = \underline{1903\ LB}$



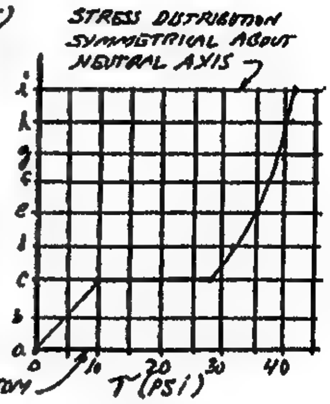
9-31 $I_x = 166\ IN^4$ FROM P7-2 : $V = 500\ LB : \tau = \frac{VQ}{It}$

SECTION	A_1	$\bar{y}_1$	A_2	$\bar{y}_2$	Q	t	τ (PSI)
a-a	0	—	—	—	0	6	0
b-b	3.0	3.75	—	—	11.25	6	5.65
c-c	6.0	3.50	—	—	21.0	6	10.54
c-c'	6.0	3.5	—	—	21.0	1	63.25
d-d	6.0	3.5	0.50	2.75	22.38	1	67.39
e-e	6.0	3.5	1.0	2.50	23.50	1	70.78
f-f	6.0	3.5	1.5	2.25	24.38	1	73.42
g-g	6.0	3.5	2.0	2.00	25.0	1	75.30
h-h	6.0	3.5	2.5	1.75	25.38	1	76.43
i-i	6.0	3.5	3.0	1.50	25.50	1	76.81



9-32 $I_x = 184\ IN^4$ FROM P7-3 : $V = 500\ LB$; $\tau = VQ/It$

SECTION	A_1	y_1	A_2	y_2	Q	t	τ (PSI)
a-a	0	—	—	—	0	6	0
b-b	3.0	3.75	—	—	11.25	6	5.10
c-c	6.0	3.50	—	—	21.0	6	9.51
c-c'	6.0	3.50	—	—	21.0	2	28.5
d-d	6.0	3.50	1.00	2.75	23.75	2	32.3
e-e	6.0	3.5	2.00	2.50	26.0	2	35.3
f-f	6.0	3.5	3.00	2.25	27.75	2	37.7
g-g	6.0	3.5	4.00	2.00	29.0	2	39.4
h-h	6.0	3.5	5.00	1.75	29.75	2	40.4
i-i	6.0	3.5	6.00	1.50	30.0	2	40.8

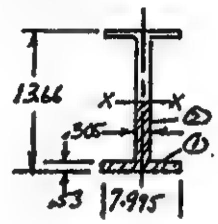


9-33 W14X43 : $I_x = 428\ in^4$

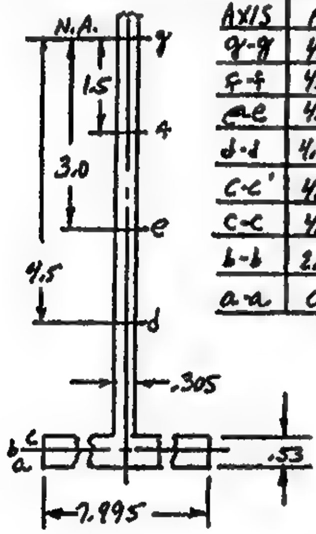
$Q = A_1 y_1 + A_2 y_2 = (7.995)(.53)(6.57) + (6.30)(0.305)(3.15)$

$Q = 33.89\ IN^3$

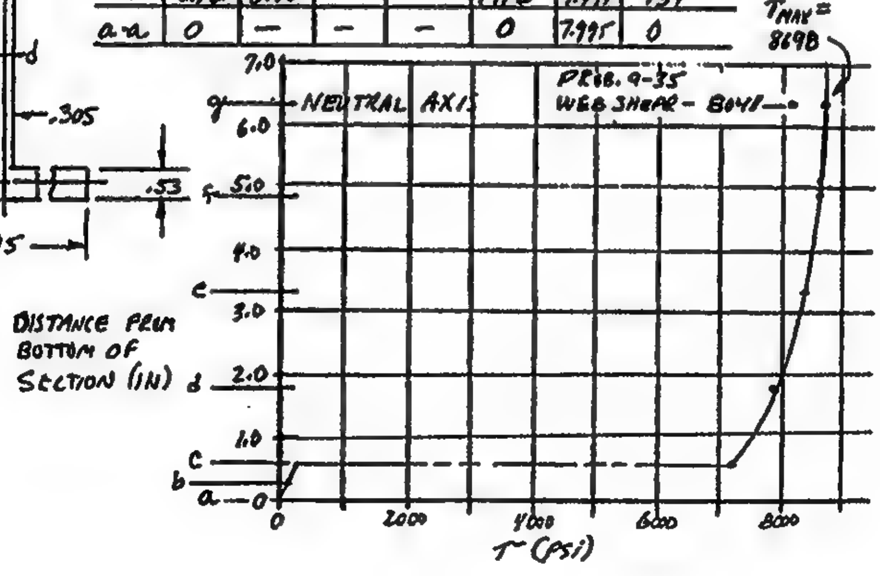
$\tau = \frac{VQ}{It} = \frac{(33\,500)(33.89)}{(428)(0.305)} = \underline{8698\ PSI}$ AT X-X



9-34 W14X43 : $I_x = 428\ IN^4$



AXIS	A_1	y_1	A_2	y_2	Q	t	τ
g-g	4.24	6.57	1.922	3.15	33.89	.305	8698
f-f	4.24	6.57	1.464	3.90	33.51	.305	8599
e-e	4.24	6.57	1.007	4.65	32.48	.305	8336
d-d	4.24	6.57	.549	5.40	30.76	.305	7895
c-c'	4.24	6.57	—	—	27.8	.305	7134
c-c	4.24	6.57	—	—	27.8	7.995	273
b-b	2.12	6.70	—	—	14.2	7.995	139
a-a	0	—	—	—	0	7.995	0



9-35 W14X43: FOR WEB SHEAR FORMULA: $t = 0.305$ IN; $h = 13.66$ IN

$$\tau_{ws} \approx \frac{V}{th} = \frac{33500 \text{ LB}}{(.305)(13.66)\text{ IN}^2} = \underline{8041 \text{ PSI}}$$ GRAPH ON PREVIOUS PAGE

FROM PROB 9-34: $\tau_{MAX} = 8698$ PSI

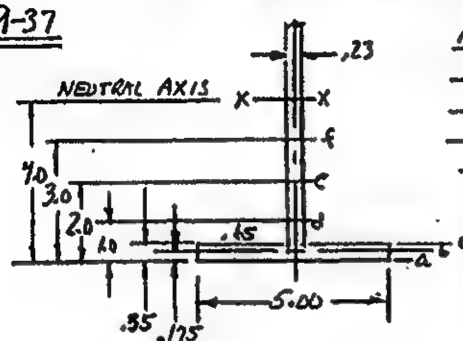
$$\tau_{WS}/\tau_{MAX} = 8041/8698 = \underline{0.92}$$

9-36 I8×6.181 ALUMINUM I-BEAM: $I_x = 59.69 \text{ IN}^4$

$$Q = A_1\bar{y}_1 + A_2\bar{y}_2 = (5.00)(0.35)(3.825) + (3.65)(0.23)(1.825) = 8.226 \text{ IN}^3$$

$$\tau = \frac{VQ}{It} = \frac{(13500 \text{ LB})(8.226 \text{ IN}^3)}{(59.69 \text{ IN}^4)(0.23 \text{ IN.})} = \underline{8089 \text{ PSI}}$$

9-37



AXIS	A_1	$\bar{y}_1$	A_2	$\bar{y}_2$	Q	t	τ
X-X	1.75	3.825	.840	1.825	8.226	.23	8089
f	1.75	3.825	.610	2.325	8.111	.23	7976
e	1.75	3.825	.380	2.825	7.767	.23	7638
d	1.75	3.825	.150	3.325	7.193	.23	7073
c′	1.75	3.826	—	—	6.694	.23	6582
c	1.75	3.825	—	—	6.694	5.00	303
b	.875	3.913	—	—	3.423	5.00	155
a	0	—	—	—	0	5.00	0

9-38 ALUMINUM I8×6.18

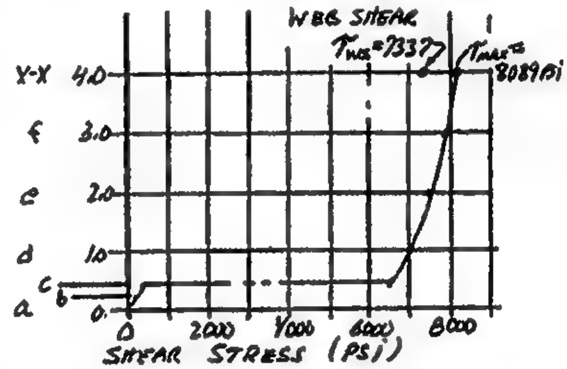
$t = 0.23$ IN; $h = 8.00$ IN

WEB SHEAR FORMULA

$$\tau_{ws} = \frac{V}{th} = \frac{13500 \text{ LB}}{(.23)(8.0)\text{ IN}^2}$$

$\tau_{ws} = \underline{7337 \text{ PSI}}$

$$\tau_{ws}/\tau_{MAX} = \frac{7337}{8089} = \underline{0.907}$$



9-39 $V_{MAX} = 10K = 10000$ LB; $M_{MAX} = 30$ K·FT $= 3.6\times10^5$ LB·IN. SEE PROB. P6-4.

W10X15: $I_x = 69.8 \text{ IN}^4$; $t = 0.23$; $h = 9.99$ IN; $S = 13.8 \text{ IN}^3$

$$\tau_{WS} = \frac{V}{th} = \frac{10000 \text{ LB}}{(.23)(9.99)\text{ IN}^2} = \underline{4352 \text{ PSI}}; \quad \tau_d = .4S_y = .4(36000) = \underline{14400 \text{ PSI}}: \underline{\text{SAFE}}$$

$$\sigma = \frac{M}{S} = \frac{3.6\times10^5 \text{ LB·IN}}{13.8 \text{ IN}^3} = \underline{26100 \text{ PSI}}: \quad \sigma_d = 0.66S_y = \underline{23760 \text{ PSI}}: \underline{\text{UNSAFE}}$$

9-40 $M_{MAX} = 3.6\times10^5$ LB·IN & $\sigma_d = 23760$ PSI (PROB 9-39). SEE ALSO P6-4.

$$S_{MIN} = \frac{M}{\sigma_d} = \frac{3.6\times10^5 \text{ LB·IN}}{23760 \text{ LB/IN}^2} = \underline{15.15 \text{ IN}^3}: \underline{W12X16}: S = 17.1 \text{ IN}^3; t = .22 \text{ IN}; h = 11.99 \text{ IN}$$

$$\tau_{WS} = \frac{V}{th} = \frac{10000 \text{ LB}}{(.22)(11.99)\text{ IN}^2} = \underline{3791 \text{ PSI}}: \quad \tau_d = 0.4S_y = \underline{14400 \text{ PSI}}: \underline{\text{SAFE}}$$

9-41 FROM PROB P6-52 : $V_{MAX} = 44.5K = 44500 LB$: $M_{MAX} = 148K\cdot FT = 1.776\times10^6 LB\cdot IN$

$S_{min} = \frac{M}{\sigma_d} = \frac{1.776\times10^6 LB\cdot IN}{23760 LB/IN^2} = \underline{74.7 IN^3}$: $\underline{W18\times55}$: $S = 98.3 IN^3$; $t = .390 IN$, $h = 18.11 IN$

$\tau = \frac{V}{th} = \frac{44500 LB}{(.39)(18.11) IN^2} = \underline{6300 PSI}$: $\tau_d = 0.4 S_y = \underline{14400 PSI}$: SAFE

9-42 FROM PROB P6-51 : $V_{MAX} = 162.9 kN \times \frac{1000 N}{kN} \times \frac{.2248 LB}{N} = 36620 LB$

$M_{MAX} = 228 kN\cdot m \times \frac{1000 N}{kN} \times \frac{8.851 LB\cdot IN}{N\cdot m} = 2.018\times10^6 LB\cdot IN$

$S_{MIN} = \frac{M}{\sigma_d} = \frac{2.018\times10^6 LB\cdot IN}{23760 LB/IN^2} = \underline{84.9 IN^3}$: $\underline{W18\times55}$: $S = 98.3 IN^3$; $t = 0.390 IN$, $h = 18.11 IN$

$\tau = \frac{V}{th} = \frac{36620 LB}{(.39)(18.11) IN^2} = \underline{5185 PSI}$: $\tau_d = 0.4 S_y = \underline{14400 PSI}$: SAFE

9-43 FROM PROB P6-51 : $V_{MAX} = 804 LB$: $M_{MAX} = 2528 LB\cdot IN$

$\sigma_d = S_y/3 = 48000 PSI/3 = 16000 PSI$: $\tau_d = 0.5 S_y/3 = 8000 PSI$

$S_{MIN} = \frac{M}{\sigma_d} = \frac{2528 LB\cdot IN}{16000 LB/IN^2} = \underline{0.158 IN^3}$: $\underline{1\tfrac{1}{4} in \text{ SCH 40 PIPE}}$: $S = 0.2346 IN^3$, $A = 0.669 IN^2$

$\tau \approx \frac{2(V)}{A} = \frac{804 LB(2)}{(0.669) IN^2} = \underline{2404 PSI}$ SAFE

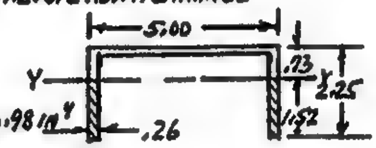
9-44 FROM PROB P6-9 : $V_{MAX} = 1557 LB$: $M_{MAX} = 6228 LB\cdot IN$

$\sigma_d = S_y/4 = 40000 PSI/4 = 10000 PSI$: $\tau_d = 0.5 S_y/4 = 5000 PSI$

$S_{MIN} = \frac{M}{\sigma_d} = \frac{6228 LB\cdot IN}{10000 LB/IN^2} = 0.623 IN^3$: C 5×2.212 ALUM. CHANNEL

$\tau = \frac{VQ}{It} = \frac{(1557)[(2)(.26)(1.52)](1.52/2)}{(0.98)(.52)}$

$\tau = \underline{1835 PSI}$ SAFE

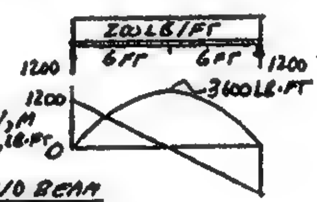


9-45 SHEAR: $\tau = \frac{3V}{2A} = \tau_d = 75 PSI$ (RECT. SECTION)

REQD $A = \frac{3V}{2\tau_d} = \frac{3(1200 LB)}{2(75 LB/IN^2)} = \underline{24 IN^2}$ $\underline{4\times8 \text{ BEAM}}$

BENDING: $\sigma = \frac{M}{S} = \sigma_d = 1150 PSI$

REQD $S = \frac{M}{\sigma_d} = \frac{(3600 LB\cdot FT)(12 IN/FT)}{1150 LB/IN^2} = \underline{37.6 IN^3}$ $\underline{4\times10 \text{ BEAM}}$



9-46 FROM PROB P6-53 : $V_{MAX} = 2950 N$: $M_{MAX} = 3350 N\cdot m$

SHEAR: $A_{MIN} = \frac{3V}{2\tau_d} = \frac{(3)(2950 N)}{(2)(0.66 N/mm^2)} = \underline{6705 mm^2}$ $\underline{2\times8 \text{ BEAM}}$ OR $\underline{4\times4 \text{ BEAM}}$

BENDING: $S = \frac{M}{\sigma_d} = \frac{3350 N\cdot m}{5.5 N/mm^2} \times \frac{10^3 mm}{m} = 609\times10^3 mm^3$: $\underline{\text{USE } 4\times10 \text{ BEAM}}$

9-47 FROM PROB 9-28 : $I_x = 148.3 IN^4$; $C = 3.50 IN$; $Q = 28.33 IN^3$. SEE ALSO P7-22.

$V_{MAX} = P$: $M_{MAX} = P(3 FT) = 36(P) LB\cdot IN$: $\tau_d = 80 PSI$; $\sigma_{MAX} = 1400 PSI$

$\tau = \frac{VQ}{It}$; $V_{MAX} = P_{MAX} = \frac{\tau I t}{Q} = \frac{(80)(148.3)(3.0)}{28.33} = \underline{1256 LB = P_{MAX}}$

$\sigma = \frac{Mc}{I} = \frac{36Pc}{I}$; $P = \frac{\sigma_d I}{36c} = \frac{(1400)(148.3)}{(36)(3.50)} = \underline{1648 LB}$

9-48 FROM PROB P6-8 : $V_{MAX} = 21.36\ kN = 21360\ N$

$I\ 9 \times 8.361$: $t = 0.27\ IN\ (25.4\ mm/IN) = 6.86\ mm$: $h = 9.00\ IN\ (25.4) = 229\ mm$

$\tau_{v} \approx \frac{V}{th} = \frac{21360\ N}{(6.86)(229)\ mm^2} = \underline{1.36\ MPa}$

9-49 FROM PROB P6-8 : $M_{MAX} = 43.2\ kN \cdot m$

$I\ 9 \times 8.361$: $S = 22.67\ IN^3 \times \frac{(25.4\ mm)^3}{IN^3} = 371\ 495\ mm^3$

$\sigma = \frac{M}{S} = \frac{43.2 \times 10^3\ N \cdot m}{371\ 495\ mm^3} \cdot \frac{10^3\ mm}{m} = \underline{116.3\ MPa}$

9-50 TOTAL LOAD $= W = wL = (80\ LB/FT)(12\ FT) = 960\ LB$: $V_{MAX} = W/2 = 480\ LB$

$\tau = \frac{3V}{2A} = \frac{(3)(480\ LB)}{2(10.87\ IN^2)} = \underline{66.2\ psi}$: $\underline{\tau_d = 70\ psi}$ $\underline{OK}$

CHECK BENDING: $M_{MAX} = wL^2/8 = \frac{(80)(12)^2}{8} = 1440\ LB \cdot FT\ (12\ IN/FT) = 17\ 280\ LB \cdot IN$

$\sigma = \frac{M}{S} = \frac{17280\ LB \cdot IN}{13.14\ IN^3} = \underline{1315\ psi}$: $\sigma_d = \underline{1000\ psi}$ $\underline{UNSAFE}$

9-51 FROM PROB P6-10 : $V_{MAX} = 1500\ LB$: $M_{MAX} = 9000\ LB \cdot IN$

(a) SHEAR: $\tau = \frac{3V}{2A} = \frac{3(1500\ LB)}{(2)(.50)(4.0)\ IN^2} = \underline{1125\ psi}$

(b) BENDING: $\sigma = \frac{M}{S} = \frac{9000\ LB \cdot IN}{(.5)(4.0)^2/6\ \ IN^3} = \underline{6750\ psi}$

(C) $\tau_d = 0.5\ S_y/3$: REQD $S_y = 3\tau_d/0.5 = 3(1125)/0.5 = 6750\ psi$

$\sigma_d = S_y/3$: REQD $S_y = 3\sigma_d = 3(6750) = \underline{20250\ psi}$ $\underline{ANY\ STEEL}$

9-52 FROM PROB P6-6 : $V_{MAX} = 1451\ N$: $M_{MAX} = 318\ N \cdot m$

(a) SHEAR: $\tau = \frac{3V}{2A} = \frac{3(1451\ N)}{(2)(16)(60)\ mm^2} = \underline{2.267\ MPa}$

(b) BENDING: $\sigma = \frac{M}{S} = \frac{318\ N \cdot m\ (10^3\ mm/m)}{(16)(60)^2/6\ \ mm^3} = \underline{33.1\ MPa}$

(C) $T_d = 0.5S_y/3$: REQD $S_y = \frac{3(\tau)}{0.5} = \frac{3(2.267)}{0.5} = \underline{13.6\ MPa}$

$\sigma_d = S_y/3$: REQD $S_y = 3\sigma = 3(33.1) = \underline{99.3\ MPa}$ 6061-T4: $S_y = 145\ MPa$ OR SEVERAL OTHERS

9-53 FROM PROB. P6-47 : $V_{MAX} = 450\ N$: $M_{MAX} = 172.5\ N \cdot m$

$\sigma_d = S_y/N = 276\ MPa/4 = 69\ MPa = M/S$:

REQD $S = \frac{M}{\sigma_d} = \frac{172\ 500\ N \cdot mm}{69\ N/mm^2} = 2500\ mm^3 = \underline{\frac{bh^2}{6}}$

REQD $h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{6(2500)\ mm^3}{12\ mm}} = \underline{35.4\ mm}$

$\tau = \frac{3V}{2A} = \frac{3(450\ N)}{(2)(12)(35.4)\ mm^2} = \underline{1.59\ MPa} = \underline{\frac{0.5S_y}{N}}$

$N = \frac{0.5S_y}{\tau} = \frac{0.5(276\ MPa)}{1.59\ MPa} = \underline{86.7}$ SAFE. VERY HIGH N.

9-54 FROM PROB P6-48 : $V_{max} = 1290\ N$: $M_{max} = 370.8\ N\cdot m$

(a) $\tau = \frac{4V}{3A} = \frac{4(1290 N)}{3(\pi)(40)^2/4\ mm^2} = \underline{1.37\ MPa} = \tau_d = \frac{0.5 S_y}{4} = \frac{S_y}{8}$

(b) $\sigma = \frac{M}{S} = \frac{370.8 \times 10^3\ N\cdot mm}{\pi(40)^3/32} = \underline{59.0\ MPa} = \sigma_d = \frac{S_y}{4}$;

(C) FOR SHEAR : REQD $S_y = 8\tau = 8(1.37) = \underline{11.0\ MPa}$

FOR BENDING : REQD $S_y = 4\sigma = 4(59.0) = \underline{236\ MPa}$ $\underline{AISI\ 1020\ HR\ S_y = 331\ MPa}$

9-55 FROM PROB P6-47 : $M_{max} = 172.5\ N\cdot m$: $V_{max} = 450\ N$

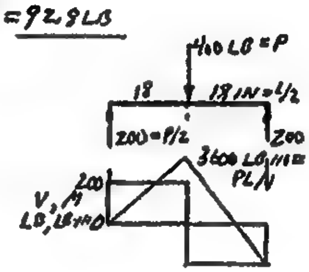
REQD. $S = \frac{M}{\sigma_d} = \frac{(172.5\ N\cdot m)(10^3\ mm/m)}{120\ N/mm^2} = 1438\ mm^3 = \pi(D)^3/32$

$D = \left[\frac{32 S}{\pi}\right]^{1/3} = \left[\frac{32(1438)}{\pi}\right]^{1/3} = \underline{24.5\ mm}$: $A = \pi D^2/4 = 470\ mm^2$

$\tau = \frac{4V}{3A} = \frac{4(450 N)}{3(470\ mm^2)} = \underline{1.28\ MPa}$

9-56 $\tau = 4V/3A$: $V_{max} = \frac{3A\tau}{4} = \frac{3(\pi)(1.50)^2 in^2 (70\ lb/in^2)}{(4)(4)} = \underline{92.9\ LB}$

9-57 $\tau_d = \frac{0.5 S_y}{6} = \frac{0.5(48000)}{6} = 4000\ PSI \approx \frac{2V}{A}$



$\sigma_d = S_y/6 = 48000/6 = 8000\ PSI = M/S$

REQD $S = \frac{M}{\sigma_d} = \frac{3600\ LB\cdot IN}{8000\ LB/IN^2} = 0.45\ IN^3$ $\underline{2\text{-}IN\ SCH\ 40\ PIPE}$

$A = 1.075\ IN^2$

$\tau = 2V/A = 2(200)/1.075 = \underline{372\ PSI}$ OK

9-58 $\sigma_d = S_y/4 = 48000/4 = 12000\ psi$; $\tau_d = 0.5S_y/4 = 6000\ PSI$; $P = 2800\ LB$

$V_{max} = P/2 = 1400\ LB$: $M_{max} = PL/4$ (PROB 9-57) : REQD $S = M/\sigma_d$: $\tau \approx 2V/A$

	L(IN)	M(LB·IN)	REQD S(IN³)	PIPE	A	$\tau \approx \frac{2V}{A}$
(a)	1.50	1050	0.0875	1	.494	5668 OK
(b)	3.00	2100	0.175	1 1/4	.669	4185 OK
(c)	4.50	3150	0.263	1 1/2	.799	3504 OK
(d)	6.00	4200	0.350	2	1.075	2605 OK

9-59 SHEAR FLOW AT JOINT = $q = VQ/I$

FROM FIG P7-14 : $I = 0.3672\ IN^4$; $\bar{Y} = 0.533\ IN$

$Q = A_p \bar{y} = (2.60)(0.20)(1.533 - 0.10) = 0.225\ IN^3$

$q = (1200\ LB)(0.225\ IN^3)/0.3672\ IN^4 = \underline{736\ LB/IN}$

ON 1.0 IN OF LENGTH, AREA OF GLUE $= (1.0\ IN)(2)(0.70\ IN) = 1.40\ IN^2 = A_s$

$\tau = \frac{736\ LB}{IN} \times \frac{1\ IN}{1.40\ IN^2} = \underline{526\ psi}$

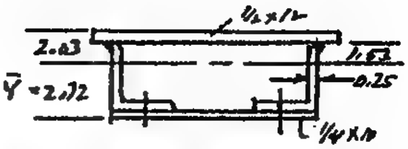
9-60 FROM PROB P7-26 : $I = 469.4\ IN^4$: $\bar{Y} = 7.90\ IN$: FLANGE WIDTH $= 5.477\ IN$ FOR S12×50

$Q = A y = (7.35)(3.81) = 28.0\ IN^3$ FOR ENTIRE CHANNEL

$q = VQ/I = (2500)(28.0)/469.4 = \underline{149\ LB/IN}$

$A_s = (1.0\ IN)(5.477\ IN) = 5.477\ IN^2/IN$

$\tau = q/A_s = 149\ LB/5.477\ IN^2 = \underline{27.2\ PSI}$

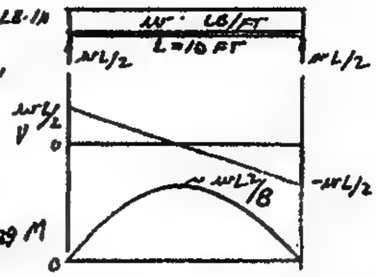
9-61 FROM PROB P7-33 : $I = 46.6\ IN^4$

$\sigma_d = S_y/4 = 21000/4 = 5250\ PSI$

$\tau_d = 0.5 S_y/4 = 2625\ PSI$

BENDING: $\sigma = Mc/I$

$M_{ALLOW} = \dfrac{\sigma_d I}{c} = \dfrac{(5250)(46.6)}{2.72} = 89\,945\ LB\cdot IN$

$w = \dfrac{8M}{L^2} = \dfrac{8(89\,945\ LB\cdot IN)}{(120\ IN)^2} = 50.0\ LB/IN$



SHEAR AT NEUTRAL AXIS :

$Q = (0.5)(12)(2.03 - 0.25) + 2(1.53)(0.25)(1.53/2)$

$Q = 11.27\ IN^3$; $t = 2(0.25) = 0.50\ IN$

$\tau = \dfrac{VQ}{It}$; $V = \dfrac{\tau_d I t}{Q} = \dfrac{(2625)(46.6)(0.50)}{11.27} = 5429\ LB$

$w = \dfrac{2(V)}{L} = \dfrac{2(5429\ LB)}{120\ IN} = 90.5\ LB/IN$

SHEAR FLOW AT WELDS : $q = \dfrac{VQ}{I}$; $V = \dfrac{q\,I}{Q} = \dfrac{(1800)(46.6)}{(6.0)(2.03 - 0.25)} = 7854\ LB$

$w = 2V/L = 2(7854)/120 = 131\ LB/IN$

RIVETS : S = SPACING = 4.0 IN ; $F_{sd} = 2(600) = 1200\ LB = S q_d$

$q_d = \dfrac{F_{sd}}{S} = \dfrac{1200\ LB}{4.0\ IN} = 300\ LB/IN$

$q = \dfrac{VQ}{I}$; $V = \dfrac{qI}{Q} = \dfrac{(300\ LB/IN)(46.6\ IN^4)}{(2.50)(2.72 - 0.25)\ IN^3} = 2155\ LB$

$w = 2V/L = 2(2155\ LB)/120\ IN = 35.9\ LB/IN\ (12\ IN/FT) = 431\ LB/FT$ LIMITING VALUE

9-62 FROM PROB P7-24 : $I = 816.3\ IN^4$; $\bar{Y} = 7.75\ IN$

#3 SOUTHERN PINE

$\sigma_d = 650\ PSI$

$\tau_d = 70\ PSI$

BENDING: $\sigma = Mc/I$

$M = \dfrac{\sigma_d I}{c} = \dfrac{(650)(816.3)}{7.75} = 68\,464\ LB\cdot IN = wL^2/8$

$w = \dfrac{8M}{L^2} = \dfrac{8(68\,464)}{(120)^2} = 38.0\ LB/IN$

SHEAR AT NEUTRAL AXIS: $Q = A\bar{y} = (3.0)(7.75)(7.75/2) = 90.09\ IN^3$

$\tau = \dfrac{VQ}{It}$; $V = \dfrac{\tau_d I t}{Q} = \dfrac{(70)(816.3)(3.0)}{90.09} = 1903\ LB = wL/2$

$w = \dfrac{2V}{L} = \dfrac{2(1903)}{120} = 31.7\ LB/IN$

NAILS : $S = 6.0\ IN$; $F_{sd} = 2(160) = 320\ LB = S q_d$; $q_d = \dfrac{F_{sd}}{S} = \dfrac{320\ LB}{6.0\ IN} = 53.3\ LB/IN$

$q_d = \dfrac{VQ}{I}$; $V = \dfrac{q_d I}{Q} = \dfrac{(53.3)(816.3)}{(1.5)(11.25)(12.75 - 7.75 - 0.75)} = 607\ LB$

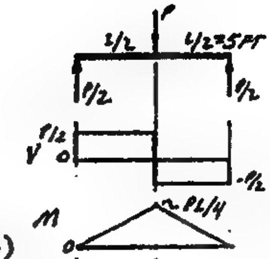
$w = \dfrac{2V}{L} = \dfrac{2(607)}{120} = 10.1\ LB/IN\ (12\ IN/FT) = 121\ LB/FT$ LIMITING

9-63 FROM PROB P7-21 : $I = 151.4\ IN^4$; $\bar{Y} = 4.25\ IN$

BENDING : $\sigma = MC/I$; $\sigma_d = 1450\ PSI$

$$M = \frac{\sigma_d I}{C} = \frac{(1450)(151.4)}{4.25} = 51\,654\ LB\cdot IN = PL/4$$

$$P = \frac{4M}{L} = \frac{4(51\,654\ LB\cdot IN)}{120\ IN} = \underline{1722\ LB}\ \ \underline{LIMITING}$$



SHEAR : $\tau = VQ/It$: $V = \tau_d It/Q$; $\tau_d = 95\ PSI$

AT NEUTRAL AXIS : $Q = (3.5)(1.5)(3.5) + (2.75)(1.5)(2.75/2)$

$Q = 24.05\ IN^3$

$$V = \frac{\tau_d I t}{Q} = \frac{(95)(151.4)(1.50)}{24.05} = 897\ LB = \frac{P}{2} : \ P = 2V = \underline{1794\ LB}$$

GLUE AT TOP OF WEB :

$$q = \frac{VQ}{I} = \frac{PQ}{2I} = \frac{P(3.5)(1.5)(3.5)}{2(151.4)} = 0.06068\ P\ LB/IN$$

$$\tau_G = \frac{q}{A_s} : q_d = \tau_G A_s = \frac{800\ LB}{IN^2} \times \frac{(0.5)(1.0)\ IN^2}{IN} = 1200\ LB/IN$$

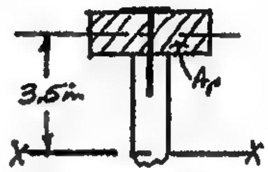
$$P = \frac{q_d}{0.0608} = \frac{1200}{0.0608} = \underline{19\,775\ LB}$$

9-64 $I = 151.4\ IN^4$: $Q = (3.5)(1.5)(3.5) = 18.375\ IN^3$

$$q = \frac{VQ}{I} = \frac{(300)(18.375)}{151.4} = 36.4\ LB/IN$$

$$S = \frac{F_{sd}}{q} = \frac{180\ LB}{36.4\ LB/IN} = \underline{4.94\ IN}$$

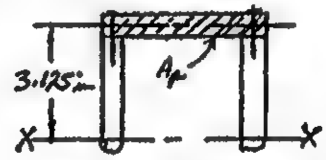
FIGURE P7-21



9-65 $I = 148.3\ IN^4$: $Q = (7.25)(0.75)(3.125) = 17.01\ IN^3$

$q = VQ/I = (600)(17.01)/148.3 = 68.8\ LB/IN$

$S = F_{sd}/q = 2(150)/68.8 = \underline{4.36\ IN}$



9-66 $I = 107.2\ IN^4$: $Q = (24)(0.5)(2.0) = 24.0\ IN^3$

$q = VQ/I = (500)(24.0)/107.2 = 112\ LB/IN$

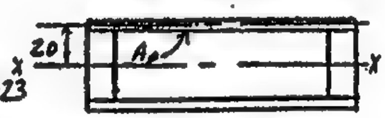


FIGURE P7-23

9-67 $V = 175\ kN = 175\,000\ N$: $V = \frac{175\,000\ N}{4.448\ N/LB} = 39\,344\ LB$

FROM PROB P7-25 : $I = 1155\ IN^4$; $\bar{Y} = 7.33\ IN$

$Q = (0.5)(14.5)(7.08) = 51.3\ IN^3$

$q = VQ/I = (39\,344)(51.3)/1155 = 1749\ LB/IN$

$$S = \frac{F_{sd}}{q} = \frac{(2)(2650)\ LB}{1749\ LB/IN} = \underline{3.03\ IN}$$

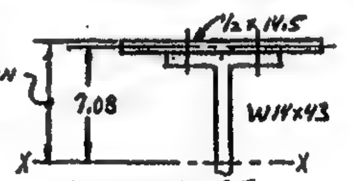


FIGURE P7-25

9-68 $V = 5000\ N(1\ LB/4.448\ N) = 1124\ LB$

FROM PROB P7-26 : $I = 469.4\ IN^4$

$Q = A\bar{y} = (7.35)(3.81) = 28.0\ IN^3$

$$q = \frac{VQ}{I} = \frac{(1124)(28.0)}{469.4} = 67\ LB/IN$$

$$S = \frac{F_{sd}}{q} = \frac{2(1750)}{67} = \underline{5.22\ IN}$$

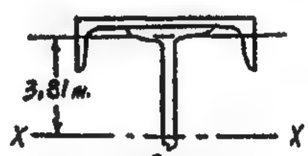


FIGURE P7-26

9-69 W18×55: $t = 0.390\,\text{IN}$, $h = 18.11\,\text{IN}$, ASTM A36 $S_y = 36\,000\,\text{PSI}$

$$\tau \cong \frac{V}{th} = \frac{36\,600\,\text{lb}}{(0.39)(18.11)\,\text{IN}^2} = \underline{5182\,\text{PSI}}\ \ \underline{\text{SAFE}}$$

$$\tau_d = 0.40\,S_y = 0.40(36\,000\,\text{PSI}) = 14\,400\,\text{PSI}\quad \text{OK}$$

9-70 W18×40: $t = 0.315\,\text{IN}$, $h = 17.90\,\text{IN}$. ASTM A242 $S_y = 50\,000\,\text{PSI}$

$$\tau \cong \frac{V}{th} = \frac{36\,600\,\text{lb}}{(0.315)(17.90)\,\text{IN}^2} = \underline{6491\,\text{PSI}}\ \ \underline{\text{SAFE}}$$

$$\tau_d = 0.40\,S_y = 0.40(50\,000\,\text{PSI}) = 20\,000\,\text{PSI}\quad \text{OK}$$

9-71 W14×26: $t = 0.255\,\text{IN}$, $h = 13.91\,\text{IN}$. ASTM A242 $S_y = 50\,000\,\text{PSI}$

$$\tau = \frac{V}{th} = \frac{10\,000\,\text{lb}}{(0.255)(13.91)\,\text{IN}^2} = \underline{2819\,\text{PSI}}\ \ \underline{\text{SAFE}}$$

$$\tau_d = 20\,000\,\text{PSI}\ \ (\text{PROB 9-70})$$

9-72 6I×4.692 ALUMINUM: $t = 0.21\,\text{IN}$ $h = 6.00\,\text{IN}$

$$\tau = \frac{V}{th} = \frac{10\,000\,\text{lb}}{(0.21)(6.00)} = \underline{7937\,\text{PSI}}\ \ \underline{\text{SAFE}}$$

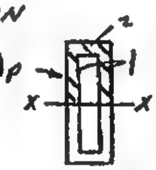
AL 6061-T6, $S_y = 40\,000\,\text{PSI}$; $\tau_d = S_y/2N = S_y/2(2) = 0.25\,S_y$

$$\tau_d = (0.25)(40\,000\,\text{PSI}) = 10\,000\,\text{PSI}\quad \text{OK.}$$

9-73 W10×13: $t = 0.190\,\text{IN}$ $h = 9.87\,\text{IN}$, ASTM A36, $\tau_d = 14\,400\,\text{PSI}$ (PROB 9-69)

$$\tau = \frac{V}{th} = \frac{6750\,\text{LB}}{(0.190)(9.87)\,\text{IN}^2} = \underline{3599\,\text{PSI}}\ \ \underline{\text{SAFE}}$$

9-74 HSS 6×2×1/4. $I = 13.8\,\text{IN}^4$, $t = 2(0.25) = 0.50\,\text{IN}$



$$Q = A_1\bar{y}_1 + A_2\bar{y}_2 = (0.50)(2.75)(1.375) + (0.25)(2.00)(2.875)$$

$$Q = 1.8906 + 1.4375 = 3.328\,\text{IN}^3$$

$$\tau = \frac{VQ}{It} = \frac{(6750\,\text{lb})(3.328\,\text{IN}^3)}{(13.8\,\text{IN}^4)(0.50\,\text{IN})} = \underline{3256\,\text{PSI}}\ \ \underline{\text{SAFE}}$$

$$\tau_d = \frac{S_y}{2N} = \frac{50\,000\,\text{PSI}}{2(2)} = 12\,500\,\text{PSI}\quad \text{OK}$$

ASTM A500, GR C
$S_y = 50\,000\,\text{PSI}$

9-75 2×8 WOOD BEAM: $A = 10.87\,\text{IN}^2$. No.2 SOUTH. PINE, $\tau_d = 70.0\,\text{PSI}$

$$\tau = \frac{3V}{2A} = \frac{3(480\,\text{lb})}{2(10.87\,\text{IN}^2)} = \underline{66.2\,\text{PSI}}\ \ \underline{\text{SAFE}}$$

9-76 FIG. P9-29: $Q = 28.59\,\text{IN}^3$ FROM PROB 9-29., $I_x = 107.2\,\text{IN}^4$

$$\tau = \frac{VQ}{It} = \frac{(750\,\text{lb})(28.59\,\text{IN}^3)}{(107.2\,\text{IN}^4)(3.0\,\text{IN})} = \underline{66.7\,\text{PSI}}\ \ \underline{\text{SAFE}}$$

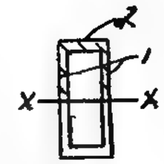
No. 2 DOUGLAS FIR, $\tau_d = 95\,\text{PSI}$ OK

9-77 HSS $8 \times 2 \times 1/4$: $I = 30.1\ in^4$, $t = (2)(0.25) = 0.50\ in.$

$Q = A_1 y_1 + A_2 y_2 = (0.50)(3.75)(1.875) + (0.25)(2.00)(3.875)$

$Q = 3.516 + 1.9375 = 5.454\ in^3$

$\tau = \dfrac{VQ}{It} = \dfrac{(1200\ lb)(5.454\ in^3)}{(30.1\ in^4)(0.50)\ in} = \underline{435\ psi}$ SAFE



ASTM A501
$s_y = 36000\ psi$

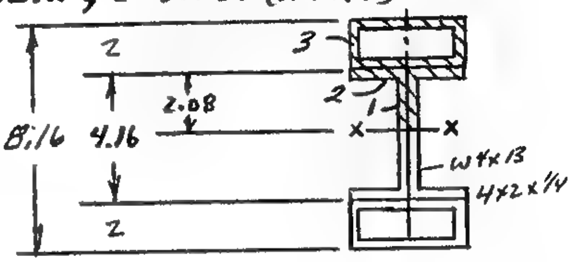
$\tau_d = s_y/2N = 36000\ psi/2(2) = 9000\ psi$ OK

9-78 FROM PROBLEM 7-42, $I = 63.52\ in^4$, $t = 0.280$ ($W4 \times 13$)

$Q = A_1 y_1 + A_2 y_2 + A_3 y_3$

	A	y	Ay
1	.486	.868	.422
2	1.401	1.908	2.673
3	2.59	3.080	7.977

$Q = 11.072\ in^3$



$\tau = \dfrac{VQ}{It} = \dfrac{(1800\ lb)(11.072\ in^3)}{(63.52\ in^4)(0.280)\ in} = \underline{1121\ psi}$ SAFE

$\tau_d = s_y/2N = 36000\ psi/2(2) = 9000\ psi$ OK

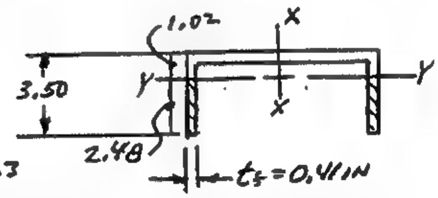
9-79 $C\ 10 \times 6.136$ ALUMINUM CHANNEL

$I_y = 6.33\ in^4$, $t_f = 0.41\ in$

Q FOR LOWER PART OF FLANGES

$Q = A y = 2(0.41)(2.48)(2.48/2) = 2.522\ in^3$

$\tau = \dfrac{VQ}{It} = \dfrac{(430\ lb)(2.522\ in^3)}{(6.33\ in^4)(2 \times 0.41\ in)} = \underline{209\ psi}$ SAFE



6061-T6 ALUM.
$s_y = 40000\ psi$

$\tau_d = s_y/2N = 40000/2(2) = 10000\ psi$ OK

9-80 DATA OF PROB. 9-78. SHEAR FLOW $q = \dfrac{VQ}{I}$

$I = 63.52\ in^4$, $V = 1800\ lb$

$Q = Ay$ FOR ONE $4 \times 2 \times 1/4$ TUBE

$Q = (2.59\ in^2)(3.08)\ in = 7.977\ in^3$

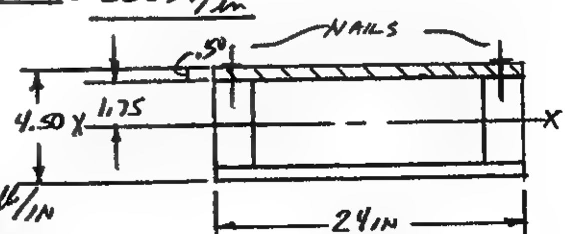
$q = \dfrac{VQ}{I} = \dfrac{(1800\ lb)(7.977\ in^3)}{63.52\ in^4} = \underline{226\ lb/in}$

9-81 FIG. P9-29: $I = 107.2\ in^4$

$Q = AY$ FOR TOP PANEL

$Q = (0.50)(24)(2.00) = 24.0\ in^3$

$q = \dfrac{VQ}{I} = \dfrac{(500\ lb)(24.0\ in^3)}{107.2\ in^4} = 112\ lb/in$



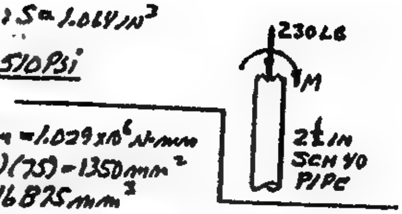
$F_{sd} = 135\ lb/NAIL \times 2\ NAILS = 270\ lb$

MAX. SPACING $= s_{max} = F_{sd}/q = 270\ lb / 112\ lb/in = \underline{2.41\ IN\ MAXIMUM}$

SPECIFY $\underline{s = 2.25\ IN}$

10-1 $M = (230\,LB)(48\,IN) = 11040\,LB\text{-}IN$; $A = 1.704\,IN^2$; $S = 1.064\,IN^3$

$$\sigma = \frac{-F}{A} - \frac{M}{S} = \frac{-230\,LB}{1.704\,IN^2} - \frac{11040\,LB\text{-}IN}{1.064\,IN^3} = \underline{-10\,510\,PSI}$$



10-2

$F_h = F\sin 30° = 1.70\,kN$

$F = 3.4\,kN$

$F_v = F\cos 30° = 2.94\,kN$

$M = F_v \cdot 350\,mm = 1.029 \times 10^6\,N\cdot mm$

$AREA = bh = (18)(75) = 1350\,mm^2$

$S = bh^2/6 = 16875\,mm^3$

σ_{MAX} OCCURS ON TOP OF BRACKET AT WALL

$$\sigma_{MAX} = \frac{F_h}{A} + \frac{M}{S} = \frac{1701\,N}{1350\,mm^2} + \frac{1.029 \times 10^6\,N\cdot mm}{16875\,mm^3} = \underline{62.2\,MPa}$$

10-3

$F_h = F\cos 40° = 4596\,LB$

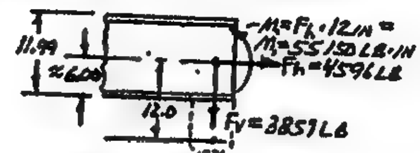
$F = 6000\,LB$

$F_v = F\sin 40° = 3857\,LB$

MOMENT DUE TO F_v: $M_2 = F_v \cdot 52\,IN$

$M_2 = 2.006 \times 10^5\,LB\cdot IN$

W12 X 16 BEAM: $A = 4.71\,IN^3$; $S = 17.1\,IN^3$



AT N: $\sigma_N = \frac{F_h}{A} + \frac{M_2}{S} - \frac{M_1}{S} = \frac{4596}{4.71} + \frac{2.006 \times 10^5}{17.1} - \frac{55150}{17.1} = \underline{9480\,PSI}$

AT M: $\sigma_M = \frac{F_h}{A} - \frac{M_2}{S} + \frac{M_1}{S} = \underline{-7530\,PSI}$

10-4 $M = 6000\,LB \cdot 52\,IN = 3.12 \times 10^5\,LB\cdot IN$

$$\sigma = \frac{M}{S} = \frac{3.12 \times 10^5\,LB\cdot IN}{17.1\,IN^3} = \underline{18246\,PSI}$$ TENSION AT N; COMPR. AT M

10-5 LOADING IS SAME AS 11-3 EXCEPT F_h ACTS TO LEFT AND M_1 IS OPPOSITE DIRECTION.

AT N: $\sigma_N = \frac{-F_h}{A} + \frac{M_2}{S} + \frac{M_1}{S} = \frac{-4596}{4.71} + \frac{2.006 \times 10^5}{17.1} + \frac{55150}{17.1} = \underline{13980\,PSI}$

AT M: $\sigma_M = \frac{-F_h}{A} - \frac{M_2}{S} - \frac{M_1}{S} = -975 - 11\,731 - 3225 = \underline{-15931\,PSI}$

10-6 $M = (125\,N)(145\,mm) = 18125\,N\cdot mm$; $S = bh^2/6 = \frac{(4)(10^2)}{6} = 66.67\,mm^3$

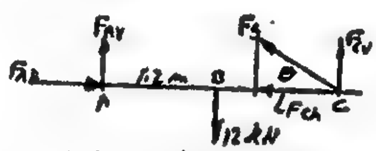
$$\sigma = \frac{-F}{A} - \frac{M}{S} = \frac{-125\,N}{40\,mm^2} - \frac{18125\,N\cdot mm}{66.67\,mm^3} = -3.125 - 271.9 = \underline{-275\,MPa}$$

10-7 $F_{AV} = F_{CV} = 12\,kN/2 = 6.0\,kN$

$\tan\theta = 1.5/2.4 = 0.625$

$F_{Ch} = F_{Ah} = \frac{F_{CV}}{\tan\theta} = \frac{6.0\,kN}{0.625} = 9.6\,kN$

$M = F_{AV} \cdot 1.2\,m = 7.2\,kN\cdot m$ AT B



W6X12: $A = 3.55\,IN^2$

$A = 2290\,mm^2$

$S = 7.31\,IN^3$

$S = 1.20 \times 10^5\,mm^3$

$$\sigma = \frac{-F_h}{A} - \frac{M}{S} = \frac{-9600\,N}{2290\,mm^2} - \frac{7.2 \times 10^6\,N\cdot mm}{1.20 \times 10^5\,mm^3}$$

$\sigma = -4.19 - 60.1 = \underline{-64.3\,MPa}$

COMPRESSION ON TOP OF BEAM

10-8 $M = 160N \cdot 80mm = 12\,800\,N\cdot mm$; $A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(12^2 - 10^2) = 34.6\ mm^2$

$S = \frac{\pi}{32}\frac{(D^4 - d^4)}{D} = \frac{\pi(12^4 - 10^4)}{32(12)} = 87.8\ mm^3$

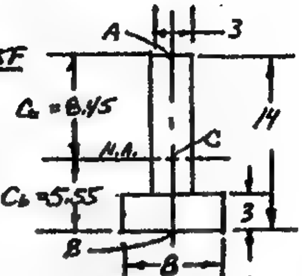
AT POINT A: $\sigma = \frac{-F}{A} - \frac{M}{S}$

$\sigma = \frac{-160N}{34.6mm^2} - \frac{12800\,N\cdot mm}{87.8\,mm^3} = \underline{150.4\ MPa}$

10-9 F.B.D. OF TOP PART OF CLAMP

$M = F(26+5.5) = \underline{31.55F}$

CAST ZINC ZA 12



DESIGN STRESS: $\sigma_d = \frac{S_u}{4} = \frac{400\ MPa}{4} = 100\ MPa$

$I = 1032\ mm^4$
$A = 57\ mm^2$

AT A: $\sigma_A = \frac{F}{A} - \frac{Mc_t}{I} = \frac{F}{57} - \frac{31.55F(8.45)}{1032}$

LET $\sigma_A = \sigma_d = 100\ MPa = -0.2408\,F$ COMPR.

$F = 100/0.2408 = \underline{415\ N}$

AT B: $\sigma_B = \frac{F}{A} + \frac{Mc_b}{I} = \frac{F}{57} + \frac{31.55F(5.55)}{1032} = 0.1872\ F$ TENSION

LET $\sigma_B = \sigma_d = 100\ MPa = 0.1872\,F$

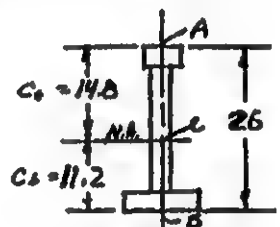
$F = 100/0.1872 = \underline{534N}$

LIMITING VALUE OF $\underline{F = 415N}$ IF S_u IS EQUAL IN TENSION AND COMPRESSION. NOTE THAT $S_{uc} > S_{ut}$ FOR SOME CAST ALLOYS.

10-10 ASTM A220 45008: $S_{ut} = 448\ MPa$, $S_{uc} = 1650\ MPa$

$\sigma_{dt} = 448/4 = 112\ MPa$; $\sigma_{dc} = 1650/4 = 412\ MPa$

$M = F(42 + 11.2) = 53.2\ F$



AT A: $\sigma_A = \frac{F}{A} - \frac{Mc_t}{I}$

$\sigma_A = \frac{F}{114} - \frac{53.2F(14.8)}{8806}$

$A = 114\ mm^2$
$I = 8806\ mm^4$

$\sigma_A = -0.0806F = \sigma_{dc} = 412\ MPa$

$F = 412/0.0806 = 5109\ N$

AT B: $\sigma_B = \frac{F}{A} + \frac{Mc_b}{I} = \frac{F}{114} + \frac{53.2F(11.2)}{8806} = 0.0764\,F = \sigma_{dt} = 112\ MPa$

$F = 112/0.0764 = \underline{1465\ N}$ LIMITING VALUE

10-11 FOR M16×2 THREADED ROD: STRESS AREA = 157 mm² (APP A3)

COMPUTE SECTION MODULUS BASED ON D_{ROOT}: $S = \frac{\pi D_R^3}{32} = \frac{\pi(13.2)^3}{32} = 226\ mm^3$

$\sigma = K_t\left[\frac{F}{A} + \frac{M}{S}\right] = 3.0\left[\frac{1200}{157} + \frac{(1200)(41)}{226}\right] = \underline{724\ MPa}$ TENSION

FOR $\sigma_d = S_y/2$; $S_y = 2\sigma = 2(724) = 1448\ MPa$

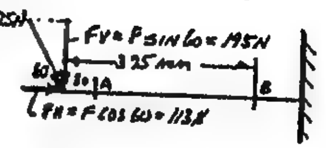
USE $\underline{\text{AISI 4140 OQT 700}}$, $S_y = 1462\ MPa$ – ONE POSSIBLE CHOICE

10-12 $M_A = F_V(30) = 5850\ N \cdot mm$

$M_B = F_V(375) = 73125\ N \cdot mm$

$S_A = \frac{bh^2}{6} = \frac{10(14)^2}{6} = 327\ mm^3;\ A_A = bh = 140\ mm^2$

$S_B = \frac{10(25)^2}{6} = 1042\ mm^3;\ A_B = bh = 250\ mm^2$



ASSUME $S_{ut} << S_{uc}$ – TENSION GOVERNS DESIGN FACTOR. $S_{ut} = 552\ MPa$

AT A: $\sigma_A = \frac{-F_h}{A_A} + \frac{M_A}{S_A} = \frac{-113}{140} + \frac{5850}{327} = 17.08\ MPa$

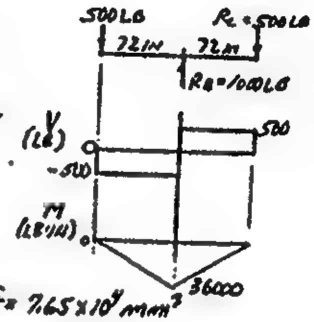
AT B: $\sigma_B = \frac{-F_h}{A_B} + \frac{M_B}{S_B} = \frac{-113}{250} + \frac{73125}{1042} = \underline{69.7\ MPa}$

$N = \frac{S_{ut}}{\sigma_B} = \frac{552\ MPa}{69.7\ MPa} = \underline{7.9}$

10-13 FOR S3×5.7: $A = 1.67\ in^2$; $S = 1.68\ in^3$

AT B-TOP: $\sigma = \frac{F}{A} + \frac{M}{S} = \frac{4600}{1.67} + \frac{36000}{1.68} = \underline{24183\ psi}$ TENSION

AT B-BOTTOM: $\sigma = \frac{F}{A} - \frac{M}{S} = \frac{4600}{1.67} - \frac{36000}{1.68} = \underline{-18674\ psi}$ COMPRESSION



10-14 FOR TUBE: $A = 2256\ mm^2$; $I = 5.74 \times 10^6\ mm^4$; $S = \frac{I}{c} = 7.65 \times 10^4\ mm^3$

LOAD: $w = m \cdot g = 1000\ kg \cdot 9.81\ m/s^2 = 9810\ N$

$\Sigma M_A = 0 = 9810(1.6) - B_V(0.6)$

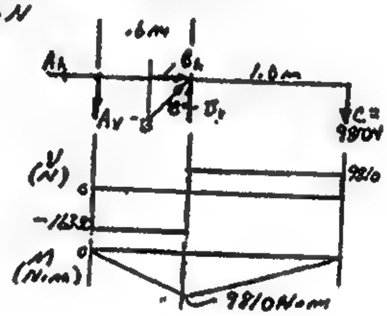
$B_V = 26160\ N = B_h = A_h$

$A_V = B_V - C_V = 26160 - 9810 = 16350\ N$

NEAR B – TO LEFT:

$\sigma = \frac{B_h}{A} + \frac{M_B}{S} = \frac{26160}{2256} + \frac{9810 \times 10^3}{7.65 \times 10^4}$

$\underline{\sigma = 140\ MPa}$



10-15 σ_{MAX} TO LEFT OF B (SEE 11-14).

$M_B = (1.0\ m)(F) = (1000\ mm)F$

$B_V = B_H = \frac{1.6F}{0.6} = 2.67F$

$\sigma_d = \frac{S_y}{3} = \frac{414\ MPa}{3} = 138\ MPa$

$\sigma = \frac{B_H}{A} + \frac{M_B}{S} = \frac{2.67F}{2256} + \frac{1000F}{7.65 \times 10^4} = 0.0143F = 138\ MPa = \sigma_d$

$F = 138/0.0143 = \underline{9650\ N}$: $m = \frac{F}{g} = \frac{9650\ kg \cdot m/s^2}{9.81\ m/s^2} = \underline{983\ kg}$

10-16

$\sigma_d = 0.6S_y = 0.6(36000) = 21600\ psi$

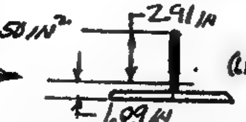
$\sigma = \frac{P}{A} + \frac{M}{S}$ USE TRIAL AND ERROR

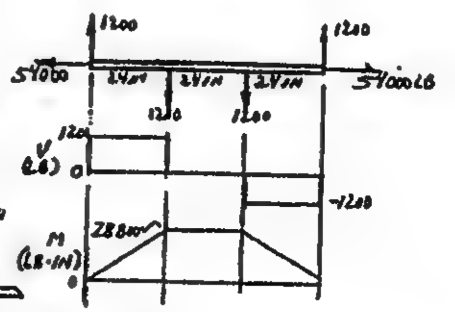
FOR BENDING ONLY:

$S = \frac{M}{\sigma_d} = \frac{28800}{21600} = 1.33\ in^3$

FOR AXIAL TENSION:

$A = \frac{P}{\sigma_d} = \frac{54000}{21600} = 2.50\ in^2$





TRY L4×4×1/4: (TWO)

$A = 2(1.94) = 3.88\ in^2$

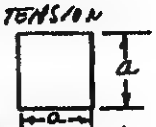
$I = 2I_x = 2(3.04) = 6.08\ in^4$; $c_b = y = 1.09\ in$; $c_t = 4.0 - 1.09 = 2.91\ in$

AT BOTTOM: $\sigma = \frac{P}{A} + \frac{Mc_b}{I} = \frac{54000}{3.88} + \frac{28800(1.09)}{6.08} = \underline{19081\ psi}$

$\underline{OK}$

10-17 FROM PROBLEM P6-77: $M = 120\ N\cdot m$; $P = 800\ N$ AXIAL TENSION

$$\sigma = \frac{P}{A} + \frac{M}{S} = \frac{P}{a^2} + \frac{M}{a^3/6} = \frac{P}{a^2} + \frac{6M}{a^3}$$



MULTIPLY BY a^3

$\sigma a^3 = Pa + 6M$; $\sigma a^3 - Pa - 6M = 0$; LET $\sigma = 42\ MPa = 42\ N/mm^2$

$42 a^3 - 800a - 6(120000) = 0$; THEN $\underline{a = 26\ mm}$

10-18 FROM P6-78: $M = 344\ LB\cdot IN$; $P = 250\ LB$ AXIAL COMPR.

$$\sigma = \frac{-P}{A} - \frac{M}{S} = \frac{-P}{a^2} - \frac{M}{a^3/6} = \frac{-P}{a^2} - \frac{6M}{a^3}$$

$\sigma a^3 = -Pa - 6M$; $\sigma a^3 + Pa + 6M = 0$; LET $\sigma = -6000\ PSI$ COMPR.

$-6000a^3 + 250a + 6(344) = 0$; THEN $\underline{a = 0.720\ IN}$

10-19 FROM P6-79: $M = 42200\ N\cdot mm$; $P = 1200\ N$ AXIAL TENSION

FROM 11-17: $\sigma a^3 - Pa - 6M = 0$; $42a^3 - 1200 - 6(42200) = 0$; $\underline{a = 18.7\ mm}$

10-20 FROM P6-80: $M = 520\ LB\cdot IN$; $P = 400\ LB$ AXIAL TENSION

FROM 11-17: $\sigma a^3 - Pa - 6M = 0$; $6000a^3 - 400a - 6(520) = 0$; $\underline{a = 0.832\ IN}$

10-21 $A = \pi(40)^2/4 = 1257\ mm^2$; $Z_p = \pi(40)^3/16 = 12566\ mm^3$

$$\sigma = \frac{P}{A} = \frac{150\,000\ N}{1257\ mm^2} = 119\ MPa \ ; \ \tau = \frac{T}{Z_p} = \frac{500\,000\ N\cdot mm}{12566\ mm^3} = 39.8\ MPa$$

$$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{119}{2}\right)^2 + 39.8^2} = \underline{71.6\ MPa}$$

10-22 $A = \pi(2.25)^2/4 = 3.98\ IN^2$; $Z_p = \pi(2.25)^3/16 = 2.24\ IN^3$

$$\sigma = \frac{P}{A} = \frac{47000\ LB}{3.98\ IN^2} = 11809\ PSI \ ; \ \tau = \frac{T}{Z_p} = \frac{8500\ LB\cdot IN}{2.24\ IN^3} = 3795\ PSI$$

$$\tau_{MAX} = \sqrt{\left(\frac{11809}{2}\right)^2 + 3795^2} = \underline{7019\ PSI}$$

10-23 $A = \pi(4.00)^2/4 = 12.57\ IN^2$; $Z_p = \pi(4.00)^3/16 = 12.57\ IN^3$

$$\sigma = \frac{-P}{A} = \frac{-40000\ LB}{12.57\ IN^2} = -3183\ PSI \ ; \ \tau = \frac{T}{Z_p} = \frac{25000\ LB\cdot IN}{12.57\ IN^3} = 1989\ PSI$$

$$\tau_{MAX} = \sqrt{\left(\frac{3183}{2}\right)^2 + 1989^2} = \underline{2548\ PSI}$$

10-24 FOR 12 IN PIPE: $A = 15.74\ IN^2$; $Z_p = 94.18\ IN^3$

$$\sigma = \frac{-P}{A} = \frac{-250\,000\ LB}{15.74\ IN^2} = -15883\ PSI \ ; \ \tau = \frac{T}{Z_p} = \frac{180\,000\ LB\cdot IN}{94.18\ IN^3} = 1911\ PSI$$

$$\tau_{MAX} = \sqrt{\left(\frac{15883}{2}\right)^2 + 1911^2} = \underline{8168\ PSI}$$

10-25 FOR 3 IN PIPE: $A = 2.228\ IN^2$; $Z_p = 3.448\ IN^3$

$$\sigma = \frac{-P}{A} = \frac{-25000\ LB}{2.228\ IN^2} = -11221\ PSI \ ; \ \tau = \frac{T}{Z_p} = \frac{15500\ LB\cdot IN}{3.448\ IN^3} = 4495\ PSI$$

$$\tau_{MAX} = \sqrt{\left(\frac{11221}{2}\right)^2 + 4495^2} = \underline{7189\ PSI}$$

10-26 $T = (20\ LB)(8\ FT)(12\ IN/FT) = 1920\ LB\cdot IN$; $M = (20\ LB)(15\ FT)(12\ IN/FT) = 3600\ LB\cdot IN$

$T_e = \sqrt{T^2 + M^2} = \sqrt{1920^2 + 3600^2} = 4080\ LB\cdot IN$

$$Z_p = \frac{\pi}{16}\ \frac{D^4 - d^4}{D} = \frac{\pi(1.50^4 - 1.375^4)}{16(1.50)} = 0.195\ IN^3$$

$$\tau = \frac{T_e}{Z_p} = \frac{4080\ LB\cdot IN}{0.195\ IN^3} = \underline{20920\ PSI}$$

LET $\tau_d = S_y/2N$; $N = \dfrac{S_y}{2\tau} = \dfrac{40000}{2(20920)} = \underline{0.96\ \text{UNSAFE}}$

10-27 NEAR SUPPORT: $T = 300\,F = 300(1200) = 3.6\times10^5\ N\cdot mm$

$M = 450\,F = 450(1200) = 5.4\times10^5\ N\cdot mm$

$T_e = \sqrt{T^2+M^2} = 6.49\times10^5\ N\cdot mm\ ;\ Z_P = \frac{\pi}{16}(40)^3 = 12\,566\ mm^3$

$\tau = \frac{T_e}{Z_P} = \frac{6.49\times10^5\ N\cdot mm}{12\,566\ mm^3} = \underline{51.6\ MPa}$

10-28 TOTAL DOWNWARD LOAD $= 200+200+300+600 = 1300\ LB$

$M = (1300\ LB)(36\ IN) = 46\,800\ LB\cdot IN$ AT SUPPORT

NET TORQUE $= 600(40)+300(20)-200(20)-200(20) = 18000\ LB\cdot IN$

$T_e = \sqrt{T^2+M^2} = \sqrt{18000^2+46800^2} = 50\,142\ LB\cdot IN$

REQD $Z_P = \frac{T_e}{\tau_d} = \frac{50\,142\ LB\cdot IN}{8000\ LB/IN^2} = \underline{6.27\ IN^3}$ USE 4 IN SCH 40 PIPE

10-29

$M_B = (2400\ N)(150\ mm) = 360\,000\ N\cdot mm\ ;\ S = \frac{\pi D^3}{32} = \frac{\pi(20)^3}{32} = 785\ mm^3$

$Z_P = \frac{\pi D^3}{16} = \frac{\pi(20)^3}{16} = 1571\ mm^3\ ;\ \tau = \frac{T}{Z_P} = \frac{1500000\ N\cdot mm}{1571\ mm^3} = 955\ MPa$

$\sigma = \frac{M}{S} = \frac{360\,000\ N\cdot mm}{785\ mm^3} = 459\ MPa$

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2+\tau^2} = \sqrt{\left(\frac{459}{2}\right)^2+955^2} = \underline{982\ MPa}$

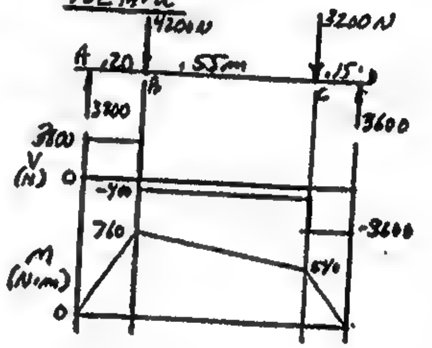
10-30 $S = \frac{\pi D^3}{32} = \frac{\pi(25)^3}{32} = 1534\ mm^3$

$Z_P = \frac{\pi D^3}{16} = 2S = 3068\ mm^3$

$\sigma = \frac{M}{S} = \frac{760\,000\ N\cdot mm}{1534\ mm^3} = 495\ MPa$

$\tau = \frac{T}{Z_P} = \frac{4500\,000}{3068} = 1467\ MPa$

$\tau_{MAX} = \sqrt{\left(\frac{495}{2}\right)^2+1467^2} = \underline{1488\ MPa}$



10-31

$T = \frac{63000(P)}{n} = \frac{63000(25)}{1150} = 1370\ LB\cdot IN$

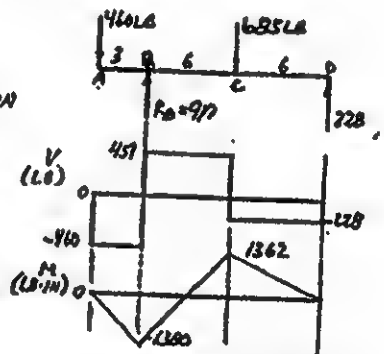
AT B: $T_e = \sqrt{T^2+M^2} = \sqrt{1370^2+1380^2} = 1945\ LB\cdot IN$

$Z_P = \frac{\pi D^3}{16} = \frac{\pi(1.0)^3}{16} = 0.196\ IN^3$

$\tau = \frac{T_e}{Z_P} = \frac{1945}{0.196} = 9923\ PSI = \tau_d = \frac{S_y}{2N}$

REQD $S_y = 2N\tau = 2(6)(9923) = 119\,000\ PSI$

AISI 1141 OQT 900 $S_y = 129\ KSI$, 15% ELONG



10-32 (a)

$T_A = (150-90)(6) = 2160\ LB\cdot IN\ CCW = T_{AC}$

$T_C = (200-240)(4) = 3840\ LB\cdot IN\ CW$

$T_E = (1050-210)(2) = \underline{1680\ LB\cdot IN\ CCW = T_{CE}}$

(b) $Z_P = \frac{\pi D^3}{16} = \frac{\pi(1.75)^3}{16} = 1.05\ IN^3;\ S = \frac{\pi}{32}D^3 = 0.526\ IN^3$

AT C: $\tau = \frac{2160(1.6)}{1.05} = 3291\ PSI$

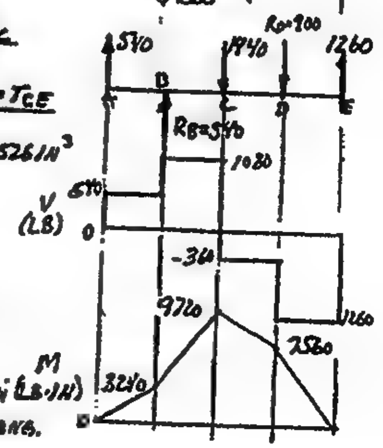
$\sigma = \frac{9720(1.6)}{0.526} = 29\,567\ PSI$

(C) $\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2+\tau^2} = 15145\ PSI = \tau_d = \frac{S_y}{2N}$

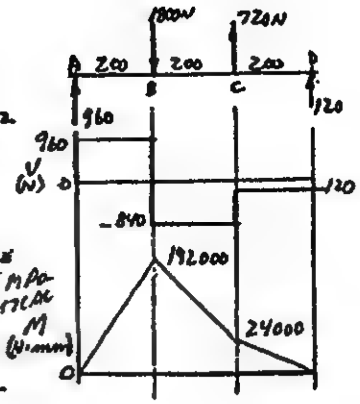
LET $N = 4$

REQD $S_y = 2N\tau = 2(4)(15145) = 121\,160\ PSI$

AISI 1141 OQT 900, $S_y = 129\ KSI$, 15% ELONG.



10-33



$T_B = (1500-300)(50) = 60000 \text{ N·mm}$

$T_C = (600-120)(125) = 60000 \text{ N·mm}$

AT B: $Z_P = \frac{\pi(28)^3}{16} = 4310 \text{ mm}^3$; $A = \frac{\pi(28)^2}{4} = 616 \text{ mm}^2$

$S = \frac{\pi(28)^3}{32} = 2155 \text{ mm}^3$

$\tau = T/Z_P = [60000/4310]1.6 = 22.3 \text{ MPa}$

$\sigma_b = \pm M/S = [\pm 192000/2155]1.6 = 142.6 \text{ MPa}$ } NET TENSILE $= 126.5$ MPa IS CRITICAL

$\sigma_c = -P/A = [-6200/616]1.6 = -16.1 \text{ MPa}$

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{126.5}{2}\right)^2 + (22.3)^2} = 67.1 \text{ MPa}$

10-34 REQD $S_Y = 2N(67.1) = \underline{537 \text{ MPa}}$ – $\underline{\text{USE AISI 1040 1100}}$

$N = 4$

10-35 8-32 UNC THREAD: $D_M = 0.164$ IN: $A_t = 0.0140 \text{ IN}^2$

IN THREADS: $P = \sigma A_t = (15000 \text{ LB/IN}^2)(0.0140 \text{ IN}^2) = 210$ LB

FOR D_M: $A_S = A_M = \pi D_M^2/4 = \pi(0.164)^2/4 = 0.0211 \text{ IN}^2$

$\sigma = P/A_M = 210/0.0211 = 9941$ PSI; $\tau = F_S/A_S = 120/0.0211 = 5681$ PSI

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{9941}{2}\right)^2 + 5681^2} = \underline{7548 \text{ PSI}}$

10-36 FOR 1/4-20 UNC THREAD: $D_M = 0.250$ IN: $A_t = 0.0318 \text{ IN}^2$

$P = \sigma A_t = (15000)(0.0318) = 477$ LB

FOR D_M: 0.250 IN: $A_M = A_S = \pi(0.25)^2/4 = 0.0491 \text{ IN}^2$

$\sigma = P/A_M = 477/0.0491 = 9717$ PSI; $\tau = F_S/A_S = 775/0.0491 = 15788$ PSI

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{9717}{2}\right)^2 + 15788^2} = \underline{16519 \text{ PSI}}$

10-37 FOR 4-48 UNF THREADS: $D_M = 0.112$ IN: $A_t = 0.006611 \text{ IN}^2$

$P = \sigma A_t = 15000(0.00661) = 99.2$ LB

FOR $D_M = 0.112$ IN: $A_M = A_S = \pi(0.112)^2/4 = 0.00985 \text{ IN}^2$

$\sigma = P/A_M = 99.2/0.00985 = 10069$ PSI; $\tau = F_S/A_S = 50/0.00985 = 5075$ PSI

$\tau_{MAX} = \sqrt{\left(\frac{10069}{2}\right)^2 + 5075^2} = \underline{7149 \text{ PSI}}$

10-38 FOR 1 1/4-12 THREAD: $D_M = 1.250$ IN: $A_t = 1.073 \text{ IN}^2$

$P = \sigma A_t = 15000(1.073) = 16095$ LB

FOR D_M: $A_M = A_S = \pi(1.25)^2/4 = 1.227 \text{ IN}^2$

$\sigma = P/A_M = 16095/1.227 = 13115$ PSI; $\tau = F_S/A_S = 2500/1.227 = 2037$ PSI

$\tau_{MAX} = \sqrt{\left(\frac{13115}{2}\right)^2 + 2037^2} = \underline{6867 \text{ PSI}}$

10-39 FOR M16×2: $D_M = 16$ mm: $A_t = 157 \text{ mm}^2$

$P = \sigma A_t = (120 \text{ N/mm}^2)(157 \text{ mm}^2) = 18840$ N

FOR $D_M = 16$ mm: $A_M = A_S = \pi(16)^2/4 = 201 \text{ mm}^2$

$\sigma = P/A_M = 18840 \text{ N}/201 \text{ mm}^2 = 93.7$ MPa; $\tau = F_S/A_S = 8000 \text{ N}/201 \text{ mm}^2 = 39.8$ MPa

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{93.7}{2}\right)^2 + 39.8^2} = \underline{61.5 \text{ MPa}}$

10-40 FOR M48×5: $D_M = 48$ mm: $A_t = 1473 \text{ mm}^2$

$P = \sigma A_t = (120 \text{ N/mm}^2)(1473 \text{ mm}^2) = 1.77 \times 10^5$ N

FOR $D_M = 48$ mm: $A_M = A_S = \pi(48)^2/4 = 1810 \text{ mm}^2$

$\sigma = P/A_M = \frac{1.77 \times 10^5 \text{ N}}{1810 \text{ mm}^2} = 97.7$ MPa; $\tau = \frac{F_S}{A_S} = \frac{80000 \text{ N}}{1810 \text{ mm}^2} = 44.2$ MPa

$\tau_{MAX} = \sqrt{\left(\frac{97.7}{2}\right)^2 + 44.2^2} = \underline{65.9 \text{ MPa}}$

10-41

AT B: $V = 2750\text{ LB}$: $M = 82500\text{ LB·IN}$

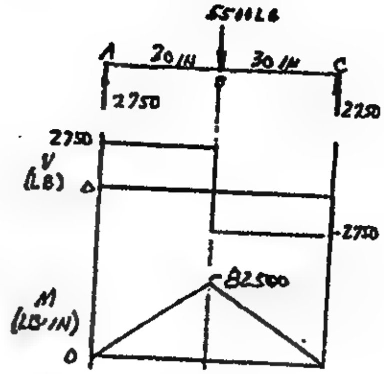
$\tau_{MAX} = \sqrt{(\sigma/2)^2 + \tau^2}$

FOR BEAM CROSS SECTION:

$A = bh = (2)(6) = 12.00\text{ IN}^2$

$S = bh^2/6 = (2)(6)^2/6 = 12.00\text{ IN}^3$

$I = bh^3/12 = (2)(6)^3/12 = 36.00\text{ IN}^4$

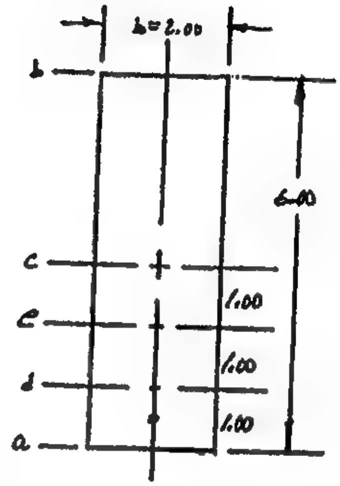


AT a: $\tau = 0$: $\sigma = M/S$

$\sigma = \dfrac{82500\text{ LB·IN}}{12.00\text{ IN}^2} = 6875\text{ PSI}$

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{\sigma}{2} = \underline{3438\text{ PSI}}$

AT b: SAME AS (a)



AT c: $\sigma = 0$: $\tau = \tau_{MAX} = 3V/2A$

$\tau_{MAX} = \dfrac{3(2750)}{2(12)} = \underline{344\text{ PSI}}$

AT d: $\sigma = \dfrac{My}{I} = \dfrac{(82500)(2.0)}{36.0} = 4583\text{ PSI}$

$\tau = \dfrac{VQ}{Ib} = \dfrac{(2750)(2.0)(1.0)(2.5)}{(36.0)(2.0)} = 191\text{ PSI}$

$\tau_{MAX} = \sqrt{\left(\frac{4583}{2}\right)^2 + 191^2} = \underline{2300\text{ PSI}}$

AT e: $\sigma = \dfrac{My}{I} = \dfrac{(82500)(1.0)}{36.0} = 2292\text{ PSI}$

$\tau = \dfrac{VQ}{Ib} = \dfrac{(2750)(2.0)(2.0)(2.0)}{(36.0)(2.0)} = 306\text{ PSI}$

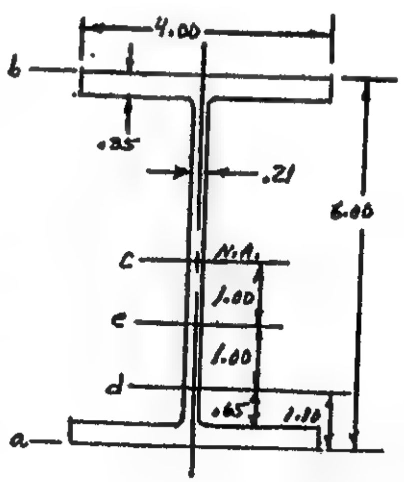
$\tau_{MAX} = \sqrt{\left(\frac{2292}{2}\right)^2 + 306^2} = \underline{1186\text{ PSI}}$

10-42

FROM 11-35: $V = 2750\text{ LB}$: $M = 82500\text{ LB·IN}$

FOR ALUM I 6×4.692 : $A = 3.990\text{ IN}^2$

$S = 8.50\text{ IN}^3$: $I = 25.50\text{ IN}^4$



AT a AND b: $\tau = 0$, $\sigma = M/S$

$\sigma = \dfrac{82500\text{ LB·IN}}{8.50\text{ IN}^3} = 9706\text{ PSI}$

$\tau_{MAX} = \sigma/2 = \underline{4853\text{ PSI}}$

AT c: $\sigma = 0$: $\tau = \tau_{MAX} = \dfrac{VQ}{It}$

$Q = (.35)(4.0)(2.825) + (.21)(2.65)(1.325)$

$Q = 3.955 + 0.737 = 4.692\text{ IN}^3$

$\tau_{MAX} = \dfrac{(2750)(4.692)}{(25.50)(.21)} = \underline{2410\text{ PSI}}$

AT d:

$\sigma = \dfrac{My}{I} = \dfrac{(82500)(2.0)}{25.50} = 6471\text{ PSI}$

$\tau = \dfrac{VQ}{It} = \dfrac{(2750)(3.999)}{(25.50)(0.21)} = 8213\text{ PSI}$

$Q = 3.955 + (.21)(0.65)(2.325) = 3.999\text{ IN}^3$

$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{6471}{2}\right)^2 + 8213^2} = \underline{8827\text{ PSI}}$

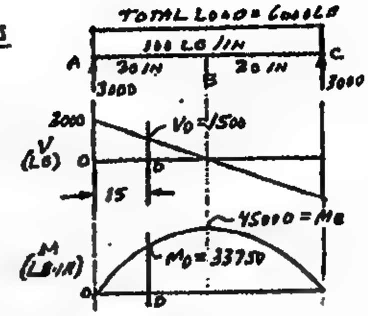
AT e:

$\sigma = \dfrac{My}{I} = \dfrac{82500(1.0)}{25.50} = 3235\text{ PSI}$: $Q = 3.955 + (0.21)(1.65)(0.825) = 4.241\text{ IN}^3$

$\tau = \dfrac{VQ}{It} = \dfrac{(2750)(4.241)}{(25.50)(0.21)} = 2178\text{ PSI}$

$\tau_{MAX} = \sqrt{\left(\frac{3235}{2}\right)^2 + 2178^2} = \underline{2713\text{ PSI}}$

10-43 SEE 11-41 FOR BEAM SECTION PROPERTIES



AT MIDDLE OF BEAM - B:

AT a AND b: $\sigma = \frac{M}{S} = \frac{45000}{12.00} = 3750$ PSI

$\tau = 0$: $\tau_{MAX} = \frac{\sigma}{2} = 1875$ PSI

AT c: $\sigma = 0$: $\tau = 0$: $\tau_{MAX} = 0$

AT d: $\sigma = \frac{My}{I} = \frac{45000(2.0)}{36.0} = 2500$ PSI

$\tau = 0$: $\tau_{MAX} = \sigma/2 = 1250$ PSI

AT e: $\sigma = \frac{My}{I} = \frac{45000(1.0)}{36.0} = 1250$ PSI : $\tau = 0$: $\tau_{MAX} = \frac{\sigma}{2} = 625$ PSI

AT SUPPORTS A AND C: $V = 3000$ LB : $M = 0$: $\sigma = 0$

AT a AND b: $\tau = 0$: $\tau_{MAX} = 0$

AT c: $\tau = 3V/2A = 3(3000)/(2)(12) = 375$ PSI $= \tau_{MAX}$

AT d: $\tau = \tau_{MAX} = \frac{VQ}{It} = \frac{(3000)(2.0)(1.0)(2.5)}{(36.0)(2.0)} = 208$ PSI

AT e: $\tau = \tau_{MAX} = \frac{VQ}{It} = \frac{(3000)(1.0)(2.0)(2.0)}{(36.0)(2.0)} = 333$ PSI

AT D - 15 IN FROM A: $V = 1500$ LB : $M = 33750$ LB·IN

AT a AND b: $\tau = 0$: $\sigma = \frac{M}{S} = \frac{33750}{12.0} = 2813$ PSI : $\tau_{MAX} = \frac{\sigma}{2} = 1406$ PSI

AT c: $\sigma = 0$: $\tau = \frac{3V}{2A} = \frac{3(1500)}{2(12)} = 188$ PSI $= \tau_{MAX}$

AT d: $\sigma = \frac{My}{I} = \frac{(33750)(2.0)}{36.0} = 1875$ PSI

$\tau = \frac{VQ}{It} = \frac{(1500)(2.0)(1.0)(2.5)}{(36.0)(2.0)} = 104$ PSI

$\tau_{MAX} = \sqrt{\left(\frac{1875}{2}\right)^2 + 104^2} = 943$ PSI

AT e: $\sigma = \frac{My}{I} = \frac{(33750)(1.0)}{36.0} = 938$ PSI

$\tau = \frac{VQ}{It} = \frac{(1500)(2.0)(2.0)(2.0)}{(36.0)(2.0)} = 167$ PSI

$\tau_{MAX} = \sqrt{\left(\frac{938}{2}\right)^2 + 167^2} = 498$ PSI

10-44 FROM FIG 5-25: $Q = 0.208a^3 = 0.208(25)^3 = 3250\ mm^3$

$\tau = T/Q = \frac{245\,000\ N\cdot mm}{3250\ mm^3} = 75.4$ MPa ; $\sigma = \frac{P}{A} = \frac{75\,000\ N}{(25)^2\ mm^2} = 120$ MPa

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{120}{2}\right)^2 + 75.4^2} = 96.4$ MPa

10-45 FROM FIG 5-25: $Q = \frac{bh^2}{3 + 1.8(h/b)} = \frac{50(30)^2}{3 + 1.8(30/50)} = 11\,029\ mm^3$

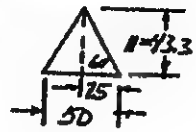
$\tau = \frac{T}{Q} = \frac{525\,000\ N\cdot mm}{11\,029\ mm^3} = 47.6$ MPa : $\sigma = \frac{P}{A} = \frac{175\,000}{(30)(50)} = 116.7$ MPa

$\tau_{MAX} = \sqrt{\left(\frac{116.7}{2}\right)^2 + 47.6^2} = 75.3$ MPa

10-46 FROM FIG 5-25: $Q = 0.050\,a^3 = 0.050(50)^3 = 6250\ mm^3$

$\tau = \frac{T}{Q} = \frac{775000\ N\cdot mm}{6250\ mm^3} = 124\ MPa$; $\sigma = \frac{P}{A} = \frac{115000}{\frac{1}{2}(50)(43.3)} = 106\ MPa$

$\tau_{MAX} = \sqrt{\left(\frac{106}{2}\right)^2 + 124^2} = \underline{135\ MPa}$



10-47 (a) $S_y = 50\ ksi$; $\sigma_d = \frac{S_y}{3} = \frac{50000}{3} = 16667\ psi = P/A$

$P = \sigma \cdot A = (16667\ LB/IN^2)(2.59\ IN^2) = 43167\ LB$

$\tau_{MAX} = \frac{\sigma}{2} = \underline{8333\ PSI}$

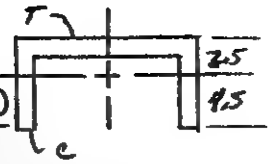
(b) FROM FIG 5-25: $Q = 2t(a-t)(b-t) = 2(0.25)(2.75)(2.75) = 3.78\ IN^3$

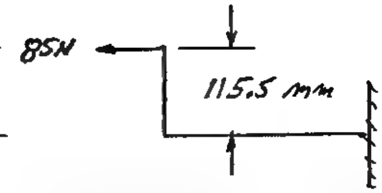
$\tau = \frac{T}{Q} = \frac{(950)(12)(LB\cdot IN)}{3.78\ IN^3} = 3015\ psi$

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{16667}{2}\right)^2 + 3015^2} = \underline{8862\ psi} = \frac{S_y}{2N}$

$N = \frac{S_y}{2\tau_{MAX}} = \frac{50000}{2(8862)} = \underline{2.82}$

10-48

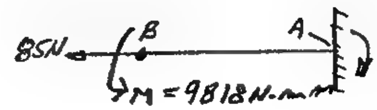




TENSILE: TOP FROM B TO A.

$$\sigma = \frac{85N}{33.0\,mm^2} + \frac{(9818\,N\cdot mm)(2.5\,mm)}{128\,mm^4}$$

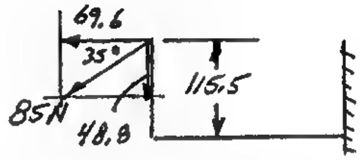
$$\sigma = 2.58 + 191.7 = 194.3\ N/mm^2 = \underline{194\ MPa}$$



COMPRESSIVE: BOTTOM FROM B TO A

$$\sigma = 2.58 - \frac{(9818\,N\cdot mm)(4.5\,mm)}{128\,mm^4} = \underline{-343\,MPa}$$

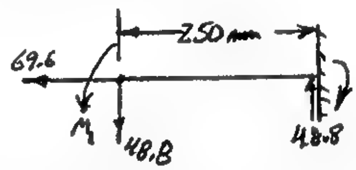
10-49 TENSILE: TOP AT A



$$\sigma = \frac{69.6N}{33.0\,mm^2} + \frac{(20239\,N\cdot mm)(2.5\,mm)}{128\,mm^2} = \underline{397\,MPa}$$

COMPRESSIVE: BOTTOM

$$\sigma = 2.11\,MPa - \frac{(20239)(4.5)}{128} = \underline{-709\,MPa}$$

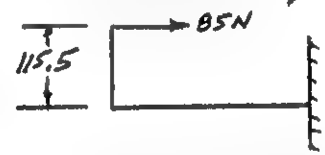


$M_1 = 69.6(115.5) = 8039\ N\cdot mm$

$M_2 = 48.8(250) = 12200\ N\cdot mm$

$M_1 + M_2 = 20239\ N\cdot mm$

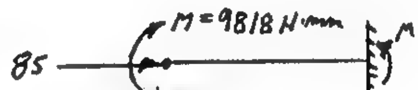
10-50 TENSION: BOTTOM



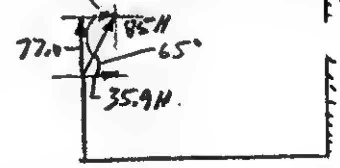
$$\sigma = \frac{-85N}{33.0\,mm^2} + \frac{(9818\,N\cdot mm)(4.5\,mm)}{128\,mm^4} = \underline{343\,MPa}$$

COMPRESSIVE: TOP

$$\sigma = \frac{-85N}{33.0\,mm^2} - \frac{(9818)(2.5)}{128} = \underline{-194\,MPa}$$



10-51 TENSION: BOTTOM



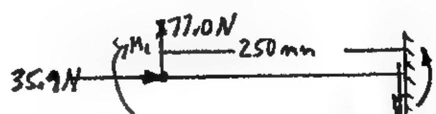
$$\sigma = \frac{-35.9N}{33.0\,mm^2} + \frac{(23396\,N\cdot mm)(4.5\,mm)}{128\,mm^4}$$

$$\sigma = -1.09 + 822.5 = \underline{821\,MPa}$$

COMPRESSIVE: TOP

$$\sigma = \frac{-35.9}{33.0} - \frac{(23396)(2.5)}{128} = -1.09 - 457.0$$

$$\sigma = \underline{-458\,MPa}$$

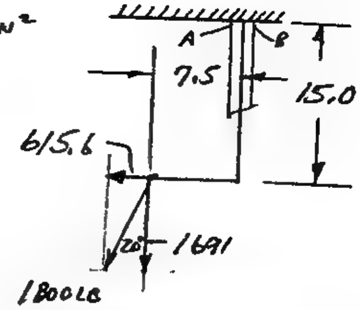


$M_1 = (35.9)(115.5) = 4146\ N\cdot mm$

$M_2 = (77.0)(250) = 19250\ N\cdot mm$

$M_1 + M_2 = 23396\ N\cdot mm$

10-52 2 x 3 x 1/4 STEEL TUBING, $S = 2.10\ in^3$, $A = 2.59\ in^2$



$M_1 = (615.6\ LB)(15.0\ IN) = 9234\ LB\cdot IN$

$M_2 = (1691\ LB)(7.5\ IN) = 12\,683\ LB\cdot IN$

AXIAL LOAD = 1691 LB ↓ TENSION

NET BENDING MOMENT = $M_2 - M_1$

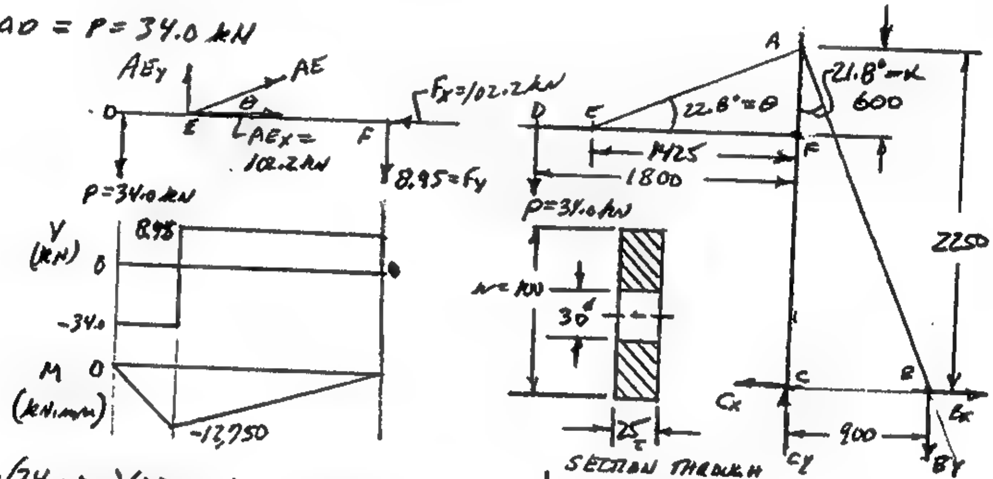
$M_{NET} = 12\,683 - 9234 = 3449\ LB\cdot IN$

PRODUCES TENSION AT A, COMPR. AT B

$\sigma_A = \dfrac{(1691\ LB)}{2.59\ IN^2} + \dfrac{(3449\ LB\cdot IN)}{2.10\ IN^3} = 653 + 1642 = \underline{2295\ PSI\ \text{TENSION}}$

$\sigma_B = 653 - 1642 = \underline{-989\ PSI\ \text{COMPRESSION}}$

10-53 LOAD = P = 34.0 kN



$\Sigma M_F = 0 = (34.0\ kN)(1800\ mm) - AE_y(1425\ mm)$

$AE_y = 42.95\ kN$

$\Sigma F_y = 0 = 42.95 - 34.0 - F_y$; $F_y = 8.95\ kN \downarrow$

$AE = AE_y/\sin\theta = 42.95\ kN / \sin 22.8° = 110.8\ kN$

$AE_x = AE\cos\theta = (110.8\ kN)\cos 22.8° = 102.2\ kN \rightarrow$

$\Sigma M_C = 0 = P(1800) - B_y(900)$

$B_y = 34.0\ kN(2) = 68\ kN$

$\Sigma F_y = 0 = C_y - 34.0 - 68.0$

$C_y = 102\ kN$

$\tan\alpha = B_x/B_y$

$B_x = B_y\tan\alpha = 68\tan 21.8° = 27.2\ kN$

$C_x = B_x$

$AB = B_y/\cos\alpha = 68/\cos 21.8° = 73.2\ kN$

AT E: FLAT PLATE WITH CENTRAL HOLE
APP. A-21-4, CURVE C. - BENDING

$\sigma_b = \dfrac{M K_t C}{I_{NET}} = \dfrac{K_t M w}{(w^3 - d^3)(t)} = \dfrac{(12\,750\ kN\cdot mm)(100\ mm)}{(100^3 - 30^3)(25)\ mm^4} \times \dfrac{1000\ N}{kN} = 52.4\ MPa$ BENDING

$d/w = 30/100 = 0.30 \Rightarrow K_t = 1.0$

TENSION-TOP; COMPR.-BOTTOM

DIRECT COMPRESSION $\sigma_c = \dfrac{K_t AE_x}{A_{NET}} = \dfrac{(3.70)(102.2\ kN)}{(100-30)(25)\ mm^2} \times \dfrac{1000\ N}{kN} = 216\ MPa$

$K_t = 3.70$ FOR CURVE B

NEAR HOLE

COMBINATION OF σ_b AND σ_c IS PROBLEMATIC BECAUSE $\sigma_{b\,MAX}$ OCCURS NEAR TOP OR BOTTOM OF SECTION WHILE $\sigma_{c\,MAX}$ OCCURS NEAR CENTERLINE NEAR HOLE.

σ_{MAX} TENSILE IS AT E TO LEFT WHERE $\sigma_b = 52.4\ MPa$ AND NO COMPR. STRESS.

σ_{MAX} COMPR. IS TO RIGHT OF E WITH A VALUE BETWEEN 216 MPa AND $216 + 52.4 = 268\ MPa$ ON LOWER PART OF THE CROSS SECTION.

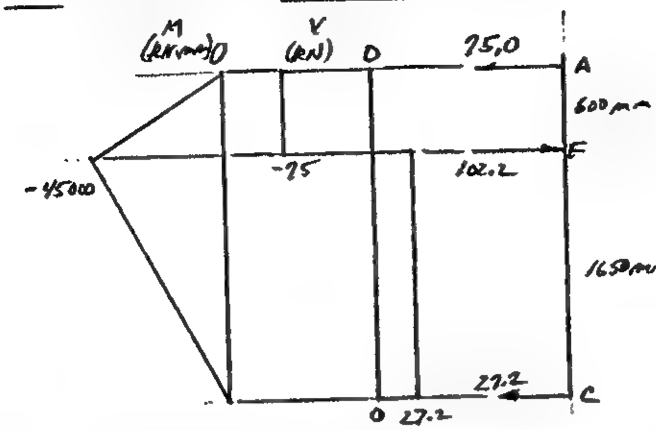
(NEXT PAGE)

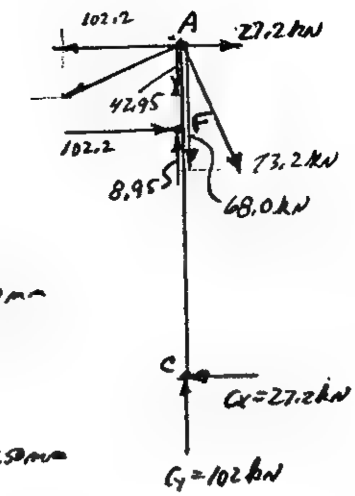
10-53 (CONTINUED)

MEMBER AFC - LOADS FROM FBD OF WHOLE STRUCTURE

AXIAL LOAD: AF = 42.95 + 68.0 = 110.95 kN COMPR.

CF = 110.95 - 8.95 = 102 kN COMPR.





BENDING STRESS AT F (SAME SECTION PROPERTIES AS AT E) ($K_t = 1.0$)

$$\sigma_{b_F} = \frac{M K_t C}{I_{NET}} = \frac{(75000\ kN\text{-}mm)w}{(w^3 - d^3)t} = \frac{(75000\ kN\text{-}mm)(100\ mm)}{(100^3 - 30^3)(25)\ mm^4} \times \frac{1000\ N}{kN}$$

$\sigma_{bF} = \pm 308\ MPa$ TENSILE ON RIGHT SIDE; COMPR. ON LEFT.

COMPRESSION NEAR F USE $P_F = 110.95$ kN ABOVE F. $K_t = 3.70$ FOR $d/w = 0.30$

$$\sigma_{CF} = \frac{-P_F K_t}{A_{NET}} = -\frac{(110.5\ kN)(3.70)}{(100-30)(25)\ mm^2} \times \frac{1000\ N}{kN} = -233.6\ MPa \text{ NEAR HOLE.}$$

COMBINED σ_b AND σ_c IS PROBLEMATIC AS AT E ON MEMBER DEF.

BECAUSE EFFECT OF K_t IS LOCALIZED NEAR HOLE, σ_c IS LIKELY TO BE MUCH LESS NEAR OUTSIDE SURFACES WHERE σ_b IS MAXIMUM.

ASSUME $\sigma_c = \dfrac{-P_F}{A_{NET}} = \dfrac{(-110.5\ kN)(1000\ N/kN)}{(100-30)(25)\ mm^2} = -63.1\ MPa$ NEAR OUTSIDE EDGE ABOVE F.

$$\sigma_c = \frac{(-102\ kN)(1000\ N/kN)}{(100-30)(25)\ mm^2} = -58.3\ MPa \text{ BELOW F.}$$

COMBINED STRESS - TENSION - BELOW F.

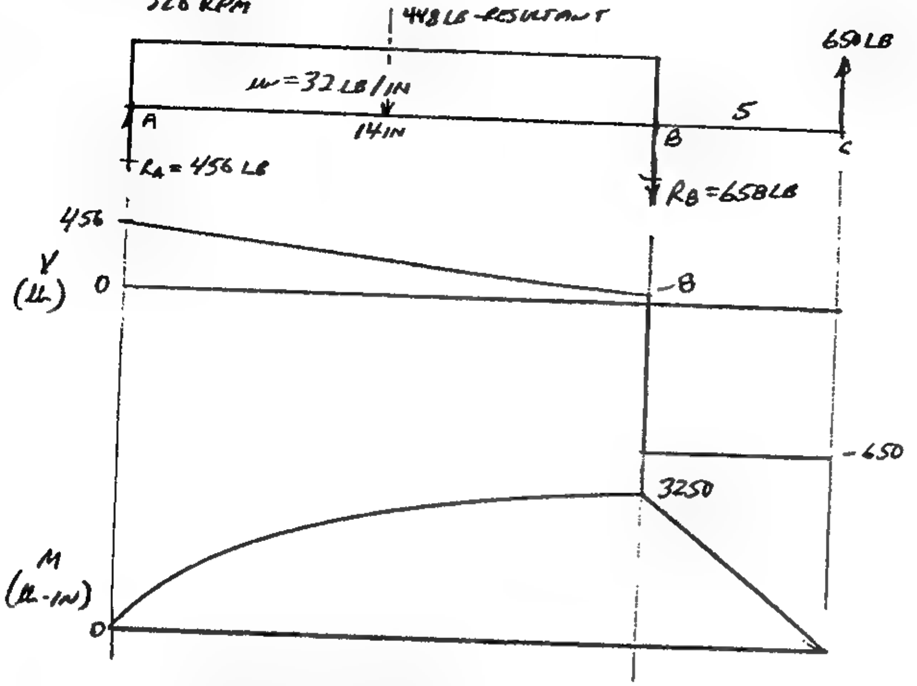
$\sigma_T = +308\ MPa - 58.3 = 249.7\ MPa$ ON RIGHT SIDE.

COMBINED STRESS - COMPRESSION - ABOVE F.

$\sigma_c = -308\ MPa - 63.1 = -371\ MPa$ ON LEFT SIDE.

10-54 SHAFT: POWER = 30.5 hp; n = 320 RPM

$$T = \frac{63\,000(30.5\ hp)}{320\ RPM} = 6064\ \text{LB-IN}$$



MAXIMUM BENDING AND TORSION OCCUR NEAR B.
USE EQUIVALENT TORQUE METHOD, EQ (10-8).
MAX STRESS OCCURS AT STEP IN 1.00 IN DIA. SHAFT

$$Z_P = \frac{\pi D^3}{16} = \frac{\pi(1.0\ in)^3}{16} = 0.196\ IN^3$$

$$\frac{r}{d} = \frac{0.03}{1.0} = 0.03;\ \frac{D}{d} = \frac{1.25}{1.0} = 1.25;\ \overset{A21\text{-}9}{K_{t_{BEND}} = 2.40};\ \overset{A21\text{-}7}{K_{t_{TORS}} = 1.77}$$

$$\underline{T_e} = \sqrt{(K_{tB}M)^2 + (K_{tT}T)^2} = \sqrt{[2.40(3250)]^2 + [1.77(6064)]^2} = \underline{13\,268\ LB\text{-}IN}$$

$$\tau_{MAX} = \frac{T_e}{Z_P} = \frac{13\,268\ LB\cdot IN}{0.196\ IN^3} = 67\,694\ psi$$

LET $N = 4$; $\tau_d = S_y/2N = S_y/8$

$\underline{\text{REQ'D } S_y} = 8\tau_{MAX} = 8(67694) = 541{,}550\ psi = \underline{542\ ksi}$

SPECIFY $\underline{\text{AISI 1040 OQT 1100}}$ S_y = 552 KSi, 24% ELONGATION

OTHER STEELS COULD BE USED WITH $S_y > 542$ KSi AND GOOD DUCTILITY.

10-55 FIND STRESS ON ELEMENTS M AND N, $P = 450\ N$

FORCE P ACTS 18 mm TO RIGHT OF CENTER LINE AND 8 mm ABOVE CL.

AT M:

$$\text{AXIAL STRESS} = \frac{P}{A} = \frac{450\ N}{(20)(28)\ mm^2} = 0.804\ MPa \text{ TENSION}$$

BENDING MOMENT: $M_1 = (450\ N)(8\ mm) = 3600\ N \cdot mm$

$$S = \frac{bh^2}{6} = \frac{(20)(28)^2}{6} = 2613\ mm^3$$

$$\sigma_{M_1} = \frac{M_1}{S} = \frac{3600\ N \cdot mm}{2613\ mm^3} = 1.377\ MPa \text{ TENSION AT M.}$$

$$\sigma_{M_{TOTAL}} = 0.804 + 1.377 = 2.181\ MPa \text{ TENSION AT M}$$

AT N:

AXIAL STRESS = 0.804 MPa AS AT M. (TENSION)

BENDING MOMENT: $M_2 = (450\ N)(18\ mm) = 8100\ N \cdot mm$

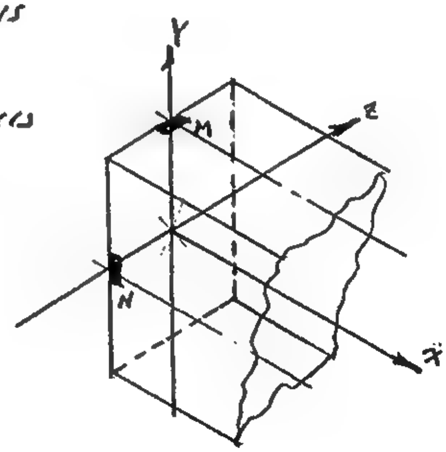
$$S = \frac{h(b)^2}{6} = \frac{(28)(20)^2}{6} = 1867\ mm^3$$

$$\sigma_{M2} = \frac{M_2}{S} = \frac{8100\ N \cdot mm}{1867\ mm^3} = 4.339\ MPa \text{ COMPRESSION AT N}$$

$$\sigma_{N_{TOTAL}} = 0.804 - 4.339 = -3.535\ MPa \text{ COMPRESSION AT N.}$$

ELEMENT M IS ON NEUTRAL AXIS FOR BENDING ABOUT Y-AXIS.

ELEMENT N IS ON NEUTRAL AXIS FOR BENDING ABOUT Z-AXIS



CHAPTER 11 The General Case of Combined Stress and Mohr's Circle

ANSWERS FOR CHAPTER 11 PROBLEMS

NOTE: The complete solution for problems 1-50 require the construction of the complete Mohr's circle and the drawing of the principal stress element and the maximum shear stress element. Listed below are the significant numerical results. Following the list are representative examples of the complete solutions. Note that the problems fall into groups of similar forms as described below.

A. For problems 1-4, the x-axis on the Mohr's circle lies in the first quadrant.

For problems 5-8, the x-axis on the Mohr's circle lies in the second quadrant.

For problems 9-12, the x-axis on the Mohr's circle lies in the third quadrant.

For problems 13-16, the x-axis on the Mohr's circle lies in the fourth quadrant.

For problems 17-24, the x-axis on the Mohr's circle could lie in any quadrant. No pattern exists.

For problems 25-28, the x-axis on the Mohr's circle lies along the original x-axis and the principal stresses are the same as the normal stresses on the given element.

B. For problems 29-40, the Mohr's circle from the given data results in both principal stresses having the same sign. For this class of problems, the supplementary circle is drawn using the procedures discussed in Section 10-11 of the text. The results include three principle stresses where $\sigma_1 > \sigma_2 > \sigma_3$. Also, the maximum shear stress is found from the radius of the circle containing σ_1 and σ_3, and is equal to $\frac{1}{2}\sigma_1$ or $\frac{1}{2}\sigma_3$, whichever has the greatest magnitude. Angles of rotation of the resulting elements are not requested.

C. For problems 41-50, the Mohr's circles from earlier problems are used to find the stress condition on the element at some specified angle of rotation. The listed results include the two normal stresses and the shear stress on the specified element.

D. For problems 51-54, specific numerical answers are requested.

CHAPTER // - PROBLEMS 1-28.

Prob. No.	σ_1	σ_2	θ (deg)	τ_{max}	σ_{avg}	θ' (deg)
1	315.4 MPa	-115.4 MPa	10.9 cw	215.4 MPa	100.0 MPa	34.1 ccw
2	255.2 MPa	-55.2 MPa	7.5 cw	155.2 MPa	100.0 MPa	37.5 ccw
3	110.0 MPa	-40.0 MPa	26.6 cw	75.0 MPa	35.0 MPa	18.4 ccw
4	202.1 MPa	-42.1 MPa	27.5 cw	122.1 MPa	80.0 MPa	17.5 ccw
5	23.5 ksi	-8.5 ksi	19.3 ccw	16.0 ksi	7.5 ksi	64.3 ccw
6	42.8 ksi	-29.8 ksi	14.9 ccw	36.3 ksi	6.5 ksi	59.9 ccw
7	79.7 ksi	-9.7 ksi	31.7 ccw	44.7 ksi	35.0 ksi	76.7 ccw
8	36.6 ksi	-54.6 ksi	13.0 ccw	45.6 ksi	-9.0 ksi	58.0 ccw
9	677.6 kPa	-977.6 kPa	77.5 ccw	827.6 kPa	-150.0 kPa	57.5 cw
10	137.8 kPa	-587.8 kPa	84.0 ccw	362.8 kPa	-225.0 kPa	51.0 cw
11	327.0 kPa	-1202.0 kPa	60.9 ccw	764.5 kPa	-437.5 kPa	74.1 cw
12	79.9 kPa	-354.9 kPa	74.8 ccw	217.4 kPa	-137.5 kPa	60.2 cw
13	570.0 psi	-2070.0 psi	71.3 cw	1320.0 psi	-750.0 psi	26.3 cw
14	1676.1 psi	-6676.1 psi	81.7 cw	4176.1 psi	-2500.0 psi	36.7 cw
15	4180.0 psi	-5180.0 psi	71.6 cw	4680.0 psi	-500.0 psi	26.6 cw
16	8600.7 psi	-150.7 psi	89.5 cw	4375.7 psi	4225.0 psi	44.5 cw
17	360.2 MPa	-100.2 MPa	27.8 ccw	230.2 MPa	130.0 MPa	72.8 ccw
18	1827.1 kPa	-377.1 kPa	24.4 cw	1102.1 kPa	725.0 kPa	20.6 ccw
19	23.9 ksi	-1.9 ksi	15.9 cw	12.9 ksi	11.0 ksi	29.1 ccw
20	7971.2 psi	-1221.2 psi	21.4 ccw	4596.2 psi	3375.0 psi	66.4 ccw
21	4.4 ksi	-32.4 ksi	20.3 cw	18.4 ksi	-14.0 ksi	24.7 ccw
22	527.6 MPa	-87.6 MPa	67.8 cw	307.6 MPa	220.0 MPa	22.8 cw
23	321.0 MPa	-61.0 MPa	66.4 ccw	191.0 MPa	130.0 MPa	68.6 cw
24	344.5 kPa	-1904.5 kPa	23.0 ccw	1124.5 kPa	-780.0 kPa	68.0 ccw

Prob. No.	σ_1	σ_2	ϕ (deg)	τ_{max}	σ_{avg}	ϕ' (deg)
25	225.0 MPa	-85.0 MPa	0.0	155.0 MPa	70.0 MPa	45.0 ccw
26	6250.0 psi	-875.0 psi	0.0	3562.5 psi	2687.5 psi	45.0 ccw
27	775.0 kPa	-145.0 kPa	0.0	460.0 kPa	315.0 kPa	45.0 ccw
28	38.6 ksi	-13.4 ksi	0.0	26.0 ksi	12.6 ksi	45.0 ccw

CHAPTER - PROBLEMS 29-40

Prob. No.	σ_1	σ_2	σ_3	τ_{max}
29	328.1 MPa	71.9 MPa	0.0 MPa	164.0 MPa
30	264.0 MPa	136.0 MPa	0.0 MPa	132.0 MPa
31	214.5 MPa	75.5 MPa	0.0 MPa	107.2 MPa
32	161.1 MPa	68.9 MPa	0.0 MPa	80.5 MPa
33	35.0 ksi	10.0 ksi	0.0 ksi	17.5 ksi
34	41.8 ksi	21.2 ksi	0.0 ksi	20.9 ksi
35	55.6 ksi	14.4 ksi	0.0 ksi	27.8 ksi
36	62.9 ksi	19.1 ksi	0.0 ksi	31.5 ksi
37	0.0 kPa	-307.9 kPa	-867.1 kPa	433.5 kPa
38	0.0 kPa	-37.5 kPa	-337.5 kPa	168.8 kPa
39	0.0 psi	-295.7 psi	-1804.3 psi	902.1 psi
40	0.0 psi	-2167.6 psi	-6832.4 psi	3416.2 psi

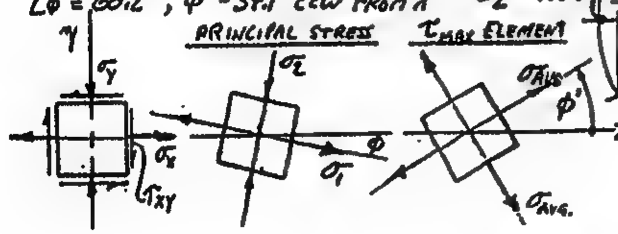
PROB. 11-1 DETAILED SOLUTION

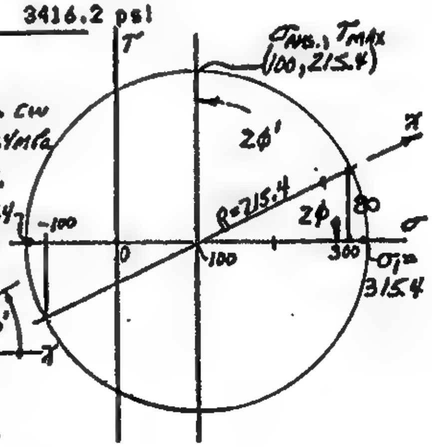
GIVEN: $\sigma_x = 300$ MPa; $\sigma_y = -100$ MPa; $\tau_{xy} = 80$ MPa cw

$\sigma_1 = 315.4$ MPa; $\sigma_2 = -115.4$ MPa; $\tau_{max} = 215.4$ MPa

$\sigma_{AVG} = 100$ MPa; $2\phi = 21.8°$; $\phi = 10.9°$ CW FROM x.

$2\phi' = 68.2°$; $\phi' = 34.1°$ ccw FROM x





CHAPTER 11 - PROBLEMS 41-50

Prob. No.	σ_A	$\sigma_{A'}$	τ_A
41	130.7 MPa	69.3 MPa	213.2 MPa cw
42	269.3 MPa	-69.3 MPa	133.2 MPa ccw
43	-37.9 MPa	197.9 MPa	31.6 MPa ccw
44	19.1 ksi	-6.1 ksi	34.0 ksi ccw
45	3.6 ksi	-21.6 ksi	43.9 ksi cw
46	-300.0 kPa	-150.0 kPa	355.0 kPa cw
47	-2010.3 psi	510.3 psi	392.6 psi cw
48	-765.5 psi	-234.5 psi	4672.5 psi cw
49	8363.5 psi	86.5 psi	1421.2 psi cw
50	894.8 kPa	555.2 kPa	1088.9 kPa ccw

CHAPTER - PROBLEMS 51-58

51 $\tau_{max} = \sqrt{(\sigma/2)^2 + (\tau)^2} = \sqrt{(260/2)^2 + (190)^2} =$ 230.2 MPa

Using a similar technique:

52 τ_{max} = 1102.1 kPa   53 τ_{max} = 12.9 ksi   54 τ_{max} = 4596.2 psi

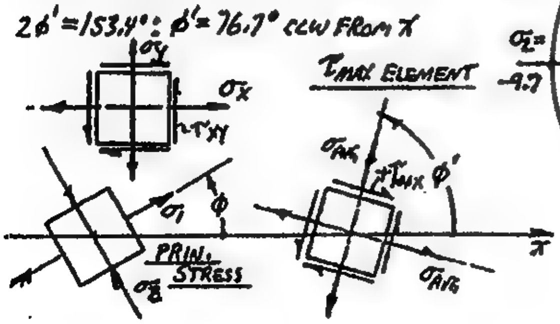
PROB 11-7 DETAILED SOLUTION

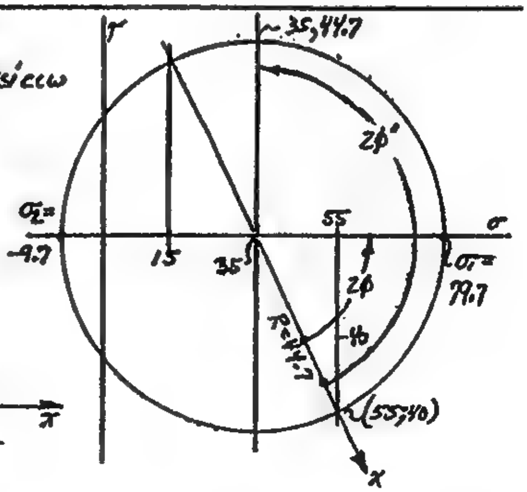
GIVEN: $\sigma_x = 55$ ksi; $\sigma_y = 15$ ksi; $\tau_{xy} = 40$ ksi ccw

$\sigma_1 = 79.7$ ksi; $\sigma_2 = -9.7$ ksi; $\tau_{MAX} = 44.7$ ksi

$2\phi = 63.4°$ & $\phi = 31.7°$ ccw FROM x

$2\phi' = 153.4°$; $\phi' = 76.7°$ ccw FROM x





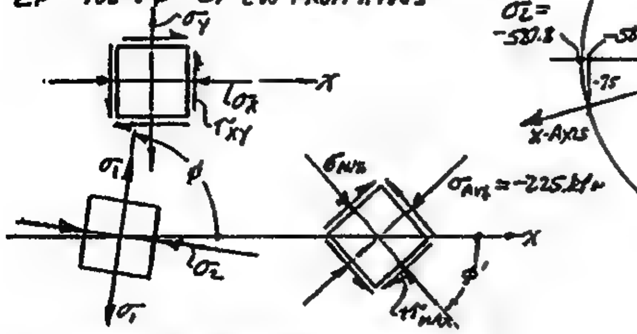
PROB. 11-10 DETAILED SOLUTION

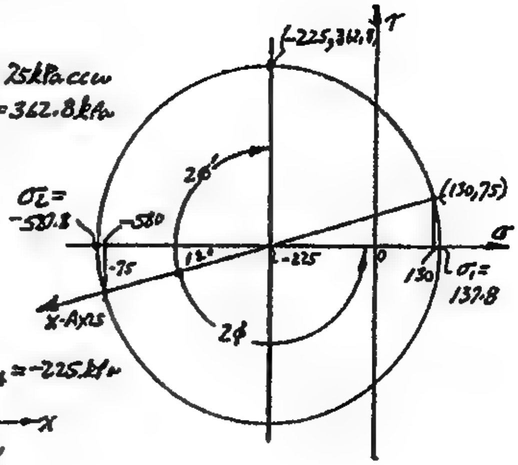
GIVEN: $\sigma_x = -580$ kPa : $\sigma_y = 130$ kPa : $\tau_{xy} = 75$ kPa ccw

$\sigma_1 = 137.8$ kPa : $\sigma_2 = -587.8$ kPa : $\tau_{MAX} = 362.8$ kPa

$2\phi = 168°$: $\phi = 84°$ CCW FROM X-AXIS

$2\phi' = 102°$: $\phi' = 51°$ CW FROM X-AXIS





PROB. 11-15 DETAILED SOLUTION

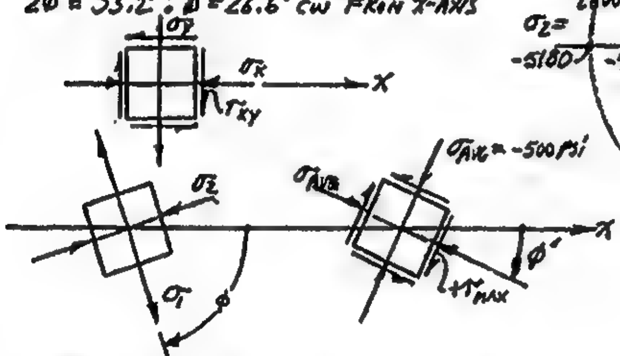
GIVEN: $\sigma_x = -4250$ psi : $\sigma_y = 3250$ psi

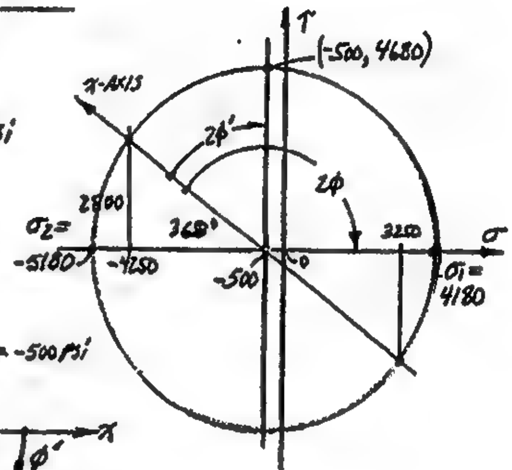
$\tau_{xy} = 2800$ psi cw

$\sigma_1 = 4180$ psi : $\sigma_2 = -5180$ psi : $\tau_{MAX} = 4680$ psi

$2\phi = 143.2°$: $\phi = 71.6°$ CW FROM X-AXIS

$2\phi' = 53.2°$: $\phi' = 26.6°$ CW FROM X-AXIS





PROB. 11-32 DETAILED SOLUTION

GIVEN: $\sigma_x = 150$ MPa : $\sigma_y = 80$ MPa

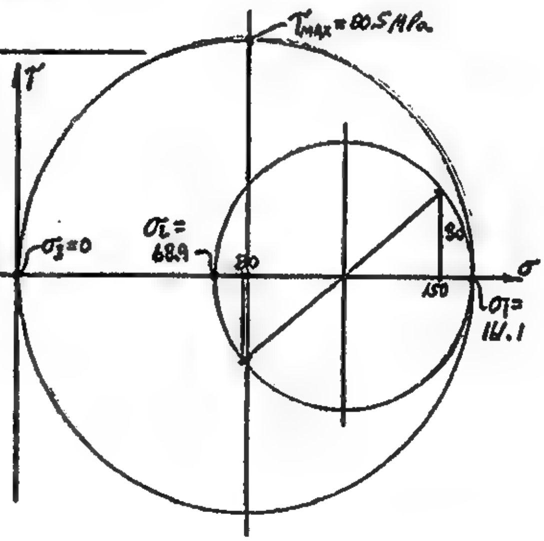
$\tau_{xy} = 30$ MPa CW

PRIMARY MOHR'S CIRCLE GIVES

$\sigma_1 = 161.1$ MPa, $\sigma_2 = 68.9$ MPa :

BECAUSE BOTH HAVE THE SAME SIGN, SECONDARY CIRCLE IS DRAWN WITH $\sigma_3 = 0$. THEN

$\tau_{MAX} = \sigma_1/2 = 80.5$ MPa



PROB 11-41 DETAILED SOLUTION

BASIC MOHR'S CIRCLE FROM PROB 11-1.

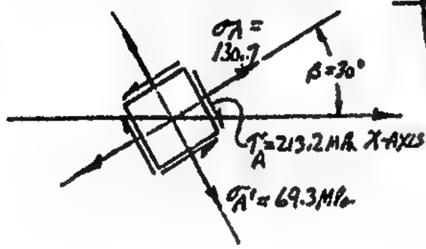
FOR $\beta = 30°$ CCW FROM X-AXIS

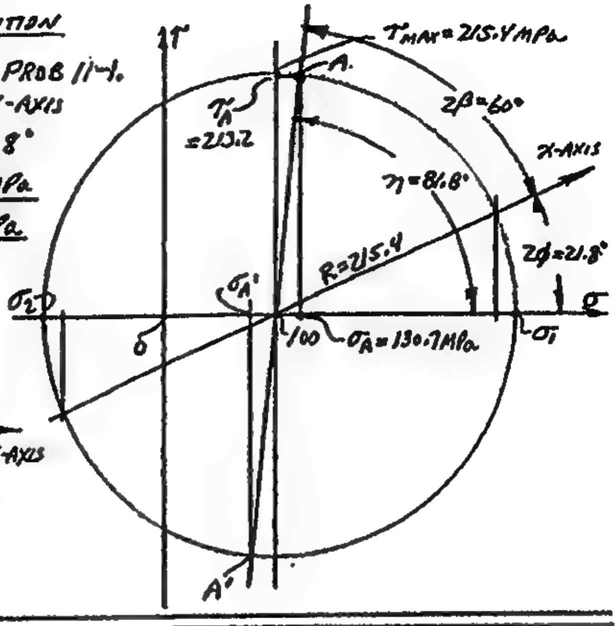
$\eta = 2\phi + 2\beta = 21.8 + 60 = 81.8°$

$\sigma_A = 100 + R\cos\eta = \underline{130.7\ MPa}$

$\sigma_{A'} = 100 - R\cos\eta = \underline{69.3\ MPa}$

$\tau_A = R\sin\eta = \underline{213.2\ MPa}$





SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.
Input data in shaded elements

Aluminum 6061-T6

Material Properties *SI Metric Units*

Modulus of Elasticity 69.0 $\times 10^{9}$ Pa
Poisson's Ratio 0.33

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

Problem 11-55.

Strain from Gage 1 1480 $\times 10^{-6}$ m/m
Strain from Gage 2 165 $\times 10^{-6}$ m/m
Strain from Gage 3 428 $\times 10^{-6}$ m/m

Results:

Max Principal Strain 1902 $\times 10^{-6}$ m/m
Min Principal Strain 6 $\times 10^{-6}$ m/m

Angle β -28.2 degrees
[From the axis of gage 1 to the nearer principal axis]
Max Principal Stress 147 Mpa
Min Principal Stress 49.1 Mpa

Max Shear Strain 1897 radians [Dimensionless]
Max Shear Stress 49.2 MPa [in plane of initial element]
******Only when Max and Min Principal Stresses have the same sign******
[Assuming stress = 0 perpendicular to plane of initial element]
True Max Shear Stress 73.7 MPa

Delta [0, 60,120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

Problem 11-63

Strain from Gage 1 1480 $\times 10^{-6}$ m/m
Strain from Gage 2 165 $\times 10^{-6}$ m/m
Strain from Gage 3 428 $\times 10^{-6}$ m/m

Results:

Max Principal Strain 1494 $\times 10^{-6}$ m/m
Min Principal Strain -112 $\times 10^{-6}$ m/m

Angle β -5.4 degrees
[From the axis of gage 1 to the nearer principal axis]
Max Principal Stress 113 Mpa
Min Principal Stress 29.5 MPa

Max Shear Strain 1607 radians [Dimensionless]
Max Shear Stress 41.7 MPa [in plane of initial element]
******Only when Max and Min principal stresses have the same sign******
[Assuming stress = 0 perpendicular to plane of initial element]
True Max Shear Stress 56.4 MPa

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.
Input data in shaded elements

Aluminum 7075-T6

Material Properties *SI Metric Units*

Modulus of Elasticity 71.7 $\times 10^9$ Pa
Poisson's Ratio 0.33

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

Problem 11-56.

Strain from Gage 1 853 $\times 10^{-6}$ m/m
Strain from Gage 2 406 $\times 10^{-6}$ m/m
Strain from Gage 3 641 $\times 10^{-6}$ m/m

Results:

Max Principal Strain 1104 $\times 10^{-6}$ m/m
Min Principal Strain 390 $\times 10^{-6}$ m/m

Angle β -36.4 degrees
[From the axis of gage 1 to the nearer principal axis]
Max Principal Stress 99.2 Mpa
Min Principal Stress 60.7 Mpa

Max Shear Strain 714 $\times 10^{-6}$ radians [Dimensionless]
Max Shear Stress 19.3 MPa [in plane of initial element]
*******Only when Max and Min Principal Stresses have the same sign******
[Assuming stress = 0 perpendicular to plane of initial element]
True Max Shear Stress 49.6 MPa

Delta [0, 60,120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

Problem 11-64.

Strain from Gage 1 853 $\times 10^{-6}$ m/m
Strain from Gage 2 406 $\times 10^{-6}$ m/m
Strain from Gage 3 641 $\times 10^{-6}$ m/m

Results:

Max Principal Strain 892 $\times 10^{-6}$ m/m
Min Principal Strain 375 $\times 10^{-6}$ m/m

Angle β -15.9 degrees
[From the axis of gage 1 to the nearer principal axis]
Max Principal Stress 81.7 Mpa
Min Principal Stress 53.9 MPa

Max Shear Strain 516 $\times 10^{-6}$ radians [Dimensionless]
Max Shear Stress 13.9 MPa [in plane of initial element]
*******Only when Max and Min principal stresses have the same sign******
[Assuming stress = 0 perpendicular to plane of initial element]
True Max Shear Stress 40.8 MPa

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.
Input data in shaded elements

AISI 1040 cold drawn steel

Material Properties *SI Metric Units*

Modulus of Elasticity	207.0 x 10^9 Pa
Poisson's Ratio	0.29

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

Problem 11-57.

Strain from Gage 1	389 x10^{-6} m/m
Strain from Gage 2	737 x10^{-6} m/m
Strain from Gage 3	-290 x10^{-6} m/m
Results:	
Max Principal Strain	816 x10^{-6} m/m
Min Principal Strain	-717 x10^{-6} m/m
Angle β	31.9 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	137.5 Mpa
Min Principal Stress	-108.6 Mpa
Max Shear Strain	1534 radians [Dimensionless]
Max Shear Stress	123.0 MPa [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 68.7 MPa

Delta [0, 60,120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

Problem 11-65.

Strain from Gage 1	389 x10^{-6} m/m
Strain from Gage 2	737 x10^{-6} m/m
Strain from Gage 3	-290 x10^{-6} m/m
Results:	
Max Principal Strain	882 x10^{-6} m/m
Min Principal Strain	-324 x10^{-6} m/m
Angle β	39.7 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	178.0 Mpa
Min Principal Stress	-15.5 MPa
Max Shear Strain	1206 radians [Dimensionless]
Max Shear Stress	96.8 MPa [in plane of initial element]

******Only when Max and Min principal stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 89.0 MPa

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.
Input data in shaded elements .

AISI 4140 OQT 900 steel

Material Properties *SI Metric Units*

Modulus of Elasticity	207.0 x 10^9 Pa
Poisson's Ratio	0.29

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

Problem 11-58.

Strain from Gage 1	925 x10^{-6} m/m
Strain from Gage 2	-631 x10^{-6} m/m
Strain from Gage 3	552 x10^{-6} m/m

Results:

Max Principal Strain	2121 x10^{-6} m/m
Min Principal Strain	-644 x10^{-6} m/m
Angle β	-41.1 degrees
	[From the axis of gage 1 to the nearer principal axis]
Max Principal Stress	437.1 Mpa
Min Principal Stress	-6.5 Mpa
Max Shear Strain	2764 radians [Dimensionless]
Max Shear Stress	221.8 MPa [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 218.5 MPa

Delta [0, 60,120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

Problem 11-66.

Strain from Gage 1	925 x10^{-6} m/m
Strain from Gage 2	-631 x10^{-6} m/m
Strain from Gage 3	552 x10^{-6} m/m

Results:

Max Principal Strain	1220 x10^{-6} m/m
Min Principal Strain	-656 x10^{-6} m/m
Angle β	-23.4 degrees
	[From the axis of gage 1 to the nearer principal axis]
Max Principal Stress	232.7 Mpa
Min Principal Stress	-68.3 MPa
Max Shear Strain	1876 radians [Dimensionless]
Max Shear Stress	150.5 MPa [in plane of initial element]

******Only when Max and Min principal stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 116.4 MPa

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.
Input data in shaded elements

Copper C14500 hard

Material Properties *U.S. Customary Unit System*

Modulus of Elasticity	17.0 x 10^6 psi
Poisson's Ratio	0.33

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

Problem 11-59

Strain from Gage 1	169 x10^{-6} in/in
Strain from Gage 2	-266 x10^{-6} in/in
Strain from Gage 3	543 x10^{-6} in/in

Results:

Max Principal Strain	1006 x10^{-6} in/in
Min Principal Strain	-294 x10^{-6} in/in
Angle β	36.6 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	17335 psi
Min Principal Stress	731 psi
Max Shear Strain	1299 radians [Dimensionless]
Max Shear Stress	8302 psi [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 8667 psi

Delta [0, 60,120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

Problem 11-67

Strain from Gage 1	169 x10^{-6} in/in
Strain from Gage 2	-266 x10^{-6} in/in
Strain from Gage 3	543 x10^{-6} in/in

Results:

Max Principal Strain	616 x10^{-6} in/in
Min Principal Strain	-319 x10^{-6} in/in
Angle β	-43.8 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	9748 psi
Min Principal Stress	-2204 psi
Max Shear Strain	935 radians [Dimensionless]
Max Shear Stress	5976 psi [in plane of initial element]

******Only when Max and Min principal stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 4874 psi

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.
Input data in shaded elements

Titanium Ti-6Al-4V, aged

Material Properties *U.S. Customary Unit System*

Modulus of Elasticity 16.5 x 10^6 psi
Poisson's Ratio 0.3

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

Problem 11-60.

Strain from Gage 1	775	x10^{-6} in/in
Strain from Gage 2	369	x10^{-6} in/in
Strain from Gage 3	-318	x10^{-6} in/in
Results:		
Max Principal Strain	793	x10^{-6} in/in
Min Principal Strain	-336	x10^{-6} in/in
Angle β	7.2	degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	12548	psi
Min Principal Stress	-1776	psi
Max Shear Strain	1129	radians [Dimensionless]
Max Shear Stress	7162	psi [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******
[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 6274 psi

Delta [0, 60,120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

Problem 11-68.

Strain from Gage 1	775	x10^{-6} in/in
Strain from Gage 2	369	x10^{-6} in/in
Strain from Gage 3	-318	x10^{-6} in/in
Results:		
Max Principal Strain	913	x10^{-6} in/in
Min Principal Strain	-363	x10^{-6} in/in
Angle β	19.2	degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	14587	psi
Min Principal Stress	-1607	psi
Max Shear Strain	1276	radians [Dimensionless]
Max Shear Stress	8097	psi [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******
[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 7294 psi

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.
Input data in shaded elements

Ductile Iron, ASTM A536, 80-55-6

Material Properties *U.S. Customary Unit System*

Modulus of Elasticity	24.0	$\times 10^6$ psi
Poisson's Ratio	0.27	

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

Problem 11-61.

Strain from Gage 1	389	$\times 10^{-6}$ in/in
Strain from Gage 2	737	$\times 10^{-6}$ in/in
Strain from Gage 3	-290	$\times 10^{-6}$ in/in

Results:

Max Principal Strain	816	$\times 10^{-6}$ in/in
Min Principal Strain	-717	$\times 10^{-6}$ in/in

Angle β 31.9 degrees
[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	16117	psi
Min Principal Stress	-12863	psi

Max Shear Strain	1534	radians [Dimensionless]
Max Shear Stress	14490	psi [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******
[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 8059 psi

Delta [0, 60,120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

Problem 11-69.

Strain from Gage 1	389	$\times 10^{-6}$ in/in
Strain from Gage 2	737	$\times 10^{-6}$ in/in
Strain from Gage 3	-290	$\times 10^{-6}$ in/in

Results:

Max Principal Strain	882	$\times 10^{-6}$ in/in
Min Principal Strain	-324	$\times 10^{-6}$ in/in

Angle β 39.7 degrees
[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	20559	psi
Min Principal Stress	-2236	psi

Max Shear Strain	1206	radians [Dimensionless]
Max Shear Stress	11397	psi [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******
[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 10280 psi

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.
Input data in shaded elements

Stainless Steel, AISI 501 OQT 1000

Material Properties *U.S. Customary Unit System*

Modulus of Elasticity 29.0 $\times 10^6$ psi
Poisson's Ratio 0.30

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

Problem 11-62.

Strain from Gage 1 1532 $\times 10^{-6}$ in/in
Strain from Gage 2 -228 $\times 10^{-6}$ in/in
Strain from Gage 3 893 $\times 10^{-6}$ in/in

Results:

Max Principal Strain 2688 $\times 10^{-6}$ in/in
Min Principal Strain -263 $\times 10^{-6}$ in/in

Angle β -38.7 degrees
[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 83147 psi
Min Principal Stress 17317 psi

Very high stress: s_y = 135 ksi
N = 1.62 Low

Max Shear Strain 2951 $\times 10^{-6}$ radians [Dimensionless]
Max Shear Stress 32915 psi [in plane of initial element]

*******Only when Max and Min Principal Stresses have the same sign*******
[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 41574 psi

Delta [0, 60,120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

Problem 11-70.

Strain from Gage 1 1532 $\times 10^{-6}$ in/in
Strain from Gage 2 -228 $\times 10^{-6}$ in/in
Strain from Gage 3 893 $\times 10^{-6}$ in/in

Results:

Max Principal Strain 1761 $\times 10^{-6}$ in/in
Min Principal Strain -296 $\times 10^{-6}$ in/in

Angle β -19.5 degrees
[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 53289 psi
Min Principal Stress 7390 psi

Max Shear Strain 2058 $\times 10^{-6}$ radians [Dimensionless]
Max Shear Stress 22949 psi [in plane of initial element]

*******Only when Max and Min Principal Stresses have the same sign*******
[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 26644 psi

CHAPTER 12 Deflection of Beams

12-1

A22-(b): $y = -\dfrac{P\,l^3}{48EI} = \dfrac{-(3000)(700)^3}{(48)(207\times10^3)(5.147\times10^4)} = \underline{-2.01\ mm}$

$E = 207\times10^9\ \dfrac{N}{m^2}\times\dfrac{1m^2}{(10^3mm)^2} = 207\times10^3\ N/mm^2$

$I = \dfrac{\pi D^4}{64} = \dfrac{\pi(32\ mm)^4}{64} = 5.147\times10^4\ mm^4$

12-2

$E = 69\ GPa = 69\times10^3\ N/mm^2$; y INVERSELY PROPORTIONAL TO E.

$y = -2.01\ mm\times\dfrac{207}{69} = \underline{-6.03\ mm}$

12-3

A-24-(b):

$y = \dfrac{-Pl^3}{192EI} = \dfrac{(-3000)(700)^3}{192(207\times10^3)(5.147\times10^4)} = \underline{-0.503\ mm}$

12-4

$y = \dfrac{-Pl^3}{48EI} = \dfrac{-3000(350)^3}{48(207\times10^3)(5.147\times10^4)} = \underline{-0.252\ mm}$

12-5

$I = \pi D^4/64 = \pi(25)^4/64 = 19\,175\ mm^4$

$y = -\dfrac{3000(700)^3}{48(207\times10^3)(19175)} = \underline{-5.40\ mm}$

12-6

A22-(b):

AT LOAD: $y = \dfrac{-Pa^2b^2}{3EIl} = \dfrac{-3000(175)^2(525)^2}{3(207\times10^3)(5.147\times10^4)(700)} = \underline{-1.13\ mm}$

AT CENTER: $y = \dfrac{-Pbx}{6EIl}(l^2-x^2-b^2)$, USE $x = 350\ mm$

$y = \dfrac{-3000(175)(350)}{6(207\times10^3)(5.147\times10^4)(700)}\left[(700)^2-(350)^2-(175)^2\right]$

$y = \underline{-1.38\ mm}$

12-7

A22-(c): W12×16; $I = 103\ in^4$; LOADING IN P6-4.

$P = 10000\ lb$; $a = 36\ in$; $l = 168\ in$; $E = 30\times10^6\ psi$

AT LOADS: $y = \dfrac{-Pa^2}{6EI}(3l-4a) = \dfrac{-(10000)(36)^2}{6(30\times10^6)(103)}\left[(3)(168)-(4)(36)\right]$

$\underline{y = -0.251\ in}$

AT CENTER: $y = \dfrac{-Pa}{24EI}(3l^2-4a^2) = \dfrac{-10000(36)}{24(30\times10^6)(103)}\left[3(168)^2-4(36)^2\right]$

$\underline{y = -0.385\ in}$

12-8

A22-(a): $I = 0.310\ in^4$

$$y = \frac{-P\ell^3}{48EI} = \frac{-650(28)^3}{48(30\times10^6)(0.310)} = \underline{-0.032\ in}$$

12-9

A22-(d): $I = 59.69\ in^4$; $E = 10\times10^6\ psi$

$W = (1125\ lb/ft)(10\ ft) = 11250\ lb$

$\ell = 10\ ft \times 12\ in/ft = 120\ in$

$$y = \frac{5w\ell^3}{384EI} = \frac{-5(11250)(120)^3}{384(10\times10^6)(59.69)} = \underline{-0.424\ in}$$

12-10

A22-(d): $x = 3.5\ FT\ (12\ IN/FT) = 42\ IN.$

$w = (1125\ LB/FT)(1\ FT/12\ IN) = 93.75\ LB/IN$

$$y = \frac{-wx}{24EI}(L^3 - 2Lx^2 + x^3) = \frac{-(93.75)(42)}{24(10\times10^6)(59.69)}(120^3 - 2(120)(42)^2 + 42^3)$$

$\underline{y = -0.379\ IN}$

12-11

LOADING IN FIGURE P6-12.

$I = 238\ in^4$: A22-(g) ; $a = 48\ in$; $\ell = 120\ in$; $P = 15000\ lb$

$$y = \frac{-Pa^2}{3EI}(a+\ell) = \frac{-15000(48)^2}{3(30\times10^6)(238)}(48+120) = \underline{-0.271\ in}$$

12-12

A22-(g): $a = 24\ IN$; $L = 144\ IN$

$$y = \frac{-Pa^2}{3EI}(a+L) = \frac{-15000(24)^2}{3(30\times10^6)(238)}(24+144) = \underline{-0.0678\ IN}$$

12-13

$+y_{MAX}$ at $x = 0.577\ (\ell) = 0.577\ (120) = 69.2\ in$ FROM A

$$y = \frac{.06415\ PaL^2}{EI} = \frac{(.06415)(15000)(48)(120)^2}{(30\times10^6)(238)} = \underline{0.093\ in}\ \text{(UPWARD)}.$$

12-14

A23-(a): $y = \dfrac{-P\ell^3}{3EI} = \dfrac{-120(8)^3}{3(30\times10^6)(0.08734)} = -0.0078\ in.$

12-15

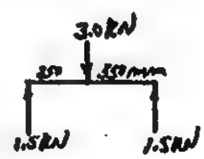
A22-(a)

$$y = \frac{-P\ell^3}{48EI}\ \therefore\ \text{REQ'D}\ I = \frac{-P\ell^3}{48Ey} = \frac{-3000(700)^3}{48(207\times10^3)(-0.12)} = 8.63\times10^5\ mm^4$$

$$I = \frac{\pi D^4}{64}\ \therefore\ D = [64I/\pi]^{1/4} = [64(8.63\times10^5)/\pi]^{1/4} = \underline{64.8\ mm}$$

12-16

$$\sigma = \frac{Mc}{I} = \frac{\overbrace{(1500N)(350mm)}^{M}\overbrace{(32.4mm)}^{c}}{8.63\times10^5\ mm^4} = \underline{19.7\ MPa}$$



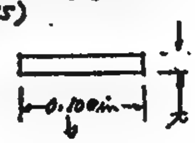
REQ'D $S_u = 8(19.7\ MPa) = \underline{158\ MPa\ \text{(ANY STEEL)}}$

12-17

A-23-(a)

$$y_{MAX} = \frac{-P\ell^3}{3EI}\ \therefore\ \text{REQ'D}\ I = \frac{-P\ell^3}{3Ey} = \frac{-0.52(1.20)^3}{3(30\times10^6)(0.15)} = 6.656\times10^{-8}\ in^4$$

$$I = \frac{bh^3}{12}\ \therefore\ t = \left[\frac{12I}{b}\right]^{1/3} = \left[\frac{12(6.656\times10^{-8})}{0.100}\right]^{1/3} = \underline{0.020\ in}$$



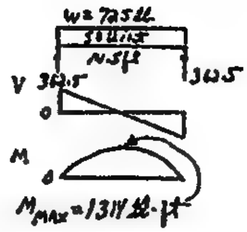
12-18

$I = \frac{bh^3}{12} = \frac{(1.50)(9.25)^3}{12} = 98.93\ in^4$

$S = \frac{bh^2}{6} = \frac{(1.50)(9.25)^2}{6} = 21.39\ in^3$

A-22-(d)

$y_{MAX} = \frac{-5wL^3}{384EI} = \frac{-5(725)(174)^3}{384(1.76\times10^6)(98.93)} = \underline{-0.286\ in}$



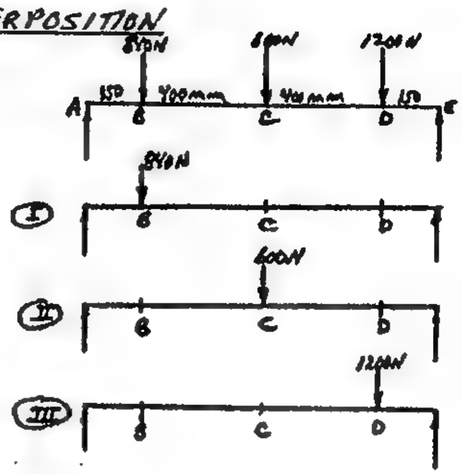
$\sigma = \frac{M}{S} = \frac{1314\ lb\text{-}ft\,(12\ in/ft)}{21.39} = \underline{737\ psi}$

$\tau = \frac{3V}{2A} = \frac{3(362.5)}{2(1.50)(9.25)} = \underline{39.2\ psi}$

OK PER TABLE A-18

12-19 SUPERPOSITION

COMBINE A22(a) AND A22-(b)



$y_B = y_{B1} + y_{B2} + y_{B3}$

$y_C = y_{C1} + y_{C2} + y_{C3}$

$y_D = y_{D1} + y_{D2} + y_{D3}$

I - A22-2

$y_{B1} = \frac{-Pa^2b^2}{3EIL}$; $b = 150, a = 950$ [I FROM P7-11]

$EI = (69\times10^3\ N/mm^2)(0.186\times10^6\ mm^4)$

$EI = 1.283\times10^{10}$

$EIL = (1.283\times10^{10})(1100) = 1.412\times10^{13}$

$y_{B1} = \frac{(-840)(150)^2(950)^2}{3(1.412\times10^{13})} = -0.403\ mm$

$y_{C1} = \frac{-Pbx}{6EIL}(L^2 - x^2 - b^2) = \frac{-840(150)(550)}{6(1.412\times10^{13})}(1100^2 - 550^2 - 150^2) = -0.724\ mm$

$y_{D1} =$ " ↓ " $= \frac{-840(150)(150)}{6(1.412\times10^{13})}(1100^2 - 150^2 - 150^2) = -0.260\ mm$

II - A22-(a)

$y_{B2} = y_{D2} = \frac{-Px}{48EI}(3L^2 - 4x^2) = \frac{-600(150)}{48(1.283\times10^{10})}(3(1100)^2 - 4(150)^2) = -0.517\ mm$

$y_{C2} = \frac{-PL^3}{48EI} = \frac{-600(1100)^3}{48(1.283\times10^{10})} = -1.297\ mm.$

III A22-(b)

$y_{B3} = \frac{-Pbx}{6EIL}(L^2 - x^2 - b^2) = \frac{-1200(150)(150)}{6(1.412\times10^{13})}(1100^2 - 150^2 - 150^2) = -0.371\ mm$

$y_{C3} =$ " ↓ " $= \frac{-1200(150)(550)}{6(1.412\times10^{13})}(1100^2 - 550^2 - 150^2) = -1.034\ mm$

$y_{D3} = \frac{-Pa^2b^2}{3EIL} = \frac{-1200(150)^2(950)^2}{3(1.412\times10^{13})} = -0.576\ mm$

TOTAL DEFLECTIONS —

$\underline{y_B} = y_{B1} + y_{B2} + y_{B3} = \underline{-1.291\ mm}$

$\underline{y_C} = y_{C1} + y_{C2} + y_{C3} = \underline{-3.055\ mm}$

$\underline{y_D} = y_{D1} + y_{D2} + y_{D3} = \underline{-1.353\ mm}$

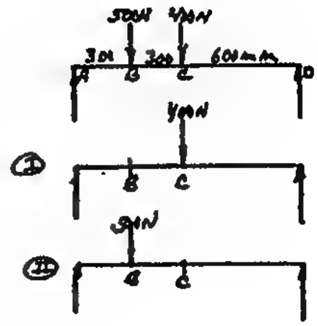
12-20

$E = 73\,GPa = 73\times10^9\,N/m^2 = 73\times10^3\,N/mm^2$

$EI = (73\times10^3)(16956) = 1.238\times10^9\,N\cdot mm^2$

$EI\ell = (1.238\times10^9)(1200) = 1.485\times10^{12}\,N\cdot mm^3$

[I FROM P7-12]



(I) A22-(a)

$$y_{B_1} = \frac{-Px}{48EI}(3\ell^2 - 4x^2)$$

$$y_{B_1} = \frac{-400(300)}{48(1.238\times10^9)}\left(3(1200)^2 - 4(300)^2\right) = -8.000\,mm$$

$$y_{C_1} = \frac{-PL^3}{48EI} = \frac{-400(1200)^3}{48(1.238\times10^9)} = -11.632\,mm$$

(II) A22-(b)

$$y_{B_2} = \frac{-Pa^2b^2}{3EI\ell} = \frac{-500(300)^2(900)^2}{3(1.485\times10^{12})} = -8.182\,mm$$

$$y_{C_2} = \frac{-Pbx}{6EI\ell}(\ell^2 - x^2 - b^2) = \frac{-500(300)(600)}{6(1.485\times10^{12})}(1200^2 - 600^2 - 300^2) = -10.000\,mm$$

TOTAL DEFLECTIONS

$$\underline{y_B} = y_{B_1} + y_{B_2} = -8.000\,mm - 8.182\,mm = \underline{-16.182\,MM}$$

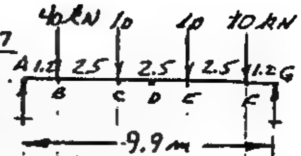
$$\underline{y_C} = y_{C_1} + y_{C_2} = -11.632\,mm - 10.000\,mm = \underline{-21.632\,mm}$$

} LARGE DEFLECTIONS

12-21 LOADING IN FIGURE P6-7. FIGURE P6-7 TOTAL LOADING

W18-55 STEEL BEAM

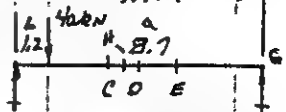
USE SUPERPOSITION



SUPERPOSITION

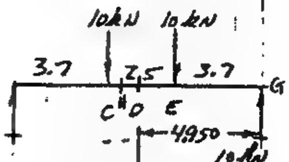
(I) 40 kN LOAD ONLY AT B.
CASE A22(b): L = 9900 mm
a = 8700 mm, b = 1200 mm

(I)



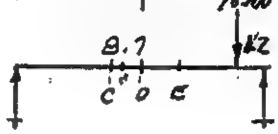
(II) TWO 10 kN LOADS AT C AND E.
CASE A-22(C): L = 9900 mm
a = 3700 mm

(II)



(III) 10 kN LOAD ONLY AT F.
a = 8700 mm, b = 1200 mm
CASE A22(b): L = 9900 mm

(III)



POINT OF MAXIMUM DEFLECTION IS NOT OBVIOUS BECAUSE EACH CASE PRODUCES A MAXIMUM DEFLECTION AT A DIFFERENT POINT. DEFLECTION AT C, D, AND E ARE COMPUTED FOR EACH LOADING, THEN SUMMED. THE MAXIMUM DEFLECTION FOR CASE I OCCURS BETWEEN C AND D AT THE POINT CALLED H, 4226 mm FROM A. DEFLECTION COMPUTED THERE ALSO.

PRODUCT OF EI APPEARS IN ALL EQUATIONS.
$E = 207 \times 10^3\ N/mm^2$
$I = 890\ in^4 \times \frac{4.162 \times 10^5\ mm^4}{in^4}$
$I = 3.70 \times 10^8\ mm^4$
$EI = 7.66 \times 10^{13}\ N\cdot mm^2$

SUMMARY OF RESULTS:

I		II		III	
	$y_C = -3.802$ mm		$y_C = -4.438$ mm		$y_C = -0.809$ mm
	$y_D = -3.76$ mm		$y_D = -4.816$ mm		$y_D = -0.940$ mm
	$y_E = -3.235$ mm		$y_E = -4.438$ mm		$y_E = -0.951$ mm
	$y_H = -3.85$ mm		$y_H = -4.689$ mm		$y_H = -0.877$ mm

BY SUPERPOSITION:

$y_C = -9.049$ mm

$\underline{y_D = -9.516\ mm}$ APPARENT MAXIMUM DEFLECTION AT MIDDLE OF BEAM AT D.

$y_E = -8.624$ mm

$y_H = -9.416$ mm

12-22

A23-(a) AND A23-(b)

(I) $y_{B_1} = \dfrac{-Px^2}{6EI}(3l - x)$

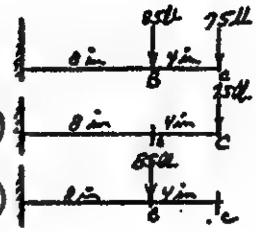
$EI = (30 \times 10^6)(0.08734) = 2.62 \times 10^6$

$y_{B_1} = \dfrac{-75(8)^2}{6(2.62 \times 10^6)}[3(12) - 8] = -0.0085 \text{ in}$

$y_{C_1} = \dfrac{-Pl^3}{3EI} = \dfrac{-75(12)^3}{3(2.62 \times 10^6)} = -0.0165 \text{ in}$

(II) $y_{B_2} = \dfrac{-Pa^3}{3EI} = \dfrac{-85(8)^3}{3(2.62 \times 10^6)} = -0.0055 \text{ in}$

$y_{C_2} = \dfrac{-Pa^2}{6EI}[3l - a] = \dfrac{-85(8)^2}{6(2.62 \times 10^6)}[3(12) - 8] = -0.0097 \text{ in}$



TOTAL DEFLECTION

$y_B = y_{B1} + y_{B2} = -0.0085 - 0.0055 = \underline{-0.0140 \text{ in}}$

$y_C = y_{C1} + y_{C2} = -0.0165 - 0.0097 = \underline{-0.0262 \text{ in}}$

12-23

$I = \dfrac{bh^3}{12} = \dfrac{20(80)^3}{12} = 8.533 \times 10^5 \text{ mm}^4$

$EI = [207 \times 10^3 \text{ N/mm}^2][8.533 \times 10^5 \text{ mm}^4] = 1.766 \times 10^{11}$

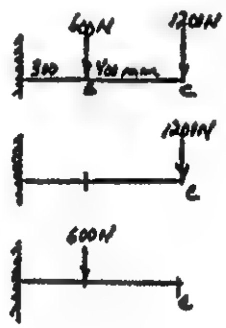
(I) A23-(a)

$y_{C1} = \dfrac{-Pl^3}{3EI} = \dfrac{-1200(700)^3}{3(1.766 \times 10^{11})} = -0.777 \text{ mm}$

(II) A23-(b)

$y_{C2} = \dfrac{-Pa^2}{6EI}[3l - a] = \dfrac{-600(300)^2}{6(1.766 \times 10^{11})}[3(700) - 300]$

$y_{C2} = -0.092 \text{ mm}$



TOTAL DEFLECTION

$y_C = y_{C1} + y_{C2} = -0.777 - 0.092 = \underline{-0.869 \text{ mm}}$

12-24

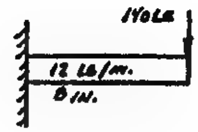
DEFLECTION INVERSELY PROPORTIONAL TO E

$y_C = -0.869 \text{ mm} \times \dfrac{E_S}{E_{AL}} = -0.869 \times \dfrac{207 \text{ GPa}}{73 \text{ GPa}} = \underline{-2.464 \text{ mm}}$

12-25

$y_C = -0.869 \text{ mm} \times \dfrac{E_S}{E_{MAG}} = -0.869 \times \dfrac{207 \text{ GPa}}{45 \text{ GPa}} = \underline{-3.997 \text{ mm}}$

12-26



$$I = \pi D^4/64 = \pi(0.80)^4/64 = 0.0201\ in^4$$

W = TOTAL LOAD = wL = (12 LB/in.)(8 in) = 96 LB

A23-(c)
$$y_1 = \frac{-wl^3}{8EI} = \frac{-96(8)^3}{8(30\times10^6)(0.0201)} = -0.0102\ in$$

A23-(a)
$$y_2 = \frac{-Pl^3}{3EI} = \frac{-140(8)^3}{3(30\times10^6)(0.0201)} = -0.0396\ in$$

TOTAL $y = y_1 + y_2 = -0.0102 - 0.0396 = \underline{-0.0498\ in}$

12-27

FROM FIG P6-7, L = 9900 mm

$L/360 = 27.5\ mm$; LET $y_{MAX} = -27.5\ mm$

FROM PROBLEM 12-21, $y_{MAX} = -9.516\ mm$

FOR A W18-55 BEAM WITH $I = 890\ IN^4$

DEFLECTION INVERSELY PROPORTIONAL TO I.

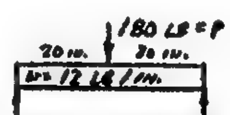
$$I_{MIN} = 890\ IN^4 \cdot \frac{9.516}{27.5} = 308\ IN^4$$

SPECIFY: W18×40 - LIGHTEST

OR W14×43 - LEAST DEPTH

RESULT COULD HAVE BEEN FOUND BY USING SUPERPOSITION APPROACH OUTLINED IN PROBLEM 12-21 WITH I TREATED AS AN UNKNOWN. THEN SET $y_{MAX} = \frac{c}{I_{MIN}}$ AND SOLVE FOR I_{MIN} FOR $y = 27.5\ mm$.

12-28



$$y_{MAX} = -0.080\ in = \underbrace{\frac{-Pl^3}{48EI}}_{A22-(a)} - \underbrace{\frac{5W\ l^3}{384EI}}_{A22-(d)}$$

$$48EI\,y_{MAX} = -Pl^3 - \frac{5W\ l^3}{8} = l^3[-P - 5W/8]$$

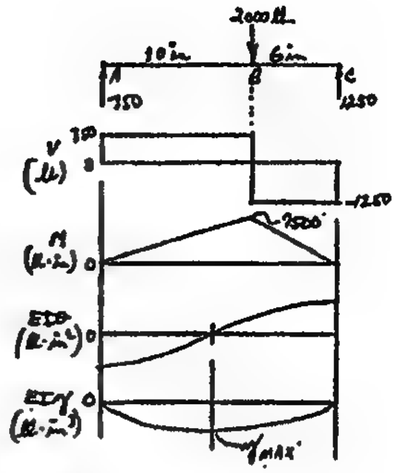
$$\text{REQ'D } I = \frac{l^3}{48E\,y_{MAX}}[-P - 5W/8] = \frac{(40)^3[-180 - 5(480)/8]}{48(10\times10^6)(-0.08)} = \underline{0.800\ in^4}$$

USE A $\underline{C4\times2.331\ (I_{yy} = 1.02\ in^4)}$

OR $\underline{C5\times2.212\ (I_{yy} = 0.98\ in^4)\ (\text{LIGHTEST})}$

12-29

$I = bh^3/12 = 1(2)^3/12 = 0.667\ in^4$



$\boxed{M_{AB} = 750x}$

$M_{BC} = -1250x + C$

at $x = 16$, $M = 0$

$0 = -1250(16) + C \quad \therefore C = 20\,000$

$\boxed{M_{BC} = -1250x + 20000}$

$\boxed{EI\theta_{AB} = \int 750x\,dx = 375x^2 + C_1}$

$\boxed{EI\theta_{BC} = \int(-1250x + 20000)\,dx = -625x^2 + 20000x + C_2}$

$EIy_{AB} = \int(375x^2 + C_1)\,dx = \boxed{125x^3 + C_1x + C_3 = EIy_{AB}}$

$EIy_{BC} = \int(-625x^2 + 20000x + C_2)\,dx = \boxed{-208.3x^3 + 10000x^2 + C_2x + C_4 = EIy_{BC}}$

BOUNDARY CONDITIONS

① at $x = 0$, $EIy_{AB} = 0$ ③ at $x = 10$, $EI\theta_{AB} = EI\theta_{BC}$

② at $x = 16$, $EIy_{BC} = 0$ ④ at $x = 10$, $EIy_{AB} = EIy_{BC}$

SUBSTITUTING:

① $0 = 0 + 0 + C_3 \quad \therefore C_3 = 0$

② $EIy_{BC} = 0 = -208.3(16)^3 + 10000(16)^2 + 16C_2 + C_4 = 1.707\times10^6 + 16C_2 + C_4 = 0$

③ $375(10)^2 + C_1 = -625(10)^2 + 20000(10) + C_2$

$C_1 - C_2 = 1.0\times10^5$

④ $125(10)^3 + 10C_1 + \cancel{C_3}^{\,0} = -208.3(10)^3 + 10000(10)^2 + 10C_2 + C_4$

$10C_1 - 10C_2 - C_4 = 6.667\times10^5$

IN ② AND ③ SOLVE FOR C_1 AND C_4 IN TERMS OF C_2

$C_1 = C_2 + 1.0\times10^5$

$C_4 = -16C_2 - 1.707\times10^6$

SUBSTITUTE INTO ④

$10[C_2 + 1.0\times10^5] - 10C_2 - (-16C_2 - 1.707\times10^6) = 6.667\times10^5$

$\cancel{10C_2} + 1.0\times10^6 \cancel{-10C_2} + 16C_2 + 1.707\times10^6 = 6.667\times10^5$

$C_2 = -2.040\times10^6/16 = -1.275\times10^5 = \underline{-127500 = C_2}$

$C_1 = C_2 + 1.0\times10^5 = -1.275\times10^5 + 1.0\times10^5 = -0.275\times10^5 = \underline{-27500 = C_1}$

$C_4 = -16C_2 - 1.707\times10^6 = -16[-1.275\times10^5] - 1.707\times10^6 = \underline{333000 = C_4}$

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12-29 (CONTINUED)

FINAL EQUATIONS

$$EI\theta_{AB} = 375X^2 - 27500$$

$$EI\theta_{BC} = -625X^2 + 20000X - 127500$$

$$EIy_{AB} = 125X^3 - 27500X$$

$$EIy_{BC} = -208.3X^3 + 10000X^2 - 127500X + 333\,000$$

MAX y OCCURS WHERE $EI\theta = 0$

SET $EI\theta_{AB} = 0 = 375X^2 - 27500$

$$X = \sqrt{27500/375} = 8.56\text{ in}$$

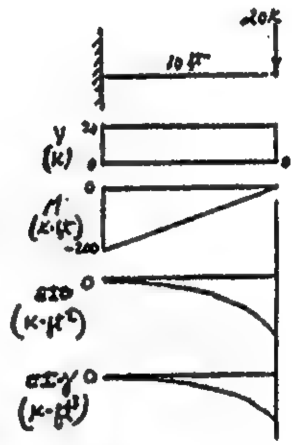
$$\text{MAX } EIy = (EIy_{AB})_{X=8.56} = 125(8.56)^3 - 27500(8.56) = -156\,997\text{ lb}\cdot\text{in}^3$$

$$\text{MAX } y = \frac{(EIy)_{MAX}}{EI} = \frac{-156997\text{ lb}\cdot\text{in}^3}{(30\times10^6)(0.667)(\text{lb/in}^2)(\text{in}^4)} = \underline{-0.0078\text{ in at } X = 8.56\text{ in}}$$

CHECK: $\sigma = \frac{Mc}{I} = \frac{(7500\text{ lb}\cdot\text{in})(1.0\text{ in})}{0.667\text{ in}^4} = 11244$ psi OK FOR STEEL

12-30

$I = 890\text{ in}^4$ W18X55



$$M = 20X - 200$$

$$EI\theta = \int(20X - 200)\,dX = 10X^2 - 200X + C_1$$

at $X = 0$, $EI\theta = 0$ ∴ $C_1 = 0$

$$EIy = \int(10X^2 - 200X)\,dX = 3.333X^3 - 100X^2 + C_2$$

at $X = 0$, $EIy = 0$ ∴ $C_2 = 0$

THEN $\boxed{EIy = 3.333X^3 - 100X^2}$

at $X = 10$ ft

$$EIy_{MAX} = 3.333(10^3) - 100(10)^2 = -6667\text{ K}\cdot\text{ft}^3$$

$$y_{MAX} = \frac{-6667\text{ K}\cdot\text{ft}^3}{(30\times10^6\text{ lb/in}^2)(890\text{ in}^4)} \times \frac{1000\text{ lb}}{\text{K}} \times \frac{1728\text{ in}^3}{\text{ft}^3} = \underline{-0.431\text{ in}}$$

CHECK: $\sigma = \frac{M}{S} = \frac{(200\text{ K}\cdot\text{ft})(1000\text{ lb/K})(12\text{ in/ft})}{98.3\text{ in}^3} = 24415$ psi

$\sigma < S_y$

BUT HIGH FOR A36 STRUCTURAL STEEL.

12-31

$V = -4x + 4$ | $I = 1.530\ in^4 \times 4.162\times10^5 \frac{mm^4}{in^4} = 6.368\times10^5\ mm^4$

$M = -2x^2 + 4x + C_1 = -2x^2 + 4x - 2$

at $x = 1.0$, $M = 0$

$0 = -2 + 4 + C_1$ $\therefore C_1 = -2$

$EI\theta = \int(-2x^2 + 4x - 2)dx = \frac{-2x^3}{3} + 2x^2 - 2x + \cancel{C_2}^{0}$

at $x = 0$, $EI\theta = 0$ $\therefore C_2 = 0$

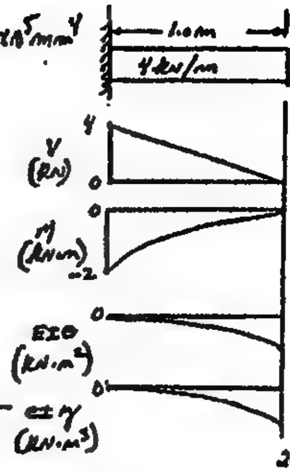
$$\boxed{EIy = \int EI\theta\,dx = -\frac{1}{6}x^4 + \frac{2x^3}{3} - x^2 + \cancel{C_3}^{0}}$$

at $x = 1.0$ m:

$EIy = -1/6 + 2/3 - 1 = -0.50\ kN{\cdot}m^3$

$$y = \frac{-0.50\ kN{\cdot}m^3\,(10^3 N/kN)(10^{15} mm^5/m^5)}{(207\times10^3 N/mm^2)(6.368\times10^5\ mm^4)} = \underline{-3.79\ mm}$$

CHECK: $\sigma = M/S = 115$ MPa - OK.



12-32

SHEAR: $V = -4x + 60$

MOMENT: $M = \int V dx = -2x^2 + 60x + C$

at $x = 0$, $M = -400$ K-ft $\therefore C_1 = -400$

$M = -2x^2 + 60x - 400$

$EI\theta = \int M dx = \frac{-2x^3}{3} + 30x^2 - 400x + \cancel{C_2}^{0}$

at $x = 0$, $EI\theta = 0$ $\therefore C_2 = 0$

$$\boxed{EIy = \int EI\theta\,dx = \frac{-x^4}{6} + 10x^3 - 200x^2 + \cancel{C_3}^{0}}$$

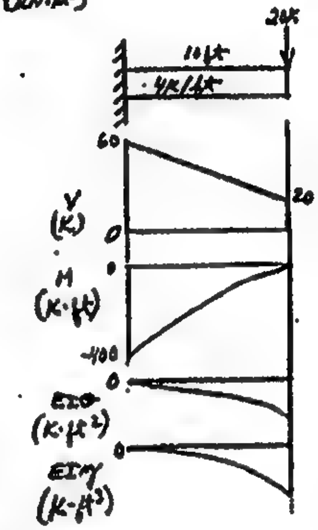
at $x = 0$, $EIy = 0$ $\therefore C_3 = 0$

at $x = 10$ ft,

$EIy_{MAX} = \frac{-10^4}{6} + 10(10^3) - 20000 = -11667\ k{\cdot}ft^3$

$$y_{max} = \frac{-11667\ k{\cdot}ft^3 (1000\ lb/k)}{(30\times10^6)\ lb/in^2\,(2100\ in^4)} \cdot \frac{(12\ in)^3}{ft^3} = \underline{-.320\ IN}$$

CHECK: $\sigma = \frac{M}{S} = 27273$ psi - HIGH



12-33

$V = -4x + 6$

$M = \int V dx = -2x^2 + 6x + C_1$

at $x = 0$, $M = -4$ $\therefore C_1 = -4$

$M = -2x^2 + 6x - 4$

$EI\theta = \int M dx = \frac{-2x^3}{3} + 3x^2 - 4x + \cancel{C_2}^{0}$

at $x = 0$, $EI\theta = 0$ $\therefore C_2 = 0$

$$\boxed{EIy = \int EI\theta\,dx = \frac{-x^4}{6} + x^3 - 2x^2 + \cancel{C_3}^{0}}$$

at $x = 0$, $EIy = 0$ $\therefore C_3 = 0$

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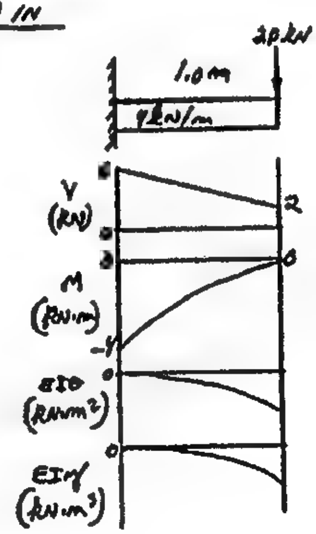


12-33 (CONTINUED)

at $x = 1.0\,m$,

$EI y_{MAX} = -\frac{1}{6} + 1 - 2 = -1.167\ kN\cdot m^3$

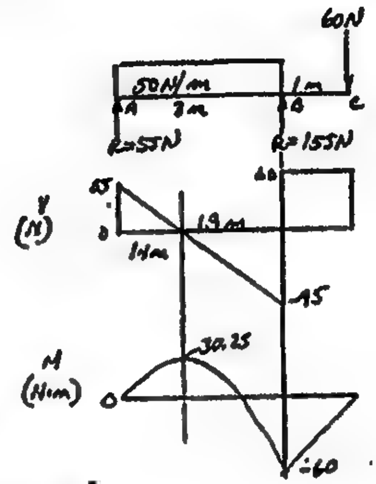
FOR $y_{MAX} = 5.0\,mm$

RCQ'D $I = \dfrac{-1.167\,kN\cdot m^3}{E y_{MAX}} = \dfrac{-1.167\times10^3\,N\cdot m^3}{(207\times10^9\,N/m^2)(5.0\,mm)} \times \dfrac{(10^3\,mm)^5}{m^5} = +1.127\times10^6\,mm^4$

$I = \pi D^4/64 \therefore D = \sqrt[4]{64 I/\pi} = [64(1.127\times10^6)/\pi]^{1/4} = \underline{69.2\,mm}$

CHECK: $\sigma = \dfrac{Mc}{I} = \dfrac{(4.0\,kN\cdot m)(34.6\,mm)}{1.127\times10^6\,mm^4} \cdot \dfrac{10^6\,N\cdot mm}{kN\cdot m} = 122.8\,MPa$ – OK FOR STEEL

12-34



$V_{AB} = -50x + 55$

$V_{BC} = 60$

$M_{AB} = \int V_{AB}\,dx = -25x^2 + 55x + \cancel{C}^{\,0}$

$M_{BC} = \int V_{BC}\,dx = 60x + C$

at $x = 4$, $M = 0$

$0 = 60(4) + C = 240 + C \therefore C = -240$

$M_{BC} = 60x - 240$

$EI\theta_{AB} = \int M_{AB}\,dx = -(25/3)x^3 + 27.5x^2 + C_1$

$EI\theta_{BC} = \int M_{BC}\,dx = 30x^2 - 240x + C_2$

$EI y_{AB} = \int EI\theta_{AB}\,dx = -(25/12)x^4 + (55/6)x^3 + C_1 x + \cancel{C_3}^{\,0}$

$EI y_{BC} = \int EI\theta_{BC}\,dx = 10x^3 - 120x^2 + C_2 x + C_4$

BOUNDARY CONDITIONS

① @ $x = 0$, $EI y_{AB} = 0$ ③ at $x = 3$, $EI y_{BC} = 0$

② @ $x = 3$, $EI y_{AB} = 0$ ④ at $x = 3$, $EI\theta_{AB} = EI\theta_{BC}$

SUBSTITUTING:

① $\boxed{C_3 = 0}$

③ $0 = 10(3)^3 - 120(3)^2 + 3C_2 + C_4$

$3C_2 + C_4 = 810$

② $0 = -(25/12)(3)^4 + (55/6)(3)^3 + 3C_1$

$C_1 = -78.75/3 = \boxed{-26.25 = C_1}$

④ $-(25/3)(3)^3 + 27.5(3)^2 - 26.25 = 30(3)^2 - 240(3) + C_2$

$\boxed{C_2 = 446.25}$

FROM ③: $C_4 = 810 - 3C_2 = 810 - 3(446.25) = \boxed{-528.75 = C_4}$

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12-34 (CONTINUED)

FINAL EQUATIONS

$$\boxed{\begin{aligned} EI\theta_{AB} &= -(25/3)x^3 + 27.5x^2 - 26.25 \\ EI\theta_{BC} &= 30x^2 - 240x + 446.25 \\ EIy_{AB} &= -(25/12)x^4 + 55/6\,x^3 - 26.25x \\ EIy_{BC} &= 10x^3 - 120x^2 + 446.25 - 528.75 \end{aligned}}$$

SHAPE OF DEFLECTION CURVE

y_{MAX} OCCURS WHERE $EI\theta = 0$ OR AT RIGHT END OF OVERHANG.

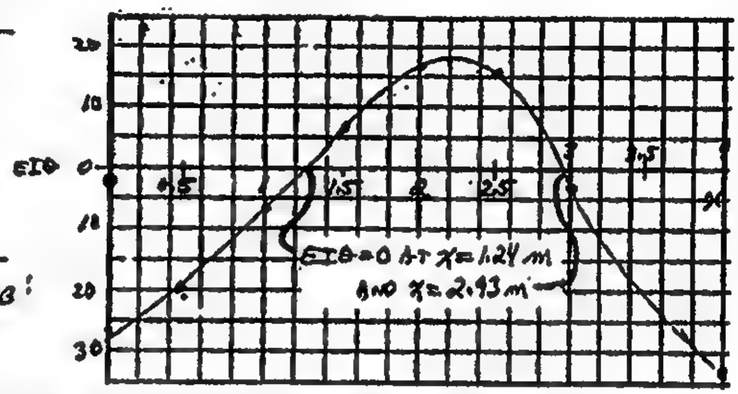
EVALUATE $EI\theta_{AB}$ AT SEVERAL POINTS + USE SOLVER.

x =	$EI\theta$ =
0	−26.25
0.5	−20.417
1.0	−7.083
1.5	7.50
2.0	17.083
2.5	15.417
3.0	−3.75

ROOTS FOR $EI\theta_{AB}$:

$x_1 = 1.2351$ m

$x_2 = 2.9341$ m



EVALUATE EIy AT $x = 1.24$ m, $x = 2.93$ m, AND $x = 4.0$ m.

$$[EIy_{AB}]_{x=1.24m} = -2.083(1.24)^4 + 9.167(1.24)^3 - 26.26(1.24) = -20.0\ N\cdot m^3$$

$$[EIy_{AB}]_{x=2.93m} = -2.083(2.93)^4 + 9.167(2.93)^3 - 26.26(2.93) = +0.121\ N\cdot m^3$$

$$[EIy_{BC}]_{x=4} = 10(4)^3 - 120(4)^2 + 446.25 - 528.7 = -23.75\ N\cdot m^3\ \underline{MAX}$$

$$\text{REQ'D } I = \frac{-23.75\ N\cdot m^3}{E\,y_{MAX}} = \frac{-23.75\ N\cdot m^3}{(207\times10^9\ N/m^2)(-1\ mm)} \times \frac{(10^3\ mm)^5}{m^5} = \underline{1.15\times10^5\ mm^4}$$

$$\text{CONVERT } I = 1.15\times10^5\ mm^4 \times 2.403\times10^{-6}\ in^4/mm^4 = \underline{0.276\ in^4}$$

POSSIBLE BEAM DESIGNS:

1. $1\frac{1}{2}$ IN. SCH 40 PIPE : $I = 0.3099\ IN^4$
2. C3×6 CHANNEL : $I_y = 0.305\ IN^4$ 
3. 2×2×1/4 HOLLOW STEEL TUBE: $I = 0.366\ IN^4$ 

CHECK: $\sigma = M/S$ − OK FOR ALL DESIGNS.

12-35

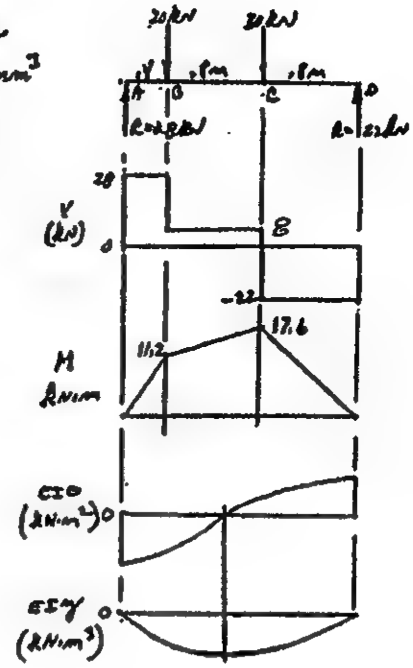
SELECT ALUMINUM I-BEAM ; $\sigma_d = 120\,MPa$

$$S = \frac{M}{\sigma_d} = \frac{17.6 \times 10^3\,N\cdot m}{120\,N/mm^2} \times \frac{10^3\,mm}{1\,m} = 1.467 \times 10^5\,mm^3$$

$$S = 1.467 \times 10^5\,mm^3 \times 6.102 \times 10^{-5}\,\frac{in^3}{\text{[redacted]}^3} = 8.95\,in^3$$

USE I 7 × 5.800 ALUM I-BEAM

$S = 12.25\,in^3$; $I = 42.89\,in^4$



MOMENT –

$M_{AB} = 28X$

$M_{BC} = 8X + C$ BUT AT $X = .4$, $M = 11.2$

$11.2 = 8(.4) + C$

$C = 11.2 - 3.2 = 8.0$

$M_{BC} = 8X + 8$

$M_{CD} = -22X + C$ BUT AT $X = 2.0$, $M = 0$

$0 = -22(2.0) + C$

$C = 44$

$M_{CD} = -22X + 44$

SLOPE –

$EI\theta_{AB} = \int M_{AB}\,dx = 14X^2 + C_1$

$EI\theta_{BC} = \int M_{BC}\,dx = 4X^2 + 8X + C_2$

$EI\theta_{CD} = \int M_{CD}\,dx = -11X^2 + 44X + C_3$

DEFLECTION –

$EIy_{AB} = \int EI\theta_{AB}\,dx = (14/3)X^3 + C_1X + C_4$

$EIy_{BC} = \int EI\theta_{BC}\,dx = (4/3)X^3 + 4X^2 + C_2X + C_5$

$EIy_{CD} = \int EI\theta_{CD}\,dx = -(11/3)X^3 + 22X^2 + C_3X + C_6$

BOUNDARY CONDITIONS

① at $X = 0$, $EIy_{AB} = 0$ ③ at $X = 0.4$, $EI\theta_{AB} = EI\theta_{BC}$ ⑤ at $X = 1.2$, $EI\theta_{BC} = EI\theta_{CD}$

② at $X = 2.0$, $EIy_{CD} = 0$ ④ at $X = 0.4$, $EIy_{AB} = EIy_{BC}$ ⑥ at $X = 1.2$, $EIy_{BC} = EIy_{CD}$

SUBSTITUTING

① $0 = C_4$

② $EIy_{CD} = 0 = -(11/3)(2)^3 + 22(2)^2 + 2C_3 + C_6$

$2C_3 + C_6 = -58.66$

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12-35 CONTINUED

③ $14(.4)^2 + C_1 = 4(0.4)^2 + 8(0.4) + C_2$

$C_1 - C_2 = 1.60$

④ $(14/3)(0.4)^3 + 0.4C_1 + \cancel{C_4}^{0} = (4/3)(0.4)^3 + 4(0.4)^2 + 0.4C_2 + C_5$

$0.4C_1 - 0.4C_2 - C_5 = 0.4266$

⑤ $4(1.2)^2 + 1.2(8) + C_2 = -11(1.2)^2 + 44(1.2) + C_3$

$C_2 - C_3 = 21.6$

⑥ $(4/3)(1.2)^3 + 4(1.2)^2 + 1.2C_2 + C_5 = -(11/3)(1.2)^3 + 22(1.2)^2 + 1.2C_3 + C_6$

$1.2C_2 + C_5 - 1.2C_3 - C_6 = 17.28$

SOLVE SIMULTANEOUSLY

$C_1 = -10.56$	$C_4 = 0$
$C_2 = -12.16$	$C_5 = 0.21\overline{33}$
$C_3 = -33.76$	$C_6 = 8.85\overline{33}$

FINAL EQUATIONS

$EI\theta_{AB} = 14x^2 - 10.56$	$EIy_{AB} = 4.667x^3 - 10.56x$
$EI\theta_{BC} = 4x^2 + 8x - 12.16$	$EIy_{BC} = (4/3)x^3 + 4x^2 - 12.16x + 0.21\overline{33}$
$EI\theta_{CD} = -11x^2 + 44x - 33.76$	$EIy_{CD} = -(11/3)x^3 + 22x^2 - 33.76x + 8.85\overline{3}$

SET $EI\theta_{BC} = 0 = 4x^2 + 8x - 12.16 = x^2 + 2x - 3.04$

$x = \frac{-2}{2} \pm \frac{1}{2}\sqrt{4 - 4(-3.04)} = -1 \pm 2.01 = +1.01\,m$ — VALID POINT

THEN MAX y OCCURS AT $x = 1.01\,m$

$\left[EIy_{BC}\right]_{x=1.01} = (4/3)(1.01)^3 + 4(1.01)^2 - 12.16(1.01) + 0.213 = -6.615\,kN \cdot m^3$

$y_{max} = \frac{-6.615\,kN \cdot m^3}{EI} = \frac{-6.615 \times 10^3\,N\,m^3}{(69 \times 10^9\,N/m^2)(1.785 \times 10^7\,mm^4)} \times \frac{(10^3\,mm)^5}{m^5} = \underline{-5.37\,mm}$

$I = 42.89\,in^4 \times 4.162 \times 10^5\,mm^4/in^4 = 1.785 \times 10^7\,mm^4$

12-36

$M_{AB} = -20x$

$M_{BC} = 20x - 160$

$M_{CD} = -10x + 80$

$M_{DE} = 20x - 280$

$EI\theta_{AB} = -10x^2 + C_1$

$EI\theta_{BC} = 10x^2 - 160x + C_2$

$EI\theta_{CD} = -5x^2 + 80x + C_3$

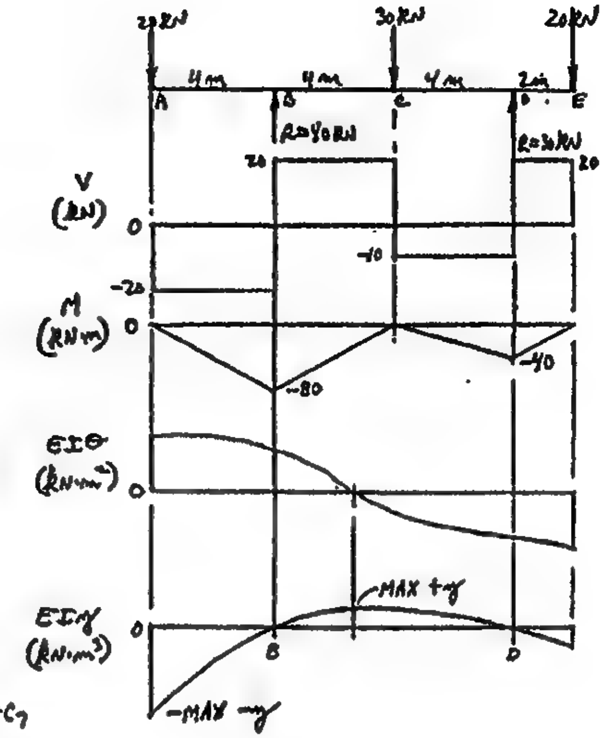
$EI\theta_{DE} = 10x^2 - 280x + C_4$

$EIy_{AB} = -(10/3)x^3 + C_1x + C_5$

$EIy_{BC} = (10/3)x^3 - 80x^2 + C_2x + C_6$

$EIy_{CD} = -(5/3)x^3 + 40x^2 + C_3x + C_7$

$EIy_{DE} = (10/3)x^3 - 140x^2 + C_4x + C_8$



BOUNDARY AND CONTINUITY CONDITIONS

① at $x = 4$; $EIy_{AB} = 0$	⑤ at $x = 4$, $EI\theta_{AB} = EI\theta_{BC}$
② at $x = 4$, $EIy_{BC} = 0$	⑥ at $x = 8$, $EI\theta_{BC} = EI\theta_{CD}$
③ at $x = 12$, $EIy_{CD} = 0$	⑦ at $x = 8$, $EIy_{BC} = EIy_{CD}$
④ at $x = 12$, $EIy_{DE} = 0$	⑧ at $x = 12$, $EI\theta_{CD} = EI\theta_{DE}$

CONSTANTS OF INTEGRATION FROM SIMULTANEOUS SOLUTION OF ① THRU ⑧

$C_1 = 306.\overline{66}$ $\quad C_5 = -1013.\overline{33}$

$C_2 = 626.\overline{66}$ $\quad C_6 = -1440$

$C_3 = -333.\overline{33}$ $\quad C_7 = 1120$

$C_4 = 1826.\overline{66}$ $\quad C_8 = -7520$

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12-36 (CONTINUED)

FINAL EQUATIONS

$EI\theta_{AB} = -10x^2 + 306.\overline{66}$	$EIy_{AB} = -(10/3)x^3 + 30\overline{6}x - 1013.\overline{33}$
$EI\theta_{BC} = 10x^2 - 160x + 626.\overline{66}$	$EIy_{BC} = (10/3)x^3 - 80x^2 + 62\overline{6}x - 1440$
$EI\theta_{CD} = -5x^2 + 80x - 333.\overline{33}$	$EIy_{CD} = -(5/3)x^3 + 40x^2 - 33\overline{3}x + 1120$
$EI\theta_{DE} = 10x^2 - 280x + 1826.\overline{66}$	$EIy_{DE} = (10/3)x^3 - 140x^2 + 182\overline{6}x - 7520$

SET $EI\theta_{BC} = 0 = 10x^2 - 160x + 626 = x^2 - 16x + 62.6$

$$x = \frac{16 \pm \sqrt{16^2 - 4(62.6)}}{2} = 8 \pm \tfrac{1}{2}(2.37) = 9.16 \text{ OR } \boxed{6.845}$$

INVALID - NOT IN SEGMENT BC

MAX y AT $x = 0$ OR $x = 6.84$ m OR $x = 14$ m

at $x = 0$; $EIy_{AB} = -101\overline{3}$ kN·m³ MAX NEG. y

at $x = 6.84$ m; $EIy_{BC} = +170$ kN·m³ MAX POS. y

at $x = 14$; $EIy_{DE} = -240$ kN·m³

$E = 207 \times 10^9$ N/m²

$I = 245$ in⁴ × 4.162×10^5 mm⁴/in⁴ = 1.020×10^8 mm⁴

$$\text{at } x=0;\ y = \frac{-1013 \times 10^3\ \text{N·m}^3\ (10^{15}\ \text{mm}^5/\text{m}^5)}{(207 \times 10^9\ \text{N/m}^2)(1.020 \times 10^8\ \text{mm}^4)} = \underline{-48.0\ \text{mm}}$$

$$\text{at } x = 6.82\ \text{m};\ y = -48.0 \times \frac{170}{-1013} = \underline{+8.07\ \text{mm}}$$

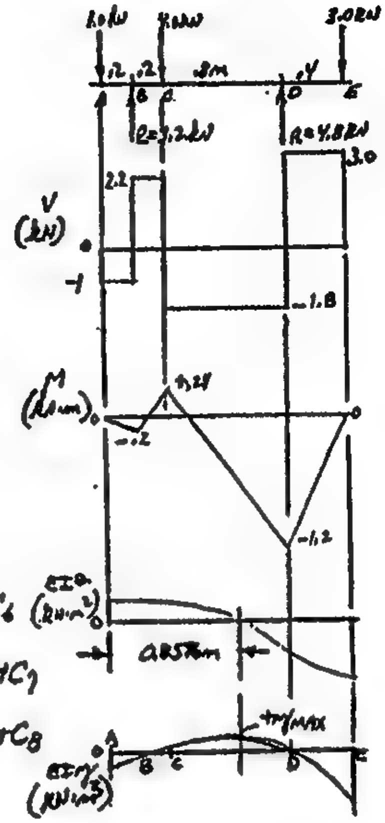
$$\text{at } x = 14\ \text{m};\ y = -48.0\ \text{mm} \times \frac{-240}{-1013} = \underline{-11.37\ \text{mm}}$$

CHECK: $\sigma = \frac{M}{S}$

$$M = 80\ \text{kN·m} \times \frac{10^3\ \text{N}}{\text{kN}} \times \frac{8.851\ \text{LB·in}}{\text{N·m}} = 7.08 \times 10^5\ \text{LB·in}$$

$$\sigma = \frac{7.08 \times 10^5\ \text{LB·in}}{35.3\ \text{in}^3} = 20{,}057\ \text{PSI} \cdot \text{OK}$$

12-37



MOMENT: $M_{AB} = -X$

$M_{BC} = 2.2X - 0.64$

$M_{CD} = -1.8X + 0.96$

$M_{DE} = 3.0X - 4.8$

$EI\theta_{AB} = \int M_{AB}\,dx = -0.5X^2 + C_1$

$EI\theta_{BC} = \int M_{BC}\,dx = 1.1X^2 - 0.64X + C_2$

$EI\theta_{CD} = \int M_{CD}\,dx = -0.9X^2 + 0.96X + C_3$

$EI\theta_{DE} = \int M_{DE}\,dx = 1.5X^2 - 4.8X + C_4$

$EIy_{AB} = \int EI\theta_{AB}\,dx = -0.1\bar{6}X^3 + C_1X + C_5$

$EIy_{BC} = \int EI\theta_{BC}\,dx = 0.36\bar{6}X^3 - 0.32X^2 + C_2X + C_6$

$EIy_{CD} = \int EI\theta_{CD}\,dx = -0.3X^3 + 0.48X^2 + C_3X + C_7$

$EIy_{DE} = \int EI\theta_{DE}\,dx = 0.5X^3 - 2.4X^2 + C_4X + C_8$

BOUNDARY CONDITIONS

1. at $X = 0.2$, $EIy_{AB} = 0$
2. at $X = 0.2$, $EIy_{BC} = 0$
3. at $X = 1.2$, $EIy_{CD} = 0$
4. at $X = 1.2$, $EIy_{DE} = 0$
5. at $X = 0.2$, $EI\theta_{AB} = EI\theta_{BC}$
6. at $X = 0.4$, $EI\theta_{BC} = EI\theta_{CD}$
7. at $X = 1.2$, $EI\theta_{CD} = EI\theta_{DE}$
8. at $X = 0.4$, $EIy_{BC} = EIy_{CD}$

CONSTANTS - FROM SIMULTANEOUS SOLUTION OF 1 THROUGH 8

$C_1 = 0.490\bar{6}$	$C_5 = -0.0968$
$C_2 = 0.158\bar{6}$	$C_6 = -0.02186\bar{6}$
$C_3 = -0.161\bar{3}$	$C_7 = 0.0208$
$C_4 = 3.294\bar{6}$	$C_8 = -1.316$

(CONTINUED NEXT PAGE)

12-31 CONTINUED

FINAL EQUATIONS

$EI\theta_{AB} = -0.5x^2 + .409\bar{6}$	$EIy_{AB} = -0.1\bar{6}x^3 + .490\bar{6}x - 0.0968$
$EI\theta_{BC} = 1.1x^2 - 0.64x + 0.158\bar{6}$	$EIy_{BC} = 0.36\bar{6}x^3 - 0.32x^2 + 0.158\bar{6}x - 0.02$
$EI\theta_{CD} = -0.9x^2 + 0.96x - 0.161\bar{3}$	$EIy_{CD} = -0.3x^3 + 0.48x^2 - 0.161x + 0.0208$
$EI\theta_{DE} = 1.5x^2 - 4.8x + 3.294\bar{6}$	$EIy_{DE} = 0.5x^3 - 2.4x^2 + 3.294\bar{6}x - 1.3616$

SET $EI\theta_{CD} = 0 = -0.9x^2 + 0.96x - 0.161\bar{3} = x^2 - 1.06\bar{6}x + 0.1792$

$$x = \frac{1.06 \pm \sqrt{1.06\bar{6}^2 - (4)(0.1792)}}{2} = 0.5335 \pm 0.325 = \underline{0.8577\ m} \text{ or } 0.2085\ m \text{ OUTSIDE CD}$$

DEFLECTION AT $x = 0.8577\ m$ IN SEGMENT CD

$$EIy_{CD} = -0.3(0.8586)^3 + 0.48(0.8586)^2 - 0.161(0.8586) + 0.0208 = 0.0462\ kN \cdot m^3$$

AT $x = 0$

$$EIy_{AB} = -0.1\bar{6}(0) + 0.4906(0) - 0.0968 = -0.0968\ kN \cdot m^3$$

AT $x = 1.6\ m$

$$EIy_{DE} = 0.5(1.6)^3 - 2.4(1.6)^2 + 3.295(1.6) - 1.362 = -0.1862\ kN \cdot m^3 \text{ MAXIMUM}$$

FOR $y_{MAX} = -0.13\ mm$; $E = 207 \times 10^3\ N/mm^2$

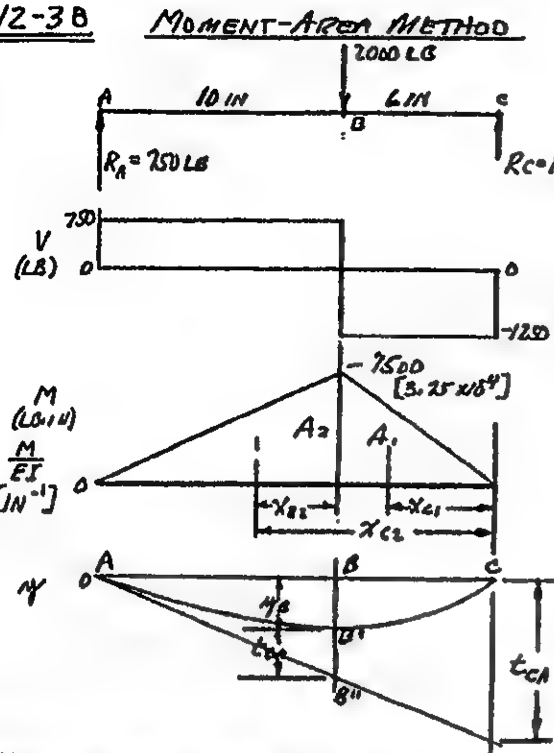
$$I = \frac{EIy}{Ey} = \frac{-0.186 \times 10^3\ N \cdot m^3}{(207 \times 10^3\ N/mm^2)(-0.13\ mm)} \frac{(10^3\ mm)^3}{m^3} = 6.91 \times 10^6\ mm^4$$

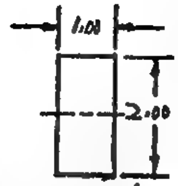
$$I = \pi D^4/64$$

$$D = [64I/\pi]^{1/4} = \left[\frac{64(6.91 \times 10^6)}{\pi}\right]^{1/4} = \underline{109\ mm}$$

CHECK: $\sigma = \frac{Mc}{I} = \frac{(1.2\ kN \cdot m)(109/2\ mm)}{6.91 \times 10^6\ mm^4} \frac{(10^3\ N)(10^3\ mm)}{kN \cdot m} = 9.46\ MPa$ - $\underline{OK}$

12-38 MOMENT-AREA METHOD





$I = \frac{bh^3}{12} = \frac{1(2)^3}{12}$

$I = 0.666\ IN^4$

$EI = 30\times10^6 \frac{LB}{IN^2} \times 0.666\ IN^4$

$EI = 2.0\times10^7\ LB\cdot IN^2$

AT B:

$\frac{M}{EI} = \frac{7500\ LB\cdot IN}{2.0\times10^7\ LB\cdot IN^2} = 3.75\times10^{-4}\ IN^{-1}$

$t_{CA} = A_1 x_{C1} + A_2 x_{C2}$

$t_{CA} = (\frac{1}{2})(3.75\times10^{-4})(6)(4) + (\frac{1}{2})(3.75\times10^{-4})(10)(9.333)$

$t_{CA} = 0.022\ IN$

$BB'' = t_{CA}\cdot\frac{AB}{AC} = 0.022\cdot\frac{10}{16}$

$BB'' = 0.01375\ IN$

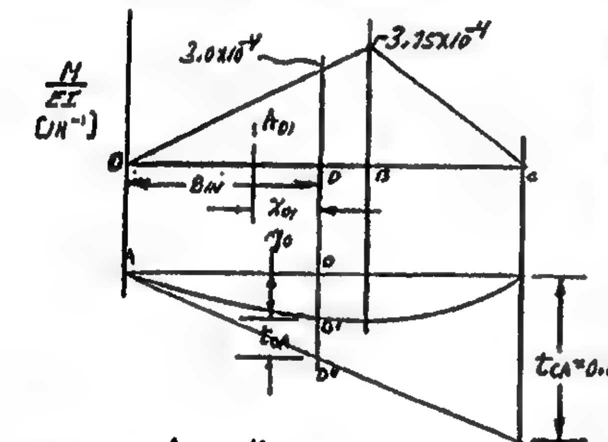
$t_{BA} = A_2 x_{B2}$

$= (.5)(3.75\times10^{-4})(10)(3.333)$

$t_{BA} = 0.00625\ IN$

$y_B = BB'' - t_{BA} = \underline{0.0075\ IN}$

12-39 SAME M/EI AS 12-38



$DD'' = t_{CA}\cdot\frac{AD}{AC} = 0.022\cdot\frac{8}{16} = 0.011\ IN$

$t_{DA} = A_{D1}\cdot x_{D1}$

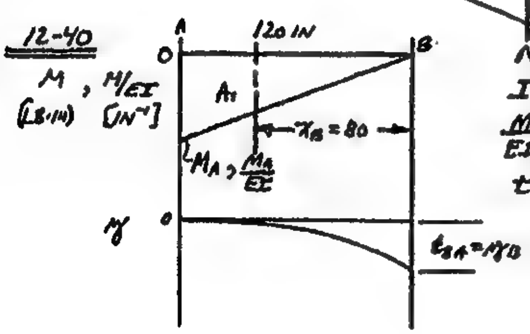
$= (0.5)(3.0\times10^{-4})(8)(2.667)$

$t_{DA} = 0.0032\ IN$

$y_D = DD'' - t_{DA} = 0.011 - 0.0032$

$\underline{y_D = 0.0078\ IN.}$

12-40



$M_A = 2.4\times10^{+6}\ LB\cdot IN$

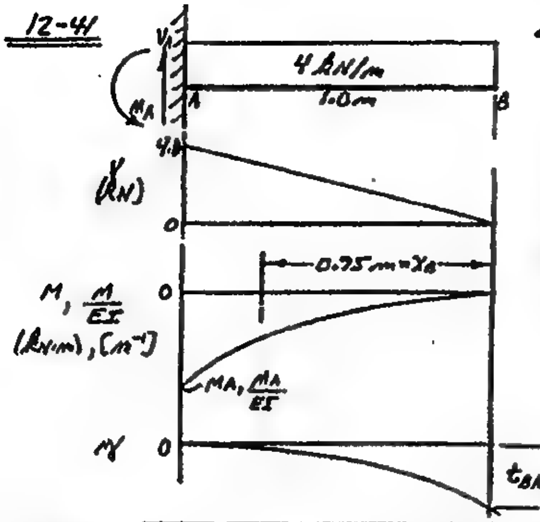
$I = 890\ IN^4$ (W18×55)

$\frac{M_A}{EI} = \frac{2.4\times10^{+6}\ LB\cdot IN}{(30\times10^6\ LB/IN^2)(890\ IN^4)} = 89.9\times10^{-6}\ IN^{-1}$

$t_{BA} = A_1 x_B = (0.5)(89.9\times10^{-6})(120)(80)$

$\underline{y_B = 0.432\ IN}$

12-41



2 ½ IN SCH 40 PIPE:

$I = 1.530\ in^4 \cdot \frac{(25.4\ mm)^4}{(1\ in)^4} \cdot \frac{1\ m^4}{(10^3\ mm)^4}$

$I = 6.37 \times 10^{-7}\ m^4$

$M_A = 4.0\ kN \cdot 0.5\ m = 2.0\ kN \cdot m$

$\frac{M_A}{EI} = \frac{2.0 \times 10^3\ N \cdot m}{(207 \times 10^9\ N/m^2)(6.37 \times 10^{-7}\ m^4)}$

$\frac{M_A}{EI} = 0.0152\ m^{-1}$

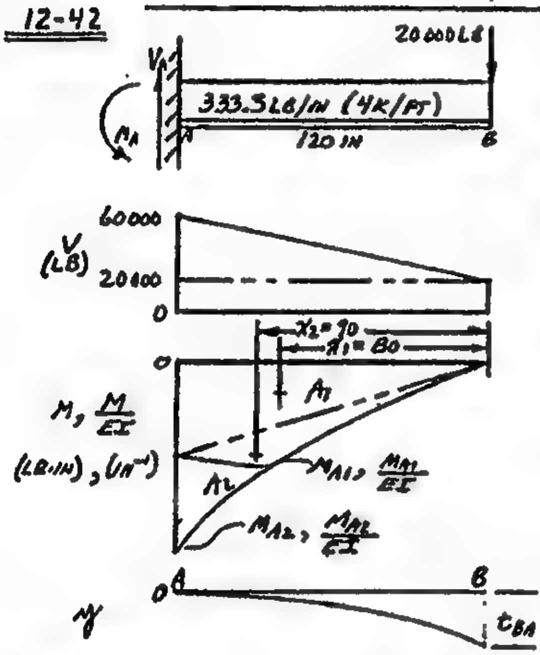
$t_{BA} = y_B = A_1 \cdot x_B$

$= (\tfrac{1}{3})(0.0152)(1.0)(0.75) =$

$t_{BA} = 0.00379\ m = 3.79 \times 10^{-3}\ m$

$\underline{y_B = 3.79\ mm}$

12-42



W24×76: $I = 2100\ in^4$

$M_{A1} = (20000\ lb)(120\ in) = 2.4 \times 10^6\ lb \cdot in$

$M_{A2} = M_{A1} + (\tfrac{1}{2})(40000\ lb)(120\ in)$

$M_{A2} = 2.4 \times 10^6 + 2.4 \times 10^6 = 4.8 \times 10^6\ lb \cdot in$

$\frac{M_{A1}}{EI} = \frac{2.4 \times 10^6\ lb \cdot in}{(30 \times 10^6\ lb/in^2)(2100\ in^4)} = 3.810 \times 10^{-5}\ in^{-1}$

$\frac{M_{A2}}{EI} = 2 \cdot \frac{M_{A1}}{EI} = 7.62 \times 10^{-5}/in$

$t_{BA} = A_1 x_1 + A_2 x_2$

$= \tfrac{1}{2}(3.810 \times 10^{-5})(120)(80) +$

$\tfrac{1}{3}(3.810 \times 10^{-5})(120)(90)$

$t_{BA} = 0.1829 + 0.1371 = \underline{0.320\ in = y_B}$

12-43 DIAGRAMS SAME FORM AS 12-42: DIA. OF BAR = 100 mm = 0.10 m

$I = \pi(0.1)^4/64 = 4.909 \times 10^{-6}\ m^4$: $EI = (69 \times 10^9\ N/m^2)(4.909 \times 10^{-6}\ m^4) = 3.39 \times 10^5\ N \cdot m^2$

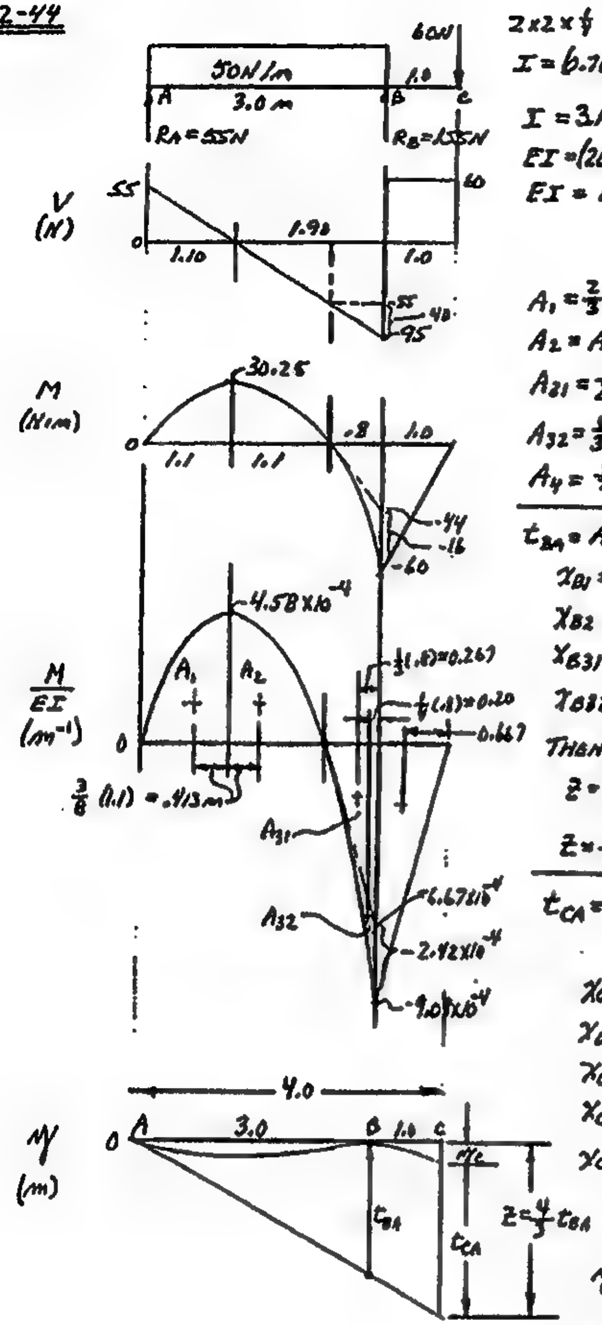
$M_{A1} = (2000\ N)(1.0\ m) = 2000\ N \cdot m$: $\frac{M_{A1}}{EI} = \frac{2000\ N \cdot m}{3.39 \times 10^5\ N \cdot m^2} = 5.91 \times 10^{-3}\ m^{-1}$

$M_{A2} = \tfrac{1}{2}(4000\ N)(1.0\ m) + M_{A1} = 4000\ N \cdot m$: $\frac{M_{A2}}{EI} = 1.18 \times 10^{-2}\ m^{-1}$

$t_{BA} = A_1 x_1 + A_2 x_2 = \tfrac{1}{2}(5.91 \times 10^{-3})(1.0)(0.667) + \tfrac{1}{3}(5.91 \times 10^{-3})(1.0)(0.75)$

$t_{BA} = 1.968 \times 10^{-3} + 1.476 \times 10^{-3} = 3.445 \times 10^{-3}\ m = \underline{3.445\ mm = y_B}$

12-44



2×2×$\frac{1}{8}$ SQ TUBE: $I = 0.766\ in^4$

$$I = (0.766\ in^4)\frac{(25.4\ mm)^4}{1\ in^4}\cdot\frac{1\ m^4}{(10^3\ mm)^4}$$

$$I = 3.19 \times 10^{-7}\ m^4$$

$$EI = (207 \times 10^9\ N/m^2)(3.19 \times 10^{-7}\ m^4) =$$

$$EI = 66.0 \times 10^3\ N\cdot m^2$$

$$A_1 = \frac{2}{3}(1.1)(4.58 \times 10^{-4}) = 3.36 \times 10^{-4}$$

$$A_2 = A_1 = 3.36 \times 10^{-4}$$

$$A_{21} = \frac{1}{2}(0.8)(6.67 \times 10^{-4}) = 2.67 \times 10^{-4}$$

$$A_{32} = \frac{1}{3}(0.8)(2.42 \times 10^{-4}) = 0.646 \times 10^{-4}$$

$$A_4 = \frac{1}{2}(1.0)(9.09 \times 10^{-4}) = 4.55 \times 10^{-4}$$

$$t_{BA} = A_1 x_{B1} + A_2 x_{B2} + A_{31} x_{B31} + A_{32} x_{B32}$$

$$x_{B1} = 1.9 + 0.413 = 2.313\ m$$

$$x_{B2} = 1.9 - 0.413 = 1.487\ m$$

$$x_{B31} = 0.267\ m$$

$$x_{B32} = 0.20\ m$$

THEN $\underline{t_{BA} = 1.193 \times 10^{-3}\ m}$

$$z = t_{BA}\cdot\frac{AC}{AB} = t_{BA}\cdot\frac{4.0}{3.0}$$

$$z = \frac{4}{3}(1.193 \times 10^{-3}) = 1.591 \times 10^{-3}\ m$$

$$t_{CA} = A_1 x_{C1} + A_2 x_{C2} + A_{31} x_{C31} + A_{32} x_{C32} + A_4 x_{C4}$$

$$x_{C1} = 2.9 + 0.413 = 3.313\ m$$

$$x_{C2} = 2.9 - 0.413 = 2.487\ m$$

$$x_{C31} = 1.0 + 0.267 = 1.267\ m$$

$$x_{C32} = 1.0 + 0.20 = 1.200\ m$$

$$x_{C4} = 0.667\ m$$

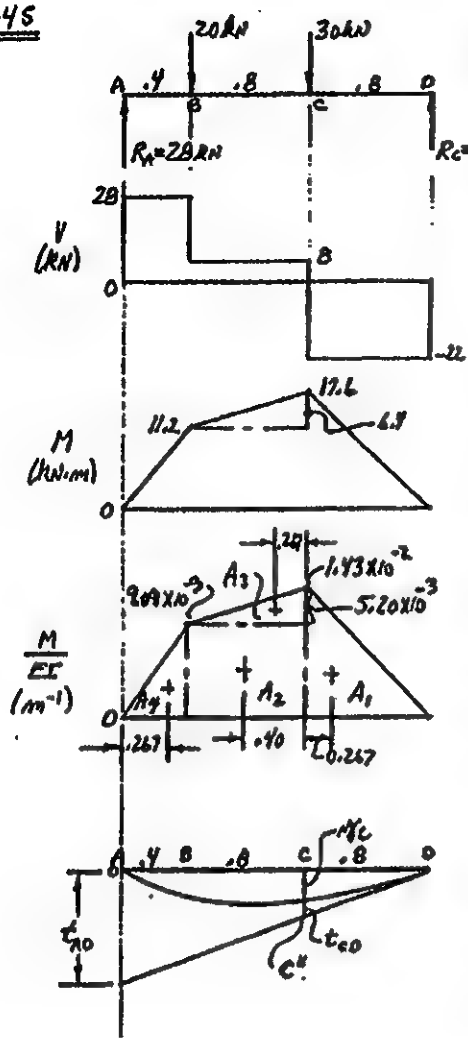
THEN $\underline{t_{CA} = 1.231 \times 10^{-3}\ m}$

$$y_C = z - t_{CA}$$

$$= 1.591 \times 10^{-3} - 1.231 \times 10^{-3}$$

$$\underline{y_C = 0.360 \times 10^{-3}\ m = 0.360\ mm}$$

12-45



ALUM $I\ 7\times5.800$: $I = 42.89\ \text{in}^4$

$$I = (42.89\ \text{in}^4)\frac{(25.4\ \text{mm})^4}{\text{in}^4}\,\frac{1\ \text{m}^4}{(10^3\ \text{mm})^4}$$

$$I = 1.785\times10^{-5}\ \text{m}^4$$

$$EI = (69\times10^9)(1.785\times10^{-5}) = 1.23\times10^6\ \text{N·m}^2$$

$$\frac{M_B}{EI} = \frac{11200\ \text{N·m}}{1.23\times10^6\ \text{N·m}^2} = 9.09\times10^{-3}\ \text{m}^{-1}$$

SIMILARLY,

$$\frac{M_C}{EI} = 1.43\times10^{-2}\ \text{m}^{-1}$$

$$A_1 = \tfrac{1}{2}(0.8)(1.43\times10^{-2}) = 5.72\times10^{-3}$$

$$A_2 = (0.8)(9.09\times10^{-3}) = 7.27\times10^{-3}$$

$$A_3 = \tfrac{1}{2}(0.8)(5.20\times10^{-3}) = 2.08\times10^{-3}$$

$$A_4 = \tfrac{1}{2}(0.4)(9.09\times10^{-3}) = 1.82\times10^{-3}$$

$$t_{AD} = A_1x_{A1} + A_2x_{A2} + A_3x_{A3} + A_4x_{A4}$$

$$x_{A1} = 0.4+0.8+0.267 = 1.467\ \text{m}$$

$$x_{A2} = 0.4+0.4 = 0.80\ \text{m}$$

$$x_{A3} = 0.4+0.8-0.267 = 0.933\ \text{m}$$

$$x_{A4} = 0.267\ \text{m}$$

THEN $t_{AD} = 16.63\times10^{-3}\ \text{m}$

$$CC'' = t_{AD}\cdot\frac{CD}{AD} = t_{AD}\cdot\frac{.8}{2.0} = 6.65\times10^{-3}\ \text{m}$$

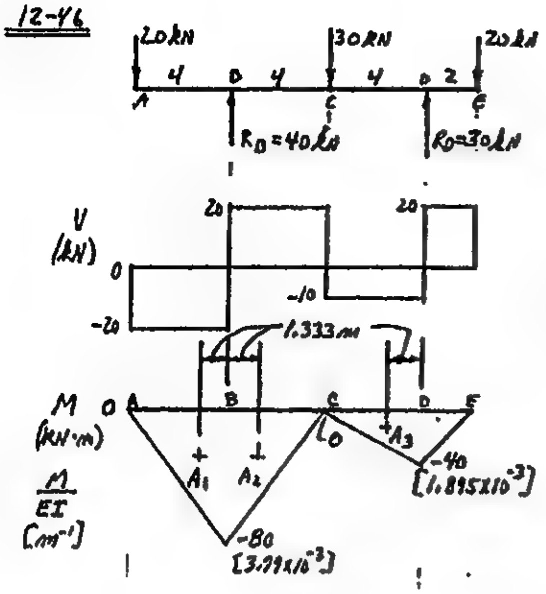
$$t_{CD} = A_1x_{C1} = (5.72\times10^{-3})(0.267\ \text{m})$$

$$t_{CD} = 1.52\times10^{-3}\ \text{m}$$

$$y_C = CC'' - t_{CD} = (6.65-1.52)10^{-3}\ \text{m}$$

$$y_C = 5.13\times10^{-3}\ \text{m} = 5.13\ \text{mm}$$

12-46



W14×26 : $I = 245\,IN^4$

$I = (245\,IN^4)\frac{(25.4\,mm)^4}{IN^4}\frac{1\,m^4}{(10^3\,mm)^4}$

$I = 1.02\times10^{-4}\,m^4$

$EI = (207\times10^9\,N/m^2)(1.02\times10^{-4}\,m^4)$

$EI = 21.1\times10^6\,N\cdot m^2$

$\frac{M_B}{EI} = \frac{80000\,N\cdot m}{21.1\times10^6\,N\cdot m^2} = 3.79\times10^{-3}\,m^{-1}$

$\frac{M_D}{EI} = \frac{1}{2}\frac{M_B}{EI} = 1.895\times10^{-3}\,m^{-1}$

$t_{BD} = A_2 x_{B2} + A_3 x_{B3}$

$A_1 = \frac{1}{2}(3.79\times10^{-3})(4) = 7.58\times10^{-3}$

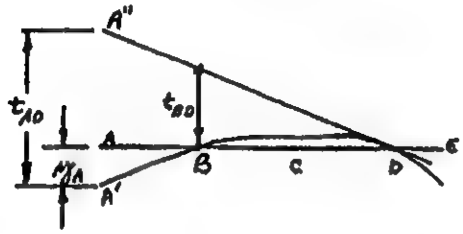
$A_2 = A_1$

$A_3 = \frac{1}{2}(1.895\times10^{-3})(4) = 3.79\times10^{-3}$

$x_{B2} = 1.333\,m$

$x_{B3} = 8.0 - 1.333 = 6.667\,m$

THEN $t_{BD} = 35.4\times10^{-3}\,m$



$AA'' = t_{BD}\cdot\frac{AD}{BD} = t_{BD}\cdot\frac{12}{8}$

$AA'' = 53.1\times10^{-3}\,m$

$t_{AD} = A_1 x_{A1} + A_2 x_{A2} + A_3 x_{A3}$

$x_{A1} = 4.00 - 1.333 = 2.667\,m$

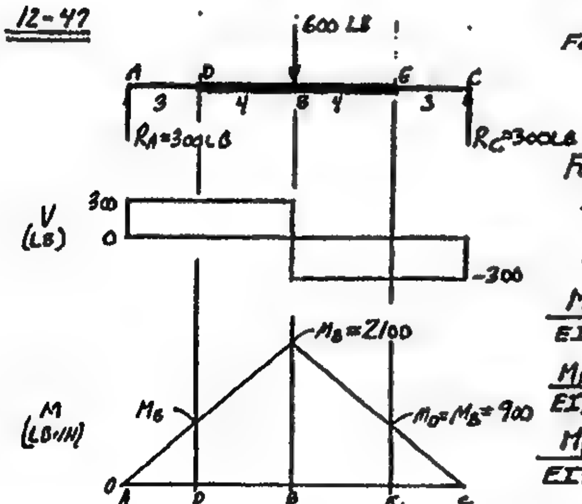
$x_{A2} = 4.00 + 1.333 = 5.333\,m$

$x_{A3} = 12.00 - 1.333 = 10.667\,m$

THEN $t_{AD} = 101.1\times10^{-3}\,m$

$y_A = t_{AD} - AA'' = (101.1 - 53.1)(10^{-3})\,m = 48.0\times10^{-3}\,m = 48.0\,mm = y_A$

12-47



FOR AD AND EC:

$I_1 = \frac{\pi(0.75)^4}{64} = 0.0155\,IN^4$

$EI_1 = (30\times10^6)(I_1) = 4.66\times10^5\,LB\cdot IN^2$

FOR DE:

$I_2 = \frac{\pi(1.40)^4}{64} = 0.1886\,IN^4$

$EI_2 = (30\times10^6)(I_2) = 5.66\times10^6\,LB\cdot IN^2$

$\frac{M_D}{EI_1} = \frac{M_E}{EI_1} = \frac{900\,LB\cdot IN}{4.66\times10^5\,LB\cdot IN^2} = 1.93\times10^{-3}\,IN^{-1}$

$\frac{M_D}{EI_2} = \frac{M_E}{EI_2} = \frac{900}{5.66\times10^6} = 0.159\times10^{-3}\,IN^{-1}$

$\frac{M_B}{EI_2} = \frac{2100}{5.66\times10^6} = 0.371\times10^{-3}\,IN^{-1}$

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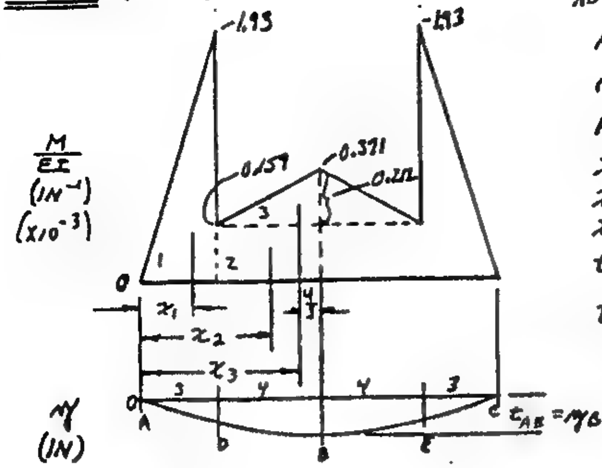
12-47 (CONT.)



$t_{AB} = y_B = A_1 x_1 + A_2 x_2 + A_3 x_3$

$A_1 = \frac{1}{2}(1.93\times10^{-3}\ in^{-1})(3\ in) = 2.90\times10^{-3}$

$A_2 = (0.159\times10^{-3})(4) = 0.636\times10^{-3}$

$A_3 = \frac{1}{2}(0.212\times10^{-3})(4) = 0.424\times10^{-3}$

$x_1 = (2/3)(3) = 2.000\ in$

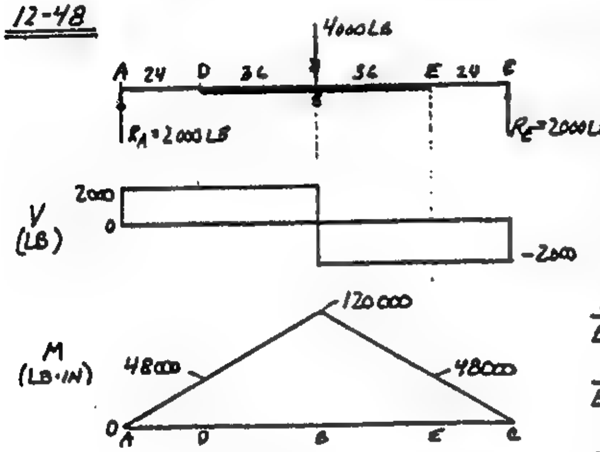
$x_2 = 3 + 2 = 5.000\ in$

$x_3 = 7 - 4/3 = 5.667\ in$

$t_{AB} = 0.00580 + 0.00318 + 0.00240$

$t_{AB} = y_B = 0.01138\ in$

12-48



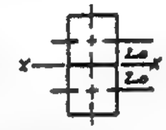
FOR AD AND EC: 4×4×1/4

$I_1 = 8.22\ in^4$

$EI_1 = (30\times10^6)(I_1) = 246.6\times10^6\ lb\cdot in^2$

$I_2 = 2[I_1 + Ad^2]$

$= 2[8.22 + (3.59)(2)^2]$



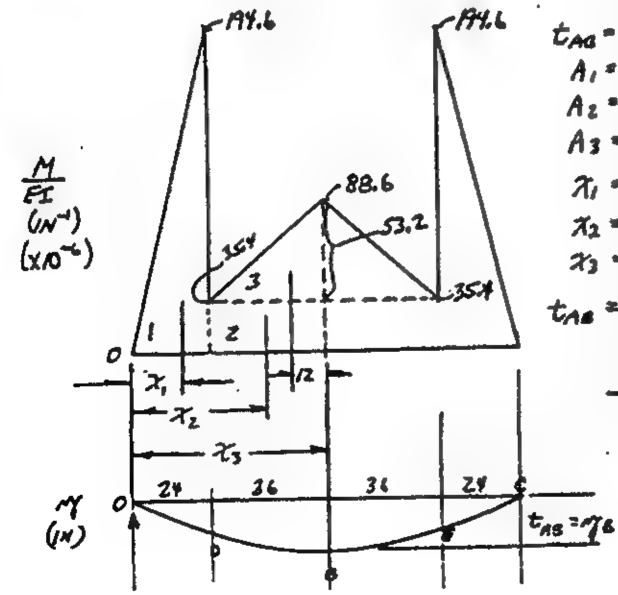
$I_2 = 45.16\ in^4$

$EI_2 = (30\times10^6)(I_2) = 1355\times10^6\ lb\cdot in^2$

$\frac{M_D}{EI_1} = \frac{48000}{246.6\times10^6} = 194.6\times10^{-6}\ in^{-1}$

$\frac{M_D}{EI_2} = \frac{48000}{1355\times10^6} = 35.4\times10^{-6}\ in^{-1}$

$\frac{M_B}{EI_2} = \frac{120000}{1355\times10^6} = 88.6\times10^{-6}\ in^{-1}$



$t_{AB} = y_B = A_1 x_1 + A_2 x_2 + A_3 x_3$

$A_1 = \frac{1}{2}(194.6\times10^{-6})(24) = 2.336\times10^{-3}$

$A_2 = (35.4\times10^{-6})(36) = 1.275\times10^{-3}$

$A_3 = \frac{1}{2}(53.2\times10^{-6})(36) = 0.957\times10^{-3}$

$x_1 = (2/3)(24) = 16.0\ in$

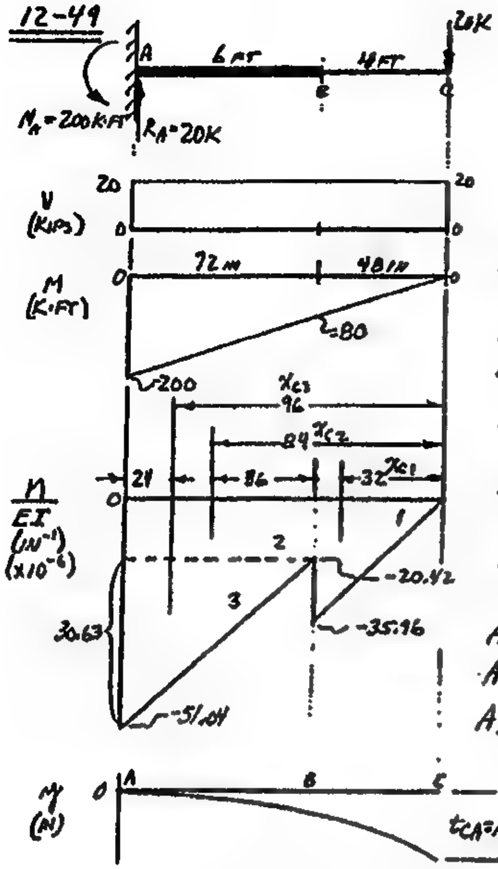
$x_2 = 24 + 18 = 42.0\ in$

$x_3 = 24 + (2/3)(36) = 48.0\ in$

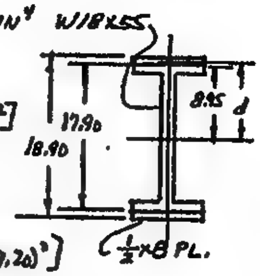
$t_{AB} = y_B = 0.0374 + 0.0536 + 0.0459$

$y_B = 0.1369\ in$

12-49



FOR BC: $I_2 = 890\ IN^4$ W18X55



$d = 8.95 + 0.25$

$d = 9.20\ IN$

$I_1 = I_2 + 2[I_{PL} + Ad^2]$

$I_{PL} = \frac{8(0.50)^3}{12}$

$I_{PL} = 0.0833\ IN^4$

$I_1 = 890 + 2[0.083 + (4.0)(9.20)^2]$

$I_1 = 1567\ IN^4$

$EI_1 = (30 \times 10^6)(I_1) = 4.70 \times 10^{10}\ LB \cdot IN^2$

$EI_2 = (30 \times 10^6)(I_2) = 2.67 \times 10^{10}\ LB \cdot IN^2$

$\frac{M_A}{EI_1} = \frac{(200\,000)(12)\ LB \cdot IN}{4.70 \times 10^{10}\ LB \cdot IN^2} = 51.04 \times 10^{-6}\ IN^{-1}$

$\frac{M_B}{EI_1} = \frac{(80\,000)(12)}{4.71 \times 10^{10}} = 20.42 \times 10^{-6}\ IN^{-1}$

$\frac{M_B}{EI_2} = \frac{(80\,000)(12)}{2.67 \times 10^{10}} = 35.96 \times 10^{-6}\ IN^{-1}$

$A_1 = \frac{1}{2}(35.96 \times 10^{-6})(48) = 0.8629 \times 10^{-3}$

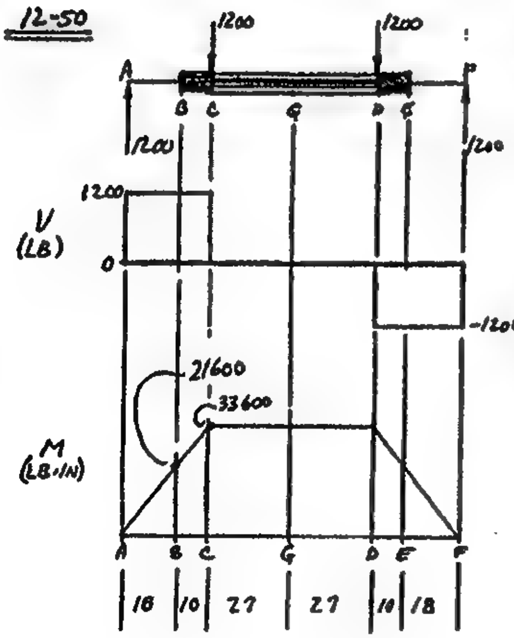
$A_2 = (20.42 \times 10^{-6})(72) = 1.470 \times 10^{-3}$

$A_3 = \frac{1}{2}(30.63 \times 10^{-6})(72) = 1.103 \times 10^{-3}$

$t_{CA} = y_C = A_1 x_{C1} + A_2 x_{C2} + A_3 x_{C3}$

$t_{CA} = y_C \quad y_C = 0.0276 + 0.1235 + 0.1058 = \underline{\underline{0.2569\ IN}}$

12-50



FOR AB AND EF: 3x3x¼; $I_1 = 3.16\ IN^4$

FOR CD: 4x4x½; $I_2 = 12.3\ IN^4$

FOR BC AND DE:

$I_3 = I_1 + I_2 = 15.46\ IN^4$

$EI_1 = (30 \times 10^6)(I_1) = 98.8 \times 10^6\ LB \cdot IN^2$

$EI_2 = 463.8 \times 10^6\ LB \cdot IN^2$

$EI_3 = 369 \times 10^6\ LB \cdot IN^2$

$\frac{M_B}{EI_1} = \frac{21600\ LB \cdot IN}{98.8 \times 10^6\ LB \cdot IN^2} = 227.8 \times 10^{-6}\ IN^{-1}$

$\frac{M_B}{EI_2} = \frac{21600}{463.8 \times 10^6} = 46.57 \times 10^{-6}\ IN^{-1}$

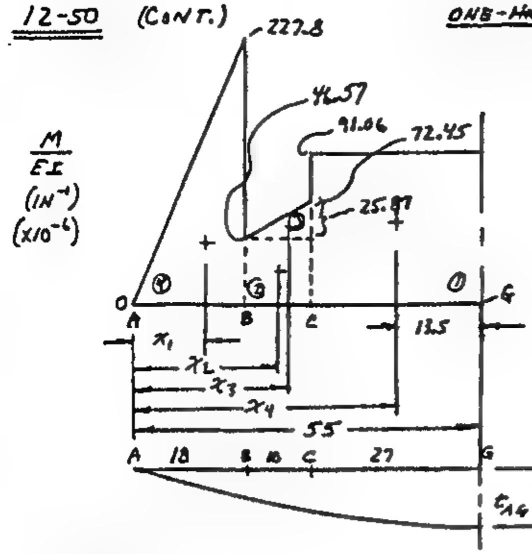
$\frac{M_C}{EI_2} = \frac{33600}{463.8 \times 10^6} = 72.45 \times 10^{-6}\ IN^{-1}$

$\frac{M_C}{EI_3} = \frac{33600}{369 \times 10^6} = 91.06 \times 10^{-6}\ IN^{-1}$

(CONT. NEXT PAGE)

12-50 (CONT.)

ONE-HALF OF DIAGRAMS DRAWN TWICE SIZE



$A_1 = (91.06 \times 10^{-6})(27) = 2.459 \times 10^{-3}$

$A_2 = (46.57 \times 10^{-6})(10) = .4657 \times 10^{-3}$

$A_3 = \frac{1}{2}(25.87 \times 10^{-6})(10) = 0.1294 \times 10^{-3}$

$A_4 = \frac{1}{2}(227.8 \times 10^{-6})(18) = 2.051 \times 10^{-3}$

$t_{AG} = y_G = A_1 x_1 + A_2 x_2 + A_3 x_3 + A_4 x_4$

$x_1 = 55 - 13.5 = 41.5$ IN.

$x_2 = 18 + 5 = 23.0$

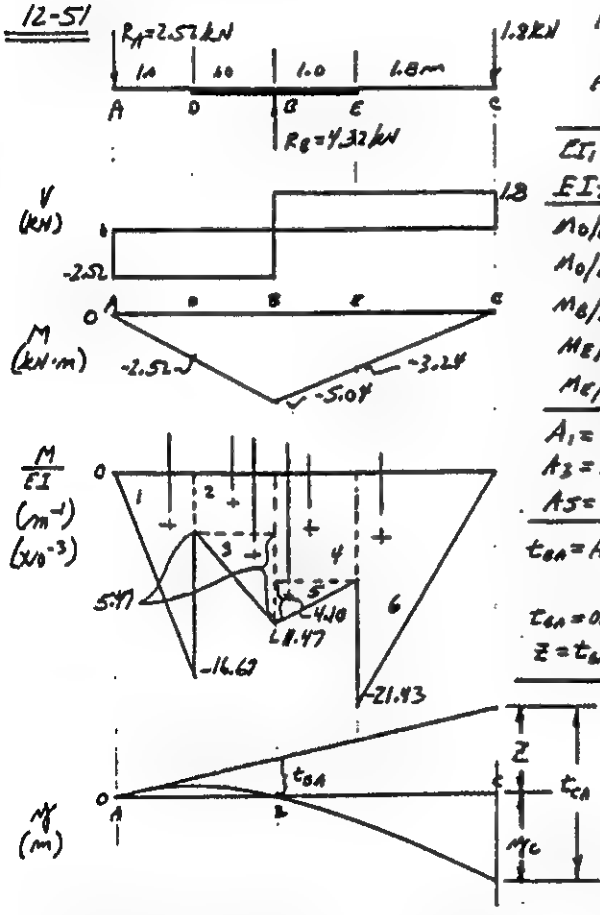
$x_3 = 18 + (2/3)(10) = 24.667$ IN

$x_4 = (2/3)(18) = 12.0$ IN

$t_{AG} = y_G = 0.1020 + 0.0107 + 0.0032 + 0.0246$

$\underline{y_G = 0.1405 \text{ IN}}$

12-51



FOR AD AND EC:

$I_1 = (0.286)(0.089)^3/12 = 16.8 \times 10^{-6} \text{ m}^4$

FOR DB AND BE:

$I_2 = (0.286)(0.127)^3/12 = 48.82 \times 10^{-6} \text{ m}^4$

$EI_1 = (9 \times 10^9 \text{ N/m}^2)(16.8 \times 10^{-6} \text{ m}^4) = 1.51 \times 10^5 \text{ N·m}^2$

$EI_2 = 4.39 \times 10^5 \text{ N·m}^2$

$M_D/EI_1 = 16.67 \times 10^{-3} \text{ m}^{-1}$

$M_D/EI_2 = 5.74 \times 10^{-3} \text{ m}^{-1}$

$M_B/EI_2 = 11.47 \times 10^{-3} \text{ m}^{-1}$

$M_E/EI_2 = 7.37 \times 10^{-3} \text{ m}^{-1}$

$M_E/EI_1 = 21.43 \times 10^{-3} \text{ m}^{-1}$

$A_1 = 8.33 \times 10^{-3}$; $A_2 = 5.74 \times 10^{-3}$

$A_3 = 2.87 \times 10^{-3}$; $A_4 = 7.37 \times 10^{-3}$

$A_5 = 2.05 \times 10^{-3}$; $A_6 = 19.28 \times 10^{-3}$

$t_{BA} = A_1 \underbrace{x_{B1}}_{1.333} + A_2 \underbrace{x_{B2}}_{0.50} + A_3 \underbrace{x_{B3}}_{0.333}$

$t_{BA} = 0.0111 + 0.00287 + 0.00096 = 0.01493$ m

$Z = t_{BA} \cdot \frac{AC}{AB} = 0.01493 \cdot \frac{4.8}{2.0} = 0.03584$ m

$t_{CA} = A_1 \overset{4.133}{x_{C1}} + A_2 \overset{3.30}{x_{C2}} + A_3 \overset{3.133}{x_{C3}}$

$+ A_4 \overset{2.30}{x_{C4}} + A_5 \overset{2.467}{x_{C5}} + A_6 \overset{1.20}{x_{C6}}$

$t_{CA} = 0.0344 + 0.0189 + 0.0090$

$+ 0.0170 + 0.0051 + 0.0231$

$t_{CA} = 0.1075$ m

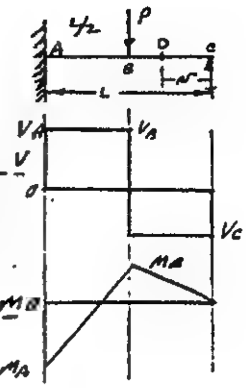
$y_C = t_{CA} - Z = 0.1075 \text{ m} - 0.0358 \text{ m} = 0.0717 \text{ m} = 71.7 \times 10^{-3} \text{ m}$

$\underline{y_C = 71.7 \text{ mm}}$

CHAPTER 13 Statically Indeterminate Beams

13-1

CASE A-24 (a); $P = 35\ kN$; $L = 4.0\ m$



$\underline{R_A} = (11/16)P = (11/16)\ 25000\ N = \underline{24{,}063\ N = V_A}$

$\underline{R_C} = (5/16)P = (5/16)\ 35000\ N = \underline{10938\ N = V_C}$

$\underline{M_A} = -3/16 \cdot PL = -3/16\,(35000\ N)(4.0\ m) = \underline{-26250\ N \cdot m}$

$\underline{M_B} = 5/32 \cdot PL = 5/32\,(35000)(4.0) = \underline{21875\ N \cdot m}$

y_B AT LOAD $= \dfrac{-7\,PL^3}{768\,EI} = \dfrac{-7(35000)(4)^3}{768\,EI} = \dfrac{-20417}{EI}$

AT D: $x = 0.447L = 0.447\,(4.0\ m) = 1.788\ m$

$y_{D\,MAX} = \dfrac{-PL^3}{107\,EI} = \dfrac{(-35000)(4.0)^3}{107\,EI} = \dfrac{20934}{EI}$

LET $y_{MAX} = L/360 = 4.0\ m/360 = 0.0111\ m$

SPECIFY STEEL BEAM $E = 207 \times 10^9\ Pa$

REQ'D. $I = \dfrac{20934\ N \cdot m^3}{E\, y_{MAX}} = \dfrac{20934\ N \cdot m^3}{207 \times 10^9\ N/m^2\,(0.0111\ m)} = 9.102 \times 10^{-6}\ m^4$

$I = 9.102 \times 10^{-6}\ m^4 \times \dfrac{(10^3\ mm)^4}{1\ m^4} \times \dfrac{1\ IN^4}{(25.4\ mm)^4} = 21.87\ IN^4$

SPECIFY W8×10: $I = 30.8\ IN^4$; $S = 7.81\ IN^3 \times \dfrac{(25.4\ mm)^3}{IN^3} = 1.28 \times 10^5\ mm^3$

CHECK STRESS:

$\sigma = \dfrac{M}{S} = \dfrac{26250\ N \cdot m}{1.28 \times 10^5\ mm^3} \times \dfrac{10^3\ mm}{1\ m} = 205\ N/mm^2 = 205\ MPa$

LET $\sigma = 0.66\,S_Y$; REQ'D $S_Y = \sigma/0.66 = \dfrac{205\ MPa}{0.66} = 311\ MPa$

SPECIFY ASTM A242, $S_Y = 345\ MPa$

SUMMARY

REACTIONS: $R_A = 24.06\ kN$, $R_C = 10.94\ kN$

$V_{MAX} = V_A = 24.06\ kN$ AT A. $M_{MAX} = 26.25\ kN \cdot m$ AT A.

REQ'D. $I = 9.102 \times 10^{-6}\ m^4$ ($21.87\ IN^4$) TO LIMIT DEFL. TO $L/360$.

SPECIFY LIGHTEST STEEL BEAM: W8×10, $I = 30.8\ IN^4$; $S = 7.81\ IN^3$

13-2 CASE A-24(b); $P = 35\text{ kN}$, $L = 4.0\text{ m}$, $a = 1.50\text{ m}$

$b = L - a = 4.0 - 1.5 = 2.5\text{ m}$

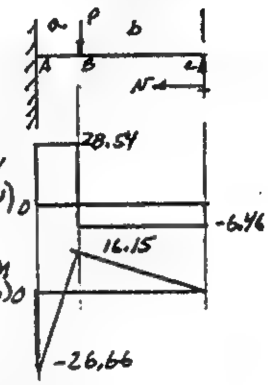
$$R_A = \frac{Pb}{2L^3}(3L^2 - b^2) = \frac{35\text{ kN}(2.5)}{2(4.0)^3}(3(4.0)^2 - 2.5^2) = \underline{28.54\text{ kN}}$$

$$R_C = \frac{Pa^2}{2L^3}(b + 2L) = \frac{35\text{ kN}(1.5)^2}{2(4)^3}(2.5 + 2(4)) = \underline{6.46\text{ kN}}$$

$$M_A = \frac{-Pab}{2L^2}(b + L) = \frac{-35\text{ kN}(1.5)(2.5)}{2(4.0)^2}(2.5 + 4.0) = \underline{-26.66\text{ kN}\cdot\text{m}}$$

$$M_B = \frac{Pa^2b}{2L^3}(b + 2L) = \frac{35\text{ kN}(1.5)^2(2.5)}{2(4.0)^3}(2.5 + 2(4.0)) = \underline{16.16\text{ kN}\cdot\text{m}}$$

DEFLECTION



MAX. DEFLECTION OCCURS IN BC.

$$y_{BC} = \frac{-Pa^2 x}{12EIL^3}[3L^2b - x^2(3L - a)]$$

x MEASURED FROM C - VARIABLE DISTANCE

REDUCE TO EXPRESSION OF THE FORM:

$$y_{BC} = \frac{a_1 x^3 - a_2 x}{EI}$$

USE $P = 35000\text{ N}$, DISTANCES IN m.

$$y_{BC} = \frac{(-35000)(1.50)^2 x}{12\,EI(4.0)^3}[3(4.0)^2(2.5) - x^2(3(4) - 1.5)]$$

$$= \frac{-102.54x}{EI}[120 - 10.5x^2] = \frac{1}{EI}[-12305x + 1076.7x^3]$$

$$y_{BC} = \frac{1}{EI}[1076.7x^3 - 12305x]$$

USING GRAPHING CALCULATOR, FUNCTION IS MINIMUM AT $x = 1.952\text{ m}$ FROM C. VALUE OF FUNCTION THERE IS:

$$y_{max} = \frac{-16010}{EI}$$

USE W8X10 — SAME AS IN PROB 13-1. $E = 207 \times 10^9\text{ N/m}^2$

$$I = 30.8\text{ in}^4 \times \frac{4.162 \times 10^5\text{ mm}^4}{\text{in}^4} = 1.28 \times 10^7\text{ mm}^4$$

$$y_{max} = \frac{-16010\text{ N}\cdot\text{m}^3}{(207 \times 10^9\text{ N/m}^2)(1.28 \times 10^7\text{ mm}^4)} \cdot \frac{(10^3)^5\text{ mm}^5}{\text{m}^5} = \underline{-6.03\text{ mm}}$$

STRESS

$$\sigma = \frac{M}{S} = \frac{26.66\text{ kN}\cdot\text{m}}{1.28 \times 10^5\text{ mm}^3} \times \frac{10^3\text{ N}}{\text{kN}} \times \frac{10^3\text{ mm}}{\text{m}} = \underline{208\text{ MPa}}$$

FOR ASTM A242, $s_y = 345\text{ MPa}$; $\sigma_d = 0.66 s_y = 227\text{ MPa}$ OK

AT LOAD, $y_B = -13939/EI = \underline{5.25\text{ mm}}$

13-3 CASE A-24(b); $P = 35\ kN$, $L = 4.0$, $a = 2.50\ m$

$b = L - a = 4.0 - 2.5 = 1.5\ m$

$$\underline{R_A} = \frac{Pb}{2L^3}(3L^2 - b^2) = \frac{35\ kN(1.5)}{2(4)^3}[3(4)^2 - (1.5)^2] = \underline{18.76\ kN} = V_{AB}$$

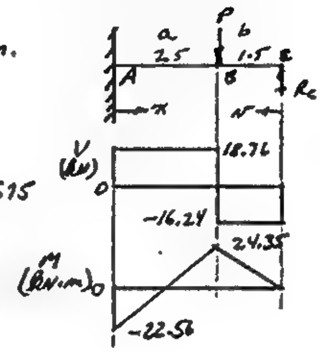
$$\underline{R_C} = \frac{Pa^2}{2L^3}(b + 2L) = \frac{35\ kN(2.5)^2}{2(4)^3}[1.5 + 2(4)] = \underline{16.24\ kN} = V_{BC}$$

$$\underline{M_A} = \frac{-Pab}{2L^2}(b + L) = \frac{-35(2.5)(1.5)}{2(4)^2}(1.5 + 4) = \underline{-22.56\ kN \cdot m}$$

$$\underline{M_B} = \frac{Pa^2 b}{2L^3}(b + 2L) = \frac{35(2.5)^2(1.5)}{2(4)^3}(1.5 + 2(4)) = \underline{24.35\ kN \cdot m}$$

DEFLECTION

FROM A TO B: $P = 35000\ N$; DISTANCES IN m.



$$y_{AB} = \frac{-Px^2 b}{12EIL^3}(3C_1 - C_2 x)$$

$$C_1 = aL(L + b) = 2.5(4)[4 + 1.5] = 55$$

$$C_2 = (L + a)(L + b) + aL = (6.5)(5.5) + 10 = 45.75$$

$$y_{AB} = \frac{(-35000)(x^2)(1.5)[3(55) - 45.75x]}{12\ EI(4)^3}$$

$$= \frac{-68.36x^2[165 - 45.75x]}{EI}$$

$$y_{AB} = \frac{3127.4x^3 - 11279x^2}{EI}$$

USING A GRAPHING CALCULATOR FUNCTION IS A MINIMUM AT $x = 2.404\ m$ FROM A. THEN y_{max} IS:

$$y_{max} = \frac{-21734}{EI}$$

$$\text{At } x = 2.5,\ y_B = \frac{-21628}{EI}$$

USING W 8x10 STEEL BEAM – SAME AS PROB 13-1,

$$\underline{y_{max}} = \frac{-21734\ N \cdot m^3}{(207 \times 10^9\ N/m^2)(1.28 \times 10^7\ mm^4)} \times \frac{10^{15}\ mm^5}{m^5} = \underline{-8.19\ mm}$$

$$y_B = \frac{-21628}{EI} = -8.15\ mm$$

STRESS:

$$\sigma = \frac{M}{S} = \frac{24.35\ kN \cdot m}{1.28 \times 10^5\ mm^3} \times \frac{10^3\ N}{kN} \times \frac{10^3\ mm}{m} = \underline{190\ MPa}$$

(OK - SEE 13-2)

13-4 CASE A-24(c): $w = 400\ lb/ft$, $L = 14.0\ ft$

$W = wL = (400\ lb/ft)(14.0\ ft) = 5600\ lb$

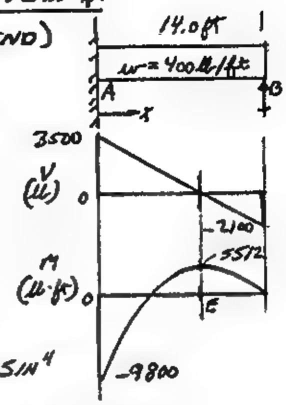
$R_A = 5/8\ W = \underline{3500\ lb} = V_A$; $R_B = 3/8\ W = \underline{2100\ lb} = V_B$

$M_A = -0.125\ WL = -0.125(5600)(14.0) = \underline{-9800\ lb\cdot ft}$

$M_E = 0.0703\ WL = 0.0703(5600)(14.0) = \underline{5512\ lb\cdot ft}$

POINT E IS 5/8 L FROM A (FIXED END)

$x_E = 5/8(14.0\ ft) = 8.75\ ft$



DEFLECTION:

AT C AT $x = 0.579L = 0.579(14.0) = 8.11\ ft$

$y_C = y_{MAX} = \dfrac{-WL^3}{185EI} = \dfrac{-(5600\ lb)(14\ ft)^3}{185EI}$

$y_{MAX} = \dfrac{-83061\ lb\cdot ft^3}{EI}$

LET $y_{MAX} \le L/360 = \dfrac{14.0\ ft\ (12\ in/ft)}{360} = 0.467\ in$

REQ'D. $I = \dfrac{-83061\ lb\cdot ft^3}{(30\times10^6\ lb/in^2)(-0.467\ in)} \times \dfrac{(12\ in)^3}{1\ ft^3} = 10.25\ in^4$

W 8×10 STEEL BEAM IS LIGHTEST, $I = 30.8\ in^4$, $S = 7.81\ in^3$

ACTUAL $y_{MAX} = \dfrac{-83061\ lb\cdot ft^3}{(30\times10^6\ lb/in^2)(30.8\ in^4)} \times \dfrac{(12\ in)^3}{ft^3} = \underline{0.155\ in.}$

STRESS: $\sigma = \dfrac{M}{S} = \dfrac{9800\ lb\cdot ft}{7.81\ in^3} \times \dfrac{12\ in}{ft} = \underline{15058\ psi}$

CAN USE ASTM A36; $\sigma_d = 0.66(36\ ksi) = 23760\ psi$ – $\underline{OK}$

13-5 CASE A-24(c): $w = 50\ lb/in$, $L = 16.0\ in$

SKETCH SIMILAR TO 13-4.

$W = wL = (50\ lb/in)(16.0\ in) = 800\ lb$

$R_A = 5/8\ W = \underline{500\ lb}$; $R_B = 3/8\ W = \underline{300\ lb}$

$M_A = -0.125\ WL = -0.125(800)(16) = \underline{-1600\ lb\cdot in}$

$M_E = 0.0703\ WL = 0.0703(800)(16) = \underline{900\ lb\cdot in}$

$x_E = 5/8\ L = 5/8(16) = 10.0\ in$

DEFLECTION:

AT $C = 0.579L = 0.579(16) = 9.264\ in$

$y_C = y_{MAX} = \dfrac{-WL^3}{185EI} = \dfrac{-800(16)^3}{185EI} = \dfrac{17712}{EI}$

DESIGN COULD RESULT IN MULTIPLE SOLUTIONS.

13-6 CASE A-24(d): $P = 350\,lb$, $L = 10.8\,in$, $a = 2.50\,in$.

$$R_A = \frac{-3Pa}{2L} = \frac{-3(350)(2.50)}{2(10.8)} = -121.5\,lb \text{ DOWN}$$

$$R_B = P\left(1 + \frac{3a}{2L}\right) = 350\,lb\left[1 + \frac{3(2.50)}{2(10.8)}\right] = 471.5\,lb \text{ UP}$$

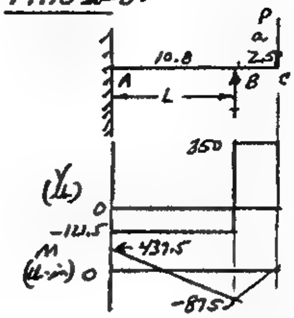
$$M_A = Pa/2 = (350)(2.50)/2 = 437.5\,lb\cdot in$$

$$M_B = -Pa = -350(2.5) = -875\,lb\cdot in$$

$$y_C = \frac{-PL^3}{EI}\left[\frac{a^2}{4L^2} + \frac{a^3}{3L^3}\right]$$

$$= \frac{-350(10.8)^3}{EI}\left[\frac{2.5^2}{4(10.8)^2} + \frac{2.5^3}{3(10.8)^3}\right]$$

$$y_C = \frac{7729\,lb\cdot in^3}{EI}$$



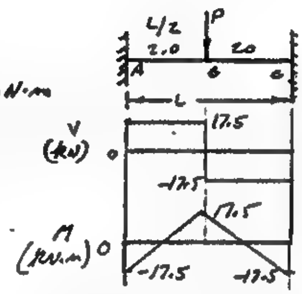
13-7 CASE A-24(e): $P = 35\,kN$; $L = 4.0\,m$

$$R_A = R_C = P/2 = 35/2 = 17.5\,kN$$

$$M_A = M_B = M_C = \frac{PL}{8} = \frac{(35\,000\,N)(4.0\,m)}{8} = 17\,500\,N\cdot m$$

$$y_B = y_{MAX} = \frac{-PL^3}{192EI} = \frac{-35\,000(4.0)^3}{192\,EI} =$$

$$y_B = \frac{11667\,N\cdot m^3}{EI}$$



13-8 CASE A-24(f): $P = 35\,kN$, $L = 4.0\,m$, $a = 1.50\,m$, $b = 4.0 - a = 2.5\,m$

THIS LOADING IS THE MIRROR IMAGE OF THAT IN 13-9. NOTATION OF CASE A-24(f) REQUIRES $a > b$. THEN CALCULATIONS ARE THE SAME.

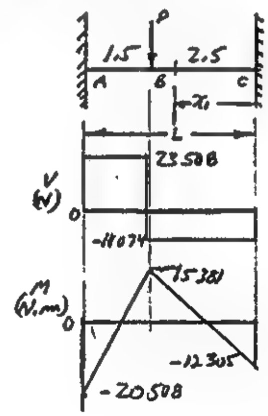
$R_A = 23926\,N$, $R_C = 11074\,N$

$M_A = -20508\,N\cdot m$ $\quad M_B = 15381\,N\cdot m$

$M_C = -12305\,N\cdot m$

$$y_{MAX} = y_D = \frac{-10127\,N\cdot m^3}{EI}$$

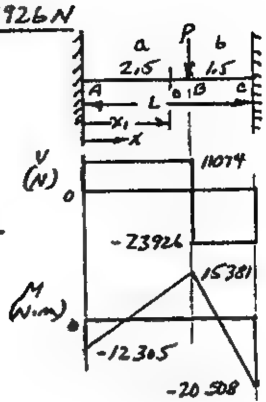
$x_1 = 2.222\,m$ FROM C TO D



13-9 CASE A-24(f): $P = 35\,kN$, $L = 4.0\,m$, $a = 2.5\,m$, $b = L - a = 1.5\,m$

$$R_A = \frac{Pb^2}{L^3}(3a+b) = \frac{35000(1.5)^2}{(4.0)^3}(3(2.5)+1.5) = \underline{11\,074\,N}$$

$$R_C = \frac{Pa^2}{L^3}(3b+a) = \frac{35000(2.5)^2}{(4.0)^3}(3(1.5)+2.5) = \underline{23\,926\,N}$$



$$M_A = \frac{-Pab^2}{L^2} = \frac{-35000(2.5)(1.5)^2}{(4.0)^2} = \underline{-12\,305\,N\cdot m}$$

$$M_B = \frac{2Pa^2b^2}{L^3} = \frac{2(35000)(2.5)^2(1.5)^2}{(4.0)^3} = \underline{15\,381\,N\cdot m}$$

$$M_C = \frac{-Pa^2b}{L^2} = \frac{-35000(2.5)^2(1.5)}{(4.0)^2} = \underline{-20\,508\,N\cdot m}$$

DEFLECTION:

$$y_{MAX} = y_D = \frac{-2Pa^3b^2}{3EI(3a+b)^2}$$

$$y_{MAX} = \frac{2(-35000)(2.5)^3(1.5)^2}{3EI[3(2.5)+1.5]^2} = \underline{\frac{-10\,127\,N\cdot m^3}{EI}}$$

$$x_1 = \frac{2aL}{3a+b} = \frac{2(2.5)(4)}{3(2.5)+1.5} = 2.222\,m \text{ FROM A TO D.}$$

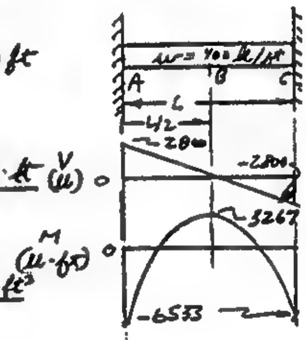
13-10 CASE A-24(g): $w = 400\,lb/ft$; $L = 14.0\,ft$

$W = wL = (400\,lb/ft)(14.0\,ft) = \underline{5600\,lb}$

$R_A = R_C = W/2 = \underline{2800\,lb}$

$M_A = M_C = -WL/12 = -5600(14)/12 = \underline{-6533\,lb\cdot ft}$

$M_B = WL/24 = \underline{3267\,lb\cdot ft}$



DEFLECTION:

$$y_B = y_{MAX} = \frac{-WL^3}{384EI} = \frac{-5600(14)^3}{384\,EI} = \underline{\frac{-40\,017\,lb\cdot ft^3}{EI}}$$

13-11 CASE A-24(g): $w = 50\,lb/in$, $L = 16.0\,in$

(SKETCH SIMILAR TO B-10.)

$W = wL = (50\,lb/in)(16\,in) = 800\,lb$

$R_A = R_C = W/2 = 400\,lb = V_A = V_C$

$M_A = M_C = -WL/12 = -800(16)/12 = \underline{-1067\,lb\cdot in}$

$M_B = WL/24 = \underline{533\,lb\cdot in}$

DEFLECTION:

$$y_B = y_{MAX} = \frac{-WL^3}{384EI} = \frac{-800(16)^3}{384\,EI} = \underline{\frac{-8533\,lb\cdot in^3}{EI}}$$

13-12 CASE A-24(h): $w = 400\ lb/ft$ $L = 7.0\ ft$

$W = wL = (400\ lb/ft)(7\ ft) = 2800\ lb$ ON 1 SPAN.

$R_A = R_C = 3W/8 = 3(2800)/8 = \underline{1050\ lb} = V_A = V_C$

$R_B = 1.25W = 1.25(2800) = \underline{3500\ lb}$

$V_B = 5W/8 = 5(2800)/8 = \underline{1750\ lb}$

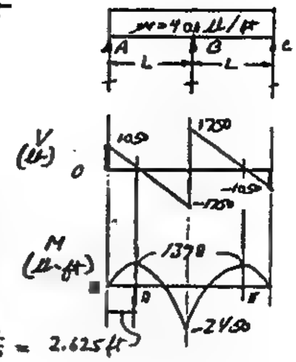
$M_D = M_E = 0.0703\ WL = 0.0703(2800)(7) = \underline{1378\ lb\cdot ft}$

$M_B = -0.125\ WL = -0.125(2800)(7) = \underline{-2450\ lb\cdot ft}$

DEFLECTION: MAX AT $x_1 = 0.4215L$ FROM A OR C.

$x_1 = 0.4215(7.0\ ft) = 2.9505\ ft$

$y_{MAX} = \frac{-wL^4}{185EI} = \frac{(-400)(7)^4}{185EI} = \frac{-5191\ lb\cdot ft^3}{EI}$



13-13 CASE A-24(h): $w = 50\ lb/in$, $L = 8.0\ in$

$W = wL = (50\ lb/in)(8\ in) = 400\ lb$ ON EACH SPAN

(SKETCH SIMILAR TO 13-12.)

$R_A = 3W/8 = 3(400)/8 = \underline{150\ lb} = R_C = V_A = V_C$

$R_B = 1.25W = 1.25(400) = \underline{500\ lb}$

$V_B = 5W/8 = 5(400)/8 = \underline{250\ lb}$

$M_D = M_E = 0.0703\ WL = 0.0703(400)(8) = \underline{225\ lb\cdot in}$

$M_B = -0.125\ WL = -0.125(400)(8) = \underline{-400\ lb\cdot in}$

DEFLECTION: MAX AT $x_1 = 0.4215L$ FROM A OR C.

$x_1 = 0.4215(8.0\ in) = 3.372\ in$

$y_{MAX} = \frac{-wL^4}{185EI} = \frac{-50(8)^4}{185EI} = \frac{-1107}{EI}$

13-14 CASE A-24(i): $w = 400\ lb/ft$, $L = 56\ in$, $W = wL = 1867\ lb$

$R_A = R_D = 0.4W = 0.4(1867\ lb) = \underline{746.7\ lb} = V_A = V_D$

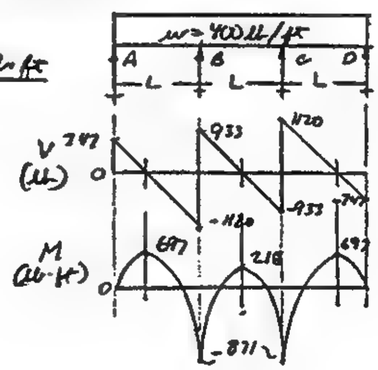
$R_B = R_C = 1.10\ W = 1.10(1867) = \underline{2053\ lb}$

$M_E = M_F = 0.08\ WL = 0.08(1867)(4.67\ ft) = \underline{697\ lb\cdot ft}$

$M_B = M_C = -0.10\ WL = \underline{-871\ lb\cdot ft}$

$M_G = 0.025\ WL = 0.025(1867)(4.67\ ft) = \underline{218\ lb\cdot ft}$

DEFLECTION FORMULAS NOT AVAILABLE



13-15 CASE A-24(i): $w = 50$ lb/in, $L = 5.333$ in, $W = wL = 266.7$ lb

$R_A = R_D = 0.4W = 106.7$ lb

$R_B = R_C = 1.10W = 293.3$ lb

$M_E = M_F = 0.08WL = 0.08(266.7)(5.333) = 113.8$ lb-in

$M_B = M_C = -0.10WL = -0.10(266.7)(5.333) = -142.2$ lb-in

$M_G = 0.025WL = 0.025(266.7)(5.333) = 35.6$ lb-in

[SKETCH SIMILAR TO 13-14.]

13-16 CASE A24(j): $w = 400$ lb/ft, $L = 3.5$ ft, $W = wL = 1400$ lb EACH SPAN

$R_A = R_E = 0.393W = 550$ lb

$R_B = R_D = 1.143W = 1600$ lb

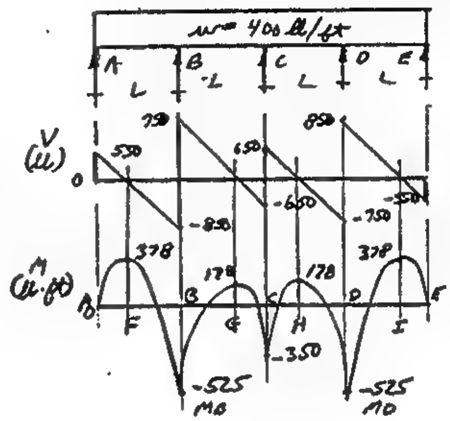
$R_C = 0.928W = 1300$ lb

$M_B = M_D = -0.1071WL = M_{MAX}$
$= -0.1071(1400)(3.5) = 525$ lb-ft

$M_F = M_I = 0.0772WL = 378$ lb-ft

$M_C = -0.0714WL = -350$ lb-ft

$M_G = M_H = 0.0364WL = 178$ lb-ft



13-17 CASE A24(j): $w = 50$ lb/in, $L = 4.0$ in $W = wL = 200$ lb

$R_A = R_E = .393W = 78.6$ lb

$R_B = R_D = 1.143W = 228.6$ lb

$R_C = 0.928W = 185.6$ lb

$M_B = M_D = -0.1071WL = -85.68$ lb·in

$M_F = M_I = 0.0772WL = 61.76$ lb·in

$M_C = -0.0714WL = -57.12$ lb·in

$M_G = M_H = 0.0364WL = 29.12$ lb·in

$V_A = R_A = 78.6$ lb

$-V_B = R_A - W = -121.4$ lb

$+V_B = -121.4 + 228.6 = 107.2$ lb

$-V_C = 107.2 - W = -92.8$ lb

$+V_C = -92.8 + 185.6 = 92.8$ lb

$-V_D = 92.8 - W = -107.2$ lb

$+V_D = -107.2 + 228.6 = 121.4$ lb

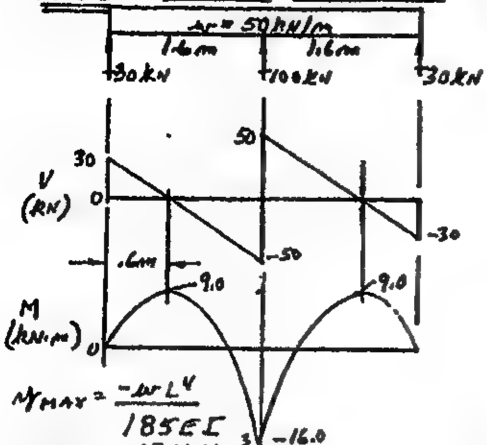
$V_E = 121.4 - W = -78.6$ lb

SKETCH SIMILAR TO 13-16.

$M_{MAX} = 85.7$ lb·in $= M_B = M_D$

$V_{MAX} = 121.4$ lb $= -V_B = +V_D$

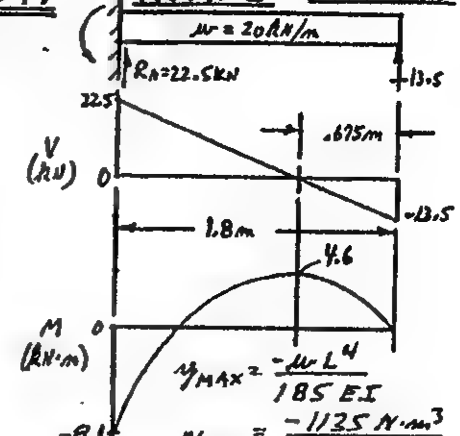
13-18 FIG P13-1 APP. A24-(b)



$y_{MAX} = \frac{-wL^4}{185EI}$

$y_{MAX} = \frac{-1771\ N\cdot m^3}{EI}$

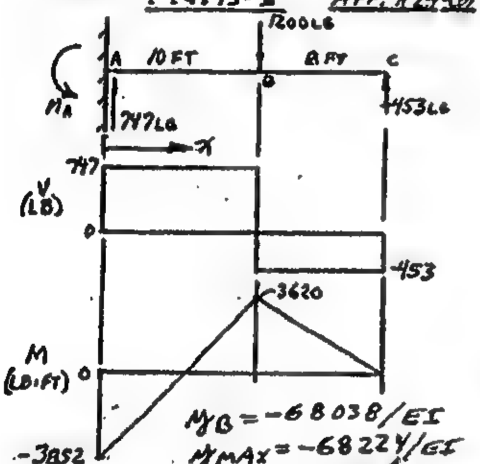
13-19 FIG P13-2 APP. A24-(c)



$y_{MAX} = \frac{-wL^4}{185EI}$

$y_{MAX} = \frac{-1125\ N\cdot m^3}{EI}$

13-20 FIG P13-2 APP. A24-(b)

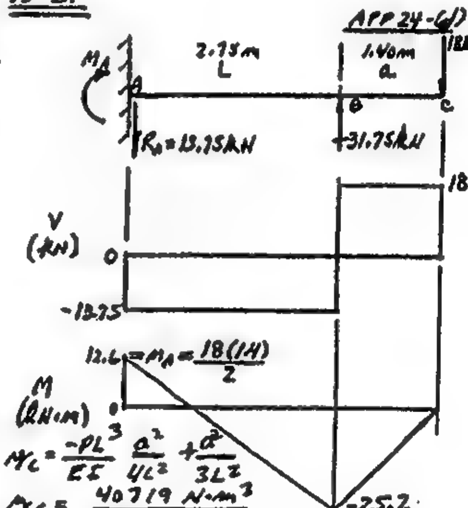


$y_B = -68038/EI$

$y_{MAX} = -68224/EI$

AT $x = 10.32$ ft

13-21 APP 24-(d)

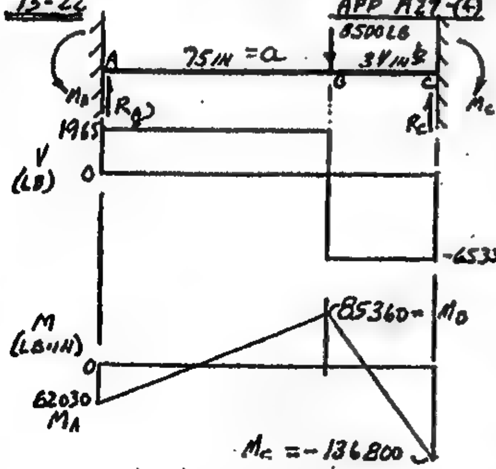


$12.6 = M_A = \frac{18(1.4)}{2}$

$y_C = \frac{-PL^3}{EI}\left[\frac{a^2}{4L^2} + \frac{a^3}{3L^3}\right]$

$y_C = \frac{40719\ N\cdot m^3}{EI}$

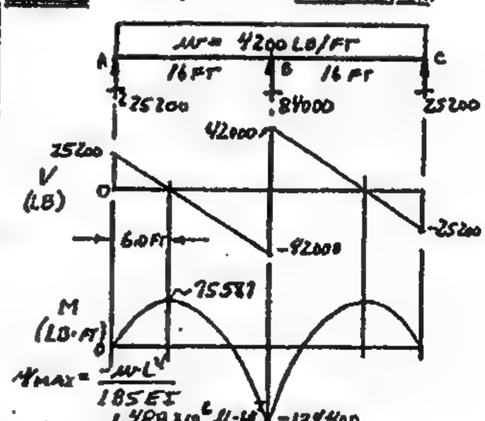
13-22 APP A24-(f)



$y_{MAX} = \frac{-2Pa^3b^2}{3EI(3a+b)^2}$

$y_{MAX} = \frac{-41.2\times10^6\ lb\cdot in^3}{EI}$

13-23 APP A24-(h)

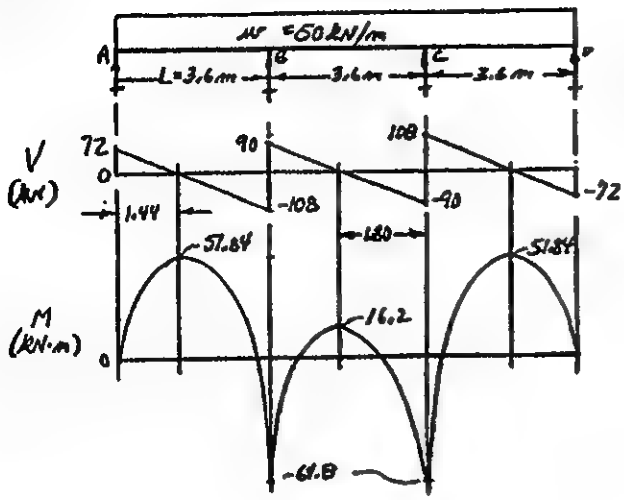


$y_{MAX} = \frac{-wL^4}{185EI}$

$y_{MAX} = \frac{1.488\times10^6\ lb\cdot ft^3}{EI}$

13-24

APP A24-(l)



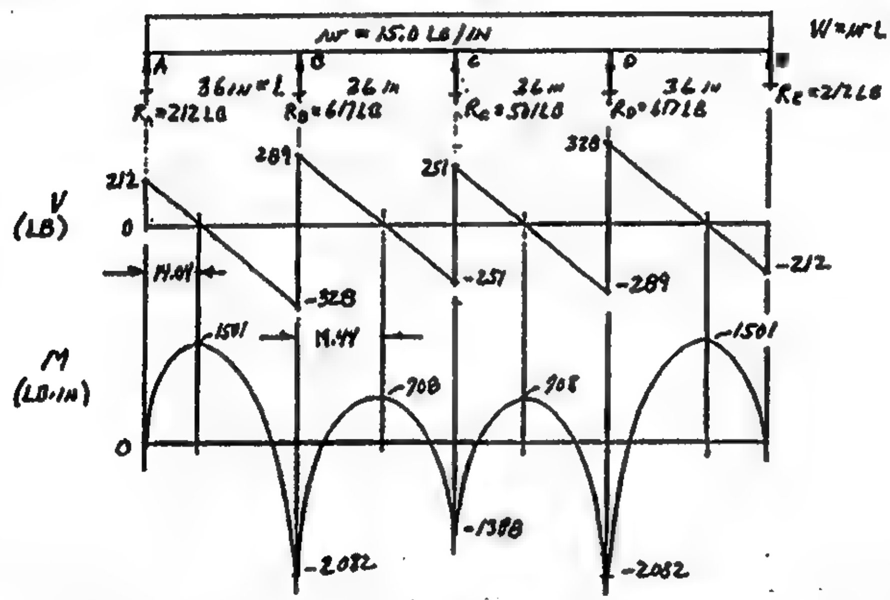
$W = wL = (50)(3.6) = 180\ kN$

$R_A = R_D = 0.4W = 72\ kN$

$R_B = R_C = 1.10W = 198\ kN$

13-25

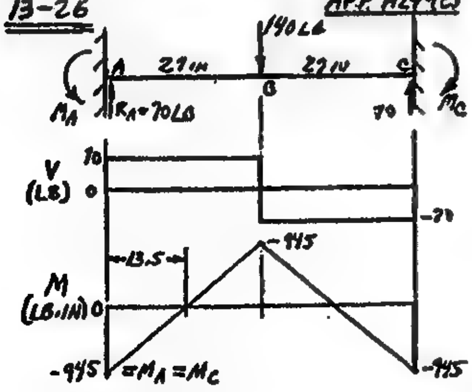
APP A24-(k)



$W = wL = 540\ LB.$

13-26

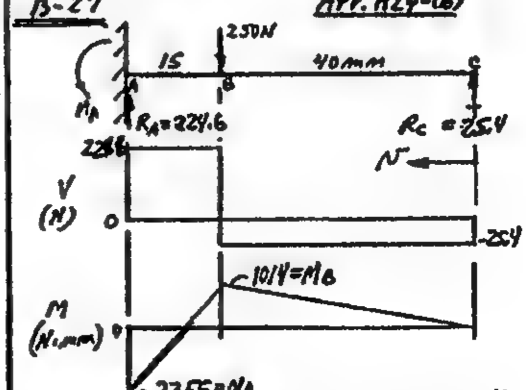
APP A24-(e)



$$y_{MAX} = \frac{-PL^3}{192EI} = \frac{1.15 \times 10^5\ lb\text{-}in^3}{EI}$$

13-27

APP. A24-(b)



$$y_B = \frac{-Pa^3b^2}{12EIL^3}(3L + b) = \frac{-1.386 \times 10^{-4}\ N\cdot m^3}{EI}$$

$y_{MAX} = -1.936 \times 10^{-4}\ N\cdot m^3/EI$

AT $x = 0.0284$ m FROM C.

13-28

The objective is to compare the results of the beam loading and support conditions for five different beams in Figures 13-4, 13-10, 13-12, 13-14, and 13-16. Details were reported earlier in this solutions manual for each problem. Note that each beam design has a total length of 14.0 ft and carries a uniformly distributed load of 400 lb/ft resulting in a total load of 5600 lb. Changing the manner of support or adding additional supports affects the shearing force, *V*, the bending moment, *M*, and the maximum deflection, *y*, for a given *EI* value for the beam material and shape. Vertical shear stress and bending stress are directly propostional to the values of *V* and *M* respectively. Therefore, a reduction in either value will result in a reduction in stress or will allow the use of a smaller or lighter section for the beam. Comparisons are shown as ratios of *V*, *M*, and *y/EI* to those values for the first design, a supported cantilever. The other designs are a fixed-end beam and continuous beams on 3, 4, and 5 supports.

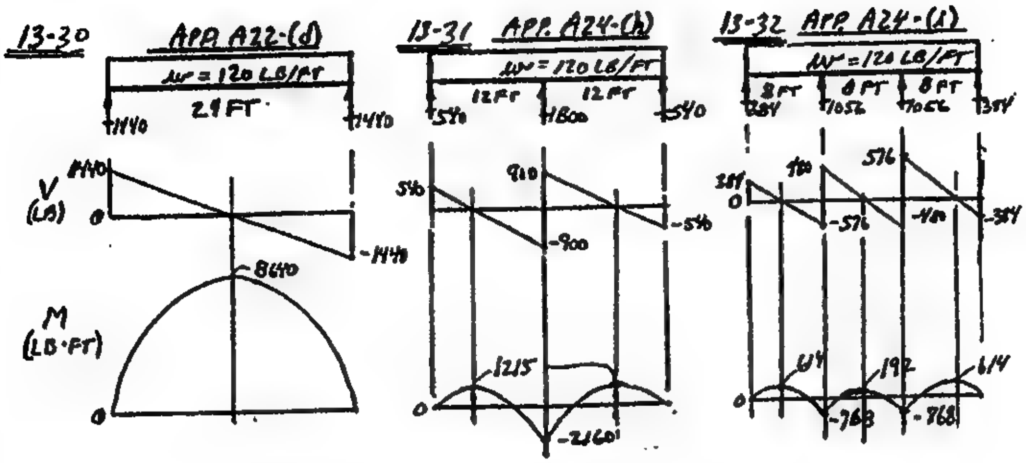
Prob.	V_{max}	V/V_1	M_{max}	M/M_1	y_{max}	y/y_1
13-4	3500 lb	1.00	9800 lb in	1.00	-83061/EI	1.00
13-10	2800 lb	0.80	6533 lb in	0.667	-40017/EI	0.482
13-12	1750 lb	0.50	2450 lb in	0.250	-5191/EI	0.0625
13-14	1120 lb	0.32	871 lb in	0.089	N/A	-
13-16	850 lb	0.24	525 lb in	0.054	N/A	-

Note that maximum shearing force, bending moment, and deflection all decrease for successive designs. Deflection formulas are not available (N/A) in this book for the last two designs. But it stands to reason that deflection would be reduced by adding additional supports and reducing the span between adjacent supports. The comparison illustrates the advantages of using fixed ends for beams and of using more supports for a given load, thus reducing the effective span between adjacent supports. Fabrication problems and costs must also be considered when selecting a method of supporting a load on a beam.

13-29

This problem has the same objective as 13-28. Refer to that problem for a discussion of the objectives and the results. Data listed here are for different beam loadings ($w = 50$ lb/in; total beam length = 16.0 in) but the support conditions are the same as in 13-28.

Problem	V_{max}	V/V_1	M_{max}	M/M_1	y_{max}	y/y_1
13-5	500 lb	1.00	1600 lb in	1.00	-17712/EI	1.00
13-11	400 lb	0.80	1067 lb in	0.667	-8533/EI	0.482
13-13	250 lb	0.50	400 lb in	0.250	-1107/EI	0.0625
13-15	160 lb	0.32	142 lb in	0.089	N/A	-
13-17	121 lb	0.24	86 lb in	0.054	N/A	-



13-33 COMPARISON OF 13-30 13-31 13-32

$w = 120\frac{LB}{FT} \times \frac{1FT}{12IN} = 10 LB/IN$; $L_a = 24 FT \times \frac{12IN}{FT} = 288 IN$

DEFLECTIONS:

[13-30] $y_{MAX} = \frac{-5wL^4}{384EI} = \frac{-5(10)(288)^4}{384EI} = \frac{-896 \times 10^6}{EI} = y_a$

[13-31] $y_{MAX} = \frac{-wL^4}{185EI} = \frac{-(10)(288)^4}{185EI} = \frac{-372 \times 10^6}{EI} = y_b = 0.415 y_a$ IF EI IS EQUAL.

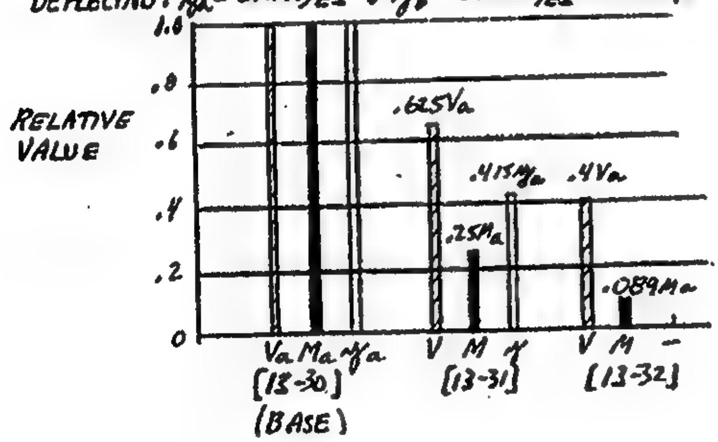
[13-32] DEFLECTION EQUATION NOT AVAILABLE; LESS THAN y_b

SUMMARY:

SHEARING FORCE: $V_{a_{MAX}} = 1440 LB$; $V_{b_{MAX}} = 900 LB = 0.625 V_a$; $V_c = 576 LB = 0.4 V_a$

MOMENT: $M_{a_{MAX}} = 8640 LB \cdot FT$; $M_b = 2160 LB \cdot FT = 0.25 M_a$; $M_c = 768 LB \cdot FT = 0.089 M_a$

DEFLECTION: $y_a = 896 \times 10^6/EI$; $y_b = 372 \times 10^6/EI = 0.415 y_a$ IF EI IS EQUAL.



(CONTINUED ON NEXT PAGE)

13-33 COMPARISON OF 13-30, 13-31, 13-32 (CONTINUED)

BEAM SIZE: FOR NO. 2 SOUTHERN PINE: $\tau_d = 70$ psi; $\sigma_d = 1000$ psi

FOR A RECTANGLE: $\tau_{MAX} = 3V/2A$; $S = th^2/6$

[13-30] $V_{MAX} = 1440$ LB: $A_{min} = \frac{3V}{2\tau_d} = \frac{3(1440)}{2(70)} = 30.86\ IN^2$

$M_{MAX} = (8640\ LB\cdot FT)(12\ IN/FT) = 103680\ LB\cdot IN$: $\sigma = M/S$

$S_{MIN} = \frac{M}{\sigma_d} = \frac{103680\ LB\cdot IN}{1000\ LB/IN^2} = 103.7\ IN^3$ 6X12 BEAM REQD $I = 697\ IN^4$

[13-31] $V_{MAX} = 900$ LB: $A_{min} = \frac{3(900)}{2(70)} = 19.3\ IN^2$

$M_{MAX} = 2160\ LB\cdot FT(12\ IN/FT) = 25920\ LB\cdot IN$

$S_{MIN} = \frac{M}{\sigma_d} = \frac{25920}{1000} = 25.9\ IN^3$ 4X8 BEAM REQD $I = 111.1\ IN^4$

[13-32] $V_{MAX} = 576$ LB: $A_{MIN} = \frac{3(576)}{2(70)} = 12.34\ IN^2$

$M_{MAX} = 768\ LB\cdot FT\ (12\ IN/FT) = 9216\ LB\cdot IN$

$S_{MIN} = \frac{M}{\sigma_d} = \frac{9216}{1000} = 9.22\ IN^3$ 2X10 BEAM REQD $I = 98.9\ IN^4$

ACTUAL DEFLECTIONS:

[13-30] $y_{MAX} = \frac{-8.96 \times 10^6}{EI} = \frac{-896 \times 10^6}{(1.3 \times 10^6)(697)} = -0.989$ IN.

[13-31] $y_{MAX} = \frac{-372 \times 10^6}{EI} = \frac{-372 \times 10^6}{(1.3 \times 10^6)(111.1)} = -2.58$ IN. (LARGE) HIGHER I MAY BE REQUIRED FOR DEFLECTION.

13-34

APPENDIX A24-(f) APP. A-18 (WOOD)

V_{MAX} AND M_{MAX} OCCUR AT SECOND SUPPORT

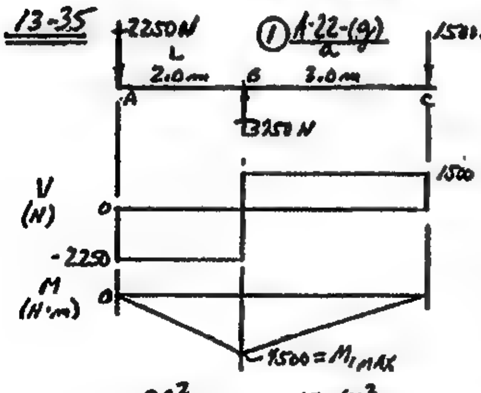
$V_{MAX} = 0.607\,wL = 0.607(100\ LB/FT)(2\ FT) = 121.4$ LB

$\tau_{MAX} = \frac{3V}{2A} = \frac{3(121.4\ LB)}{2(1.50)(5.50)\ IN^2} = 22.07$ PSI : $\tau_{ALLOW} = 70$ PSI OK

$M_{MAX} = 0.107\,wL^2 = 0.107(100\ LB/FT)(2.0\ FT)^2 = 42.8\ LB\cdot FT \times 12\ IN/FT = 513.6\ LB\cdot IN$

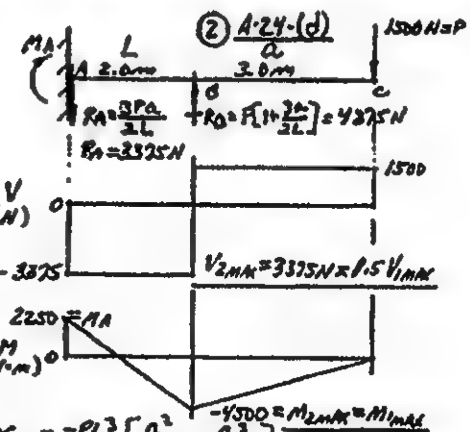
$\sigma = \frac{M}{S} = \frac{513.6\ LB\cdot IN}{(5.50)(1.50)^2/6} = 249$ PSI : $\sigma_{ALLOW} = 1000$ PSI OK

13-35



$y_{C1} = \frac{-Pa^2}{3EI}(L+a) = \frac{-1500(3)^2}{3EI}(2+3)$

$y_{C1} = \frac{22500}{EI}$



$y_{C2} = \frac{-PL^3}{EI}\left[\frac{a^2}{4L^2} + \frac{a^3}{3L^3}\right]$

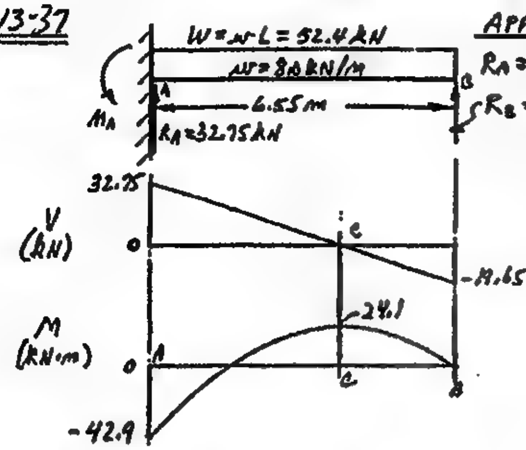
$y_{C2} = \frac{-1500(2)^3}{EI}\left[\frac{3^2}{4(2)^2} + \frac{3^3}{3(2)^3}\right] = \frac{20250}{EI} = 0.9\,y_{C1}$

NO MAJOR DIFFERENCE BETWEEN DESIGNS. ② IS 10% STIFFER. ② HAS 50% HIGHER SHEARING FORCE.

13-36

MULTIPLE DESIGNS POSSIBLE: USE V_{MAX} AND M_{MAX} FROM PROB. 13-35. SPECIFY MATERIAL, DESIGN STRESS, DESIGN FACTOR, SHAPE OF CROSS SECTION, DIMENSIONS. BECAUSE BENDING MOMENT IS EQUAL FOR BOTH DESIGNS, BOTH MAY BE THE SAME. BUT CHECK SHEAR.

13-37



APP. A-24-(C)

$R_A = \frac{5}{8}W = 32.75\ kN$

$R_B = \frac{3}{8}W = 19.65\ kN$

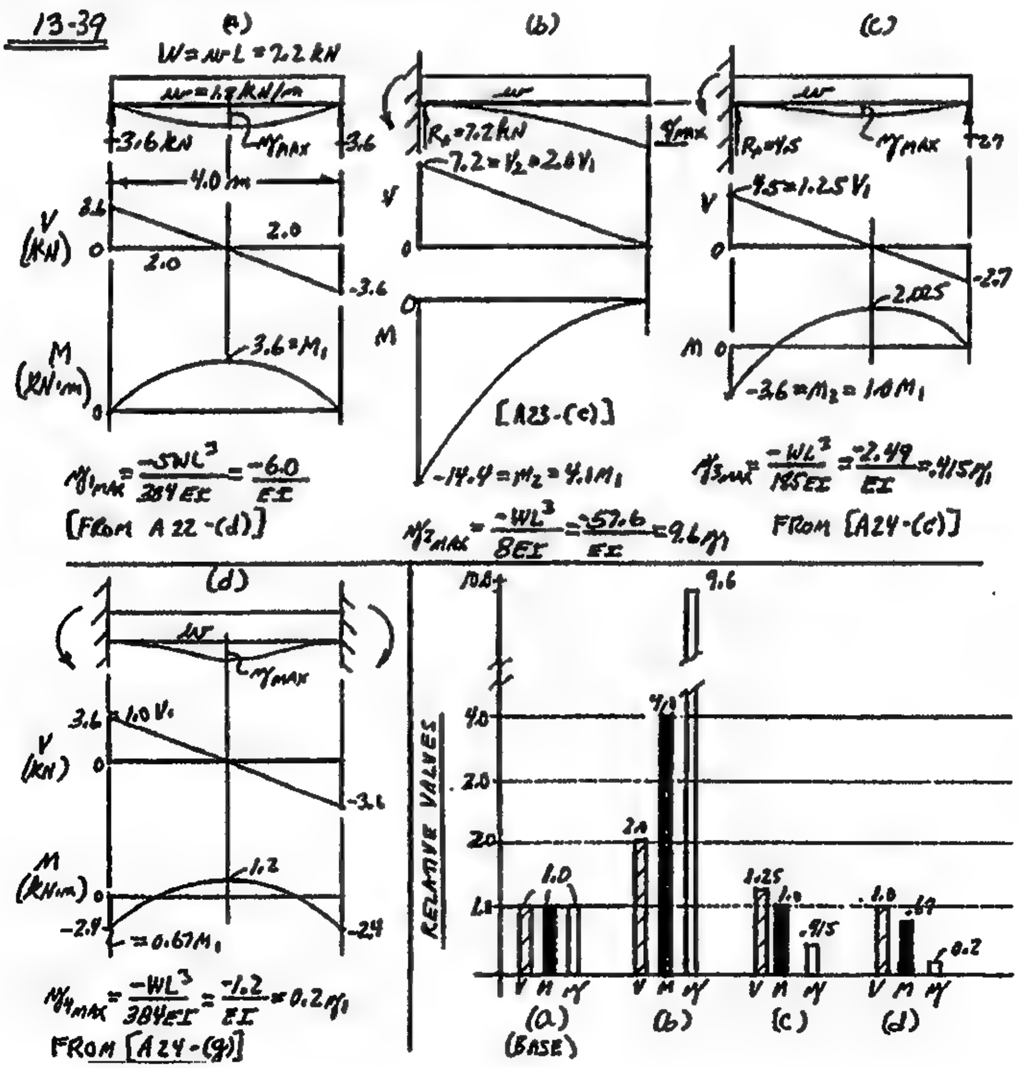
$M_A = -0.125WL = -0.125(52.4)(6.55)$

$M_A = -42.9\ kN\cdot m$

$M_C = 0.0703\,WL = 24.1\ kN\cdot m$

13-38 MULTIPLE DESIGNS POSSIBLE

13-39



SUPERPOSITION METHOD

13-40 FIG P13-1

$y_{C1} = \frac{-5wL^3}{384EI}$ (CASE(d)) TABLE A-22

$y_{C1} = \frac{-5(160\times10^3)(3200)^3}{384EI} = -6.827\times10^{13}$

$y_{C2} = \frac{+PL^3}{48EI} = \frac{+R_B(3200)^3}{48EI} = 6.827\times10^8(R_B)$ (A22(a))

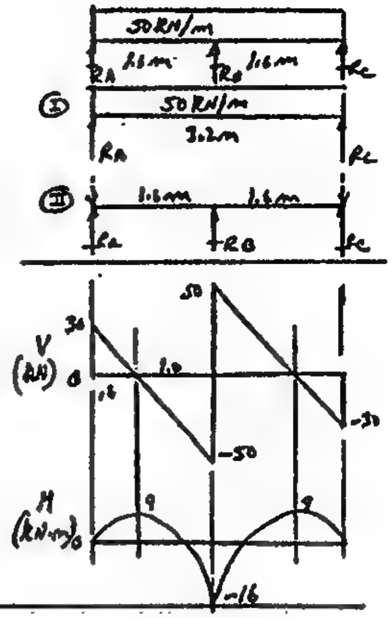
$y_{C1} + y_{C2} = 0$

$-6.827\times10^{13} + 6.827\times10^8(R_B) = 0$

$R_B = 6.827\times10^{13}/6.827\times10^8 = 1.0\times10^5 N$

$R_B = 100 kN$

THEN $R_A = R_C = 30 kN$



13-41 FIG. P13-2

$y_{B1} = \frac{-wL^4}{8EI} = \frac{(-36\times10^3)(1800)^3}{8EI} = \frac{-2.624\times10^{13}}{EI}$ [A23-(c)]

$y_{B2} = \frac{+PL^3}{3EI} = \frac{R_B(1800)^3}{3EI} = \frac{R_B(1.944\times10^9)}{EI}$ [A23-(a)]

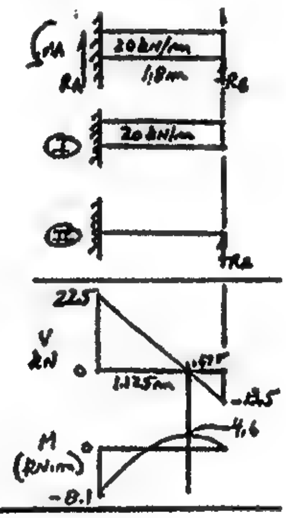
$y_{B1} + y_{B2} = 0$

$-2.624\times10^{13}/EI + R_B(1.944\times10^9)/EI = 0$

$R_B = 2.624\times10^{13}/1.944\times10^9 = 13.50 kN = R_B$

THEN $R_A = 36 - 13.5 = 22.5 kN = R_A$

$M_A = 36(0.9) - 13.5(1.8) = 8.1 kN\cdot m$ (NEGATIVE)



13-42 FIG. P13-3

$y_{C1} = \frac{-Pa^2}{6EI}(3L-a) = \frac{-(1200)(120)^2}{6EI}(3(216)-120) = \frac{-1.521\times10^9}{EI}$ [A23 (b)]

$y_{C2} = \frac{R_C L^3}{3EI} = \frac{R_C(216)^3}{3EI} = \frac{3.359\times10^6(R_C)}{EI}$ [A23-(a)]

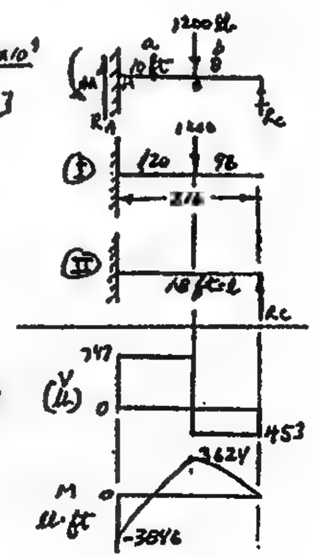
$y_{C1} + y_{C2} = 0$

$-1.521\times10^9/EI + 3.359\times10^6(R_C)/EI = 0$

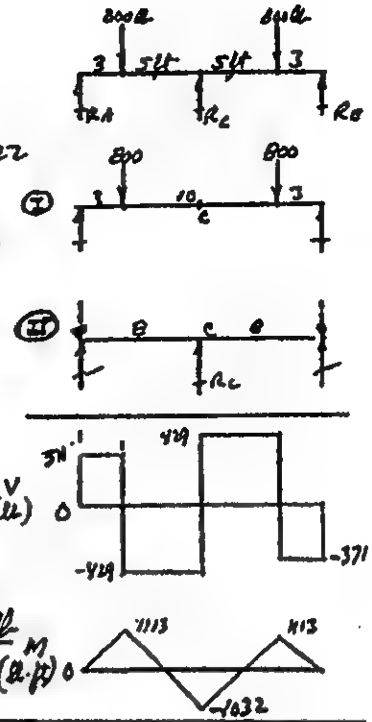
$R_C = 1.521\times10^9/3.359\times10^6 = 453 lb = R_C$

THEN $R_A = 1200 - 453 = 747 lb = R_A$

$M_A = 1200(10) - R_C(18) = 12000 - 453(18) = 3846 lb\cdot ft$ (NEGATIVE)



13-43 FIG. P13-7



SUPERPOSITION: $y_{c1} + y_{c2} = 0$

$y_{c1} = \frac{-Pa}{24EI}(3l^2 - 4a^2)$ (CASE (c)) TABLE A-22

$y_{c1} = \frac{-800(36)}{24EI}\left[(3)(192)^2 - 4(36)^2\right] = \frac{-1.265\times10^8}{EI}$

$y_{c2} = +\frac{Pl^3}{48EI}$ (CASE (a))

$y_{c2} = \frac{R_c(192)^3}{48EI} = \frac{1.475\times10^5 R_c}{EI}$

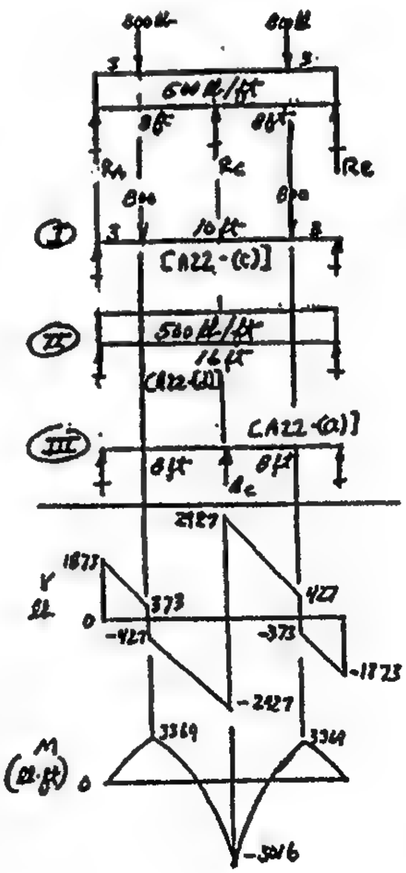
THEN $y_{c1} + y_{c2} = 0$

$-\frac{1.265\times10^8}{EI} + \frac{1.475\times10^5 R_c}{EI} = 0$

$R_c = 1.265\times10^8 / 1.475\times10^5 = \underline{858 \text{ lb}}$

BECAUSE OF SYMMETRY; $\underline{R_A = R_E = 371 \text{ lb}}$

13-44 FIG. P13-8



$y_{c1} = \frac{-Pa}{24EI}(3l^2 - 4a^2) = \frac{-1.265\times10^8}{EI}$

$y_{c2} = \frac{-5wl^3}{384EI} = \frac{-5(8000)(192)^3}{384EI} = \frac{-7.37\times10^8}{EI}$

$y_{c3} = \frac{+Pl^3}{48EI} = \frac{R_c(192)^3}{48EI} = \frac{(1.475\times10^5)R_c}{EI}$

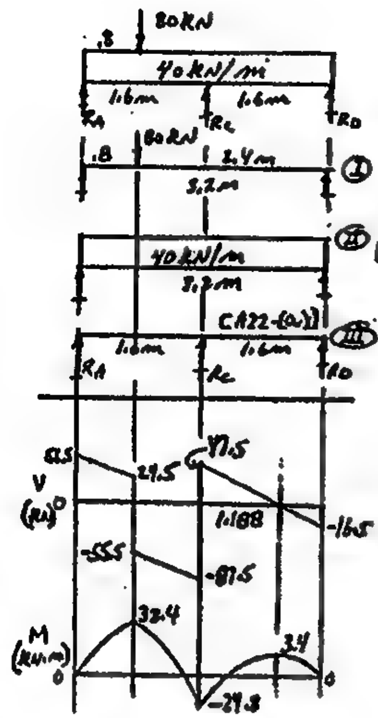
THEN $y_{c1} + y_{c2} + y_{c3} = 0$

$\frac{-1.265\times10^8}{EI} - \frac{7.37\times10^8}{EI} + \frac{R_c(1.475\times10^5)}{EI} = 0$

$R_c = \frac{1.265\times10^8 + 7.37\times10^8}{1.475\times10^5} = \underline{5854 \text{ lb} = R_c}$

THEN $\underline{R_A = R_E = 1873 \text{ lb}}$

13-45 FIG. P13-9



$$y_{C1} = \frac{-(80\,kN)(800)(1600)}{6EI(3200)}\left[3200^2 - 1600^2 - 800^2\right] = \frac{-3.755\times10^{13}}{EI}$$

$$y_{C2} = \frac{-5wL^3}{384EI} = \frac{-5(128\times10^3)(3200)^3}{384EI} = \frac{-5.461\times10^{13}}{EI}$$

$$y_{C3} = \frac{+PL^3}{48EI} = \frac{R_C(3200)^3}{48EI} = \frac{R_C(6.827\times10^8)}{EI}$$

THEN $y_{C1} + y_{C2} + y_{C3} = 0$

$$\frac{-3.755\times10^{13}}{EI} - \frac{5.461\times10^{13}}{EI} + \frac{R_C(6.827\times10^8)}{EI} = 0$$

$$R_C = \frac{+3.755\times10^{13} + 5.461\times10^{13}}{6.827\times10^8} = \underline{135\,kN = R_C}$$

$\Sigma M_A = 0 = 80(.8) + 128(1.6) - 135(1.6) - R_D(3.2)$

$\underline{R_D = 16.5\,kN}$

$\Sigma M_D = 0 = 80(2.4) + 128(1.6) - 135(1.6) - R_A(3.2)$

$\underline{R_A = 56.5\,kN}$

13-46 FIG. P13-10

$W = w \cdot L = (40\,kN/m)(3.6\,m) = 144\,kN$

$$y_{C1} = \frac{-WL^3}{8EI} = \frac{-144(3600)^3}{8EI} = -8.398\times10^{11}/EI \quad [A23\text{-}(c)]$$

$$y_{C2} = \frac{-Pa^2}{6EI}(3L-a) = \frac{-60(2000)^2}{6EI}\left[3(3600)-2000\right] \quad [A23\text{-}(b)]$$

$$y_{C2} = 3.52\times10^{11}/EI$$

$$y_{C3} = \frac{+R_C(3600)^3}{3EI} = R_C(1.555\times10^{10})/EI \quad [A23\text{-}(a)]$$

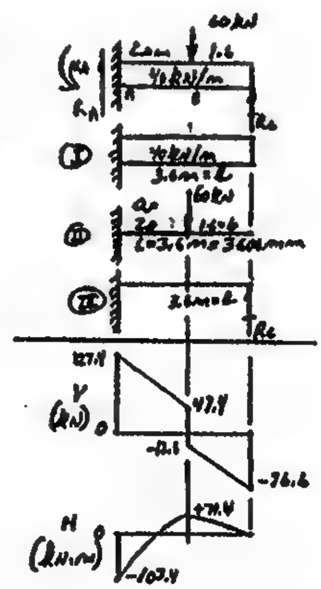
$y_{C1} + y_{C2} + y_{C3} = 0$

$-8.398\times10^{11}/EI - 3.52\times10^{11}/EI + R_C(1.555\times10^{10})/EI = 0$

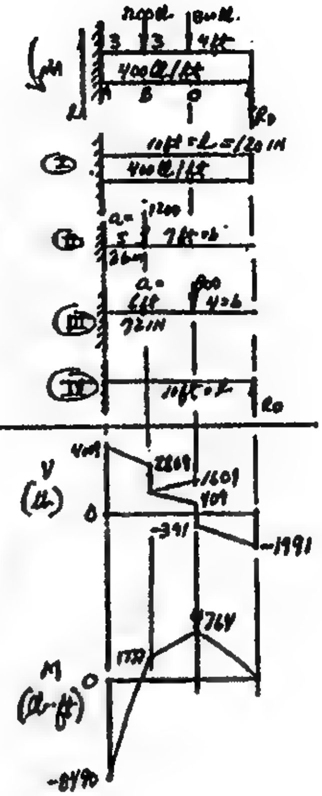
$\underline{R_C = 76.6\,kN}$

$R_A = 60\,kN + 144\,kN - 76.6\,kN = \underline{127.4\,kN = R_A}$

$M_A = 60(2.0) + 144(1.8) - R_C(3.6) = 103.44\,kN\cdot m$ (NEGATIVE)



13-47 FIG. P13-11



$$y_{D1} = \frac{-wL^3}{8EI} = \frac{-4000(120)^3}{8EI} = -8.64\times10^8/EI \quad [A23-(c)]$$

$$y_{D2} = \frac{-Pa^2}{6EI}[3(L)-a] = \frac{-1200(36)^2}{6EI}[3(120)-36] \quad [A23-(b)]$$

$$y_{D2} = -8.398\times10^7/EI$$

$$y_{D3} = \frac{-800(72)^2}{6EI}[3(120)-72] = -1.991\times10^8/EI \quad [A23(b)]$$

$$y_{D4} = \frac{+R_D(120)^3}{3EI} = 5.76\times10^5(R_D)/EI \quad [A23-(a)]$$

$$y_{D1} + y_{D2} + y_{D3} + y_{D4} = 0$$

$$\frac{-8.64\times10^8}{EI} - \frac{8.398\times10^7}{EI} - \frac{1.991\times10^8}{EI} + \frac{R_D(5.76\times10^5)}{EI} = 0$$

$R_D = 1991$ lb

$R_A = 1200 + 800 + 4000 - 1991 = 4009$ lb $= R_A$

$M_A = 1200(3) + 800(6) + 4000(5) - 1991(10) = 8490$ lb·ft (NEGATIVE)

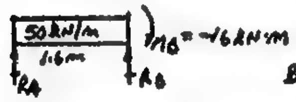
THEOREM OF THREE MOMENTS

13-48 FIG. P13-1 USE EQ.(13-3) WITH $M_A = M_C = 0$

SEE PROB 13-40 FOR SHEAR AND MOMENT DIAGRAMS.

$0 + 4M_B + 0 = -50(1.6)^3/2 = -64$

$M_B = -64/4 = -16$ kN·m



$\Sigma M_B = 0 = 80(.8) - R_A(1.6) - 16$

$R_A = 30$ kN

BECAUSE OF SYMMETRY, $R_C = R_A = 30$ kN

$R_B = 50(3.2) - 2(30) = 100$ kN

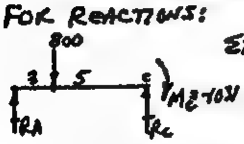
13-49 FIG P13-7 USE EQ(13-4) WITH $M_A = M_E = 0$

$$0 + 2M_C(8+8) + 0 = -\frac{800(3)}{8}(8^2-3^2) - \frac{800(3)}{8}(8^2-3^2)$$

$32M_C = -33000$; $M_C = -33000/32 = -1031$ lb·ft

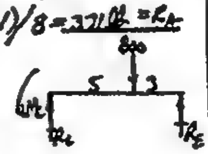
NOTE: M_C IS THE MOMENT AT THE MIDDLE SUPPORT. SUBSCRIPTS IN EQ(13-4) WERE ADJUSTED TO MATCH FIG. P13-7.

FOR REACTIONS:



$\Sigma M_C = 0 = 800(5) - R_A(8) - 1031$

$R_A = (4000 - 1031)/8 = 371$ lb $= R_A$



SHEAR AND MOMENT DIAGRAMS SAME AS PROB 13-43.

$\Sigma M_C = 0 = 800(5) - R_E(8) - 1031$

$R_E = 371$ lb

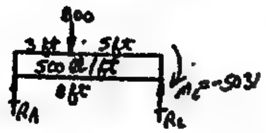
$R_C = 1600 - 2(371) = 858$ lb $= R_C$

13-50 FIG P13-8

USE EQ.(13-6) WITH $M_A = M_E = 0$

$$0 + 2M_C(8+8) + 0 = \frac{-800(3)}{8}(8^2-3^2) - \frac{800(3)}{8}(8^2-3^2) - \frac{500(8)^3}{4} - \frac{500(8)^3}{4}$$

$32M_C = -161000$; $M_C = -161000/32 = -5031$ lb·ft



$\Sigma M_C = 0 = 800(5) + 4000(4) - R_A(8) - 5031$

$R_A = 1871$ lb $= R_E$ BECAUSE OF SYMMETRY

$R_C = 800 + 8000 + 800 - 2(1871) = 5858$ lb $= R_C$

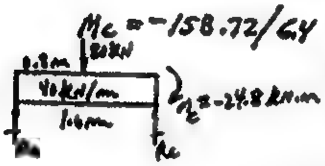
SEE PROB 13-44 FOR SHEAR AND MOMENT DIAGRAMS

13-51 FIG P13-9

USE EQ.(13-6) WITH $M_A = M_D = 0$

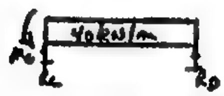
$$0 + 2M_C(1.6+1.6) = \frac{-80(0.8)}{1.6}(1.6^2 - 0.8^2) - \frac{40(1.6)^3}{4} - \frac{40(1.6)^3}{4}$$

$M_C = -158.72/6.4 = -24.8$ kN·m



$\Sigma M_C = 0 = 80(0.8) + 64(0.8) - 24.8 - R_A(1.6)$

$R_A = 56.5$ kN



$\Sigma M_C = 0 = 64(0.8) - 24.8 - R_D(1.6)$

$R_D = 16.5$ kN

$R_C = 80 + 128 - 56.5 - 16.5 = 135$ kN $= R_C$

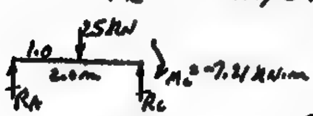
SEE PROB 13-45 FOR SHEAR AND MOMENT DIAGRAMS.

13-52 FIG. P13-12

EQ.(13-6) WITH $M_A = M_D = 0$

$$0 + 2M_C(3.2) + 0 = \frac{-25(1.0)}{2.0}(2.0^2 - 1.0^2) - \frac{20(1.2)^3}{4} = -46.14$$

$M_C = -46.14/6.4 = -7.21$ kN·m



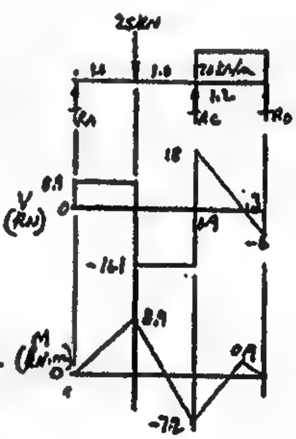
$\Sigma M_C = 0 = 25(1.0) - 7.21 - R_A(2)$

$R_A = 8.90$ kN

$\Sigma M_C = 0 = 24(0.6) - 7.21 - R_D(1.2)$

$R_D = 6.00$ kN

$R_C = 25 + 24 - 8.90 - 6.00 = 34.1$ kN



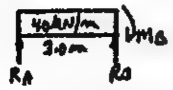
13-53 FIG. P13-13

EQ (13-3) WITH $M_A = 0$

$M_C = (40\,kN/m)(1.5\,m)\left(\frac{1.5\,m}{2}\right) = 45\,kN \cdot m$

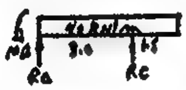
$0 + 4M_B - 45 = -40(3)^2/2 = -180$

$M_B = (-180 + 45)/4 = -33.75\,kN \cdot m$



$\Sigma M_B = 0 = 120(1.5) - 33.75 - R_A(3)$

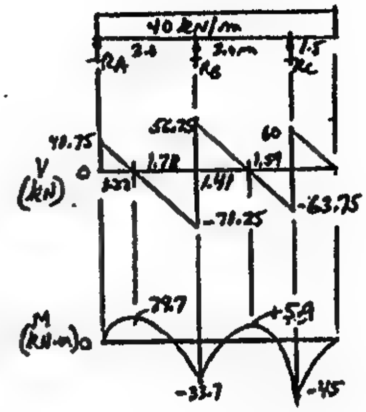
$\underline{R_A = 48.75\,kN}$



$\Sigma M_B = 0 = 180(2.25) - 33.75 - R_C(3)$

$\underline{R_C = 123.75\,kN}$

$\underline{R_B} = 300 - 48.75 - 123.75 = \underline{127.5\,kN}$

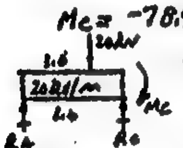


13-54 FIG. P13-14

Eq. (13-6) WITH $M_A = M_B = 0$

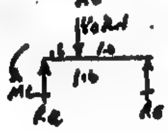
$0 + 2M_C(3.2) + 0 = \frac{-20(1)}{1.6}(1.6^2 - 1^2) - \frac{20(1.6)^3}{4} - \frac{40(1.0)}{1.6}(1.6^2 - 1^2)$

$M_C = -78.98/6.4 = -12.34\,kN \cdot m$



$\Sigma M_C = 0 = 20(1.6) + 32(0.8) - 12.34 - R_A(1.6)$

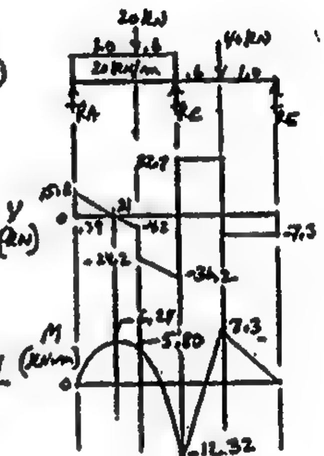
$\underline{R_A = 15.8\,kN}$



$\Sigma M_C = 0 = 40(0.6) - 12.34 - R_E(1.6)$

$\underline{R_E = 7.3\,kN}$

$\underline{R_C} = 20 + 32 + 40 - 15.8 - 7.3 = \underline{68.9\,kN}$



13-55 FIG. P13-15

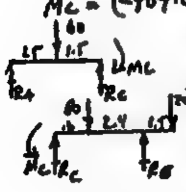
EQ. (13-4) WITH $M_A = 0$

$M_E = -20(1.5) = -30\,kN \cdot m$

$0 + 2M_C(6) - 30(3) = \frac{-60(1.5)}{3}(3^2 - 1.5^2) - \frac{80(2.4)}{3.0}(3^2 - 2.4^2)$

$12M_C - 90 = -409.86$

$M_C = (-409.86 + 90)/12 = -26.7\,kN \cdot m$



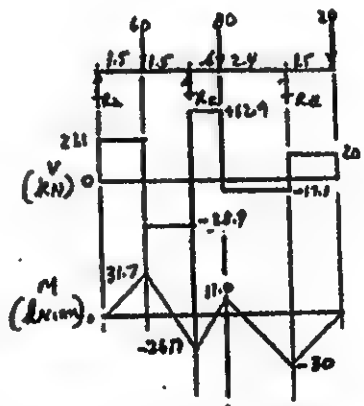
$\Sigma M_C = 0 = 60(1.5) - 26.7 - R_A(3)$

$\underline{R_A = 21.1\,kN}$

$\Sigma M_C = 0 = 80(0.6) + 20(4.5) - 26.7 - R_E(3)$

$\underline{R_E = 37.1\,kN}$

$\underline{R_C} = 60 + 80 + 20 - 21.1 - 37.1 = \underline{101.8\,kN}$



14-1 $\frac{Le}{r} = \frac{(1.0)(800)}{(20/4)} = 160$: FOR $S_y = 331$ MPa, $C_c = 110$ (FIG. 14-3) : LONG COLUMN

USE EULER EQ. (14-4) : $A = \pi D^2/4 = \pi(20)^2/4 = 314 \text{ mm}^2$

$$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2(207\times10^3 N/m^2)(314 mm^2)}{(160)^2} \times \frac{1 m^2}{10^6 mm^2} = \underline{25.1 kN}$$

14-2 $Le/r = (1.0)(350)/(20/4) = 70$: $C_c = 110$. – SHORT COLUMN : EQ(14-6)

$$P_{CR} = AS_y\left[1 - \frac{S_y(Le/r)^2}{4\pi^2 E}\right] = (314 mm^2)(331 N/mm^2)\left[1 - \frac{331\times10^6 Pa(70)^2}{4\pi^2(207\times10^9 Pa)}\right]$$

$\underline{P_{CR} = 83.3 \text{ kN}}$

14-3 $Le/r = 160$: FOR ALUM, $S_y = 276$ MPa – $C_c = 70$: FIG 14-4 : LONG COLUMN

$$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2(69\times10^9 N/m^2)(314 mm^2)}{(160)^2} \times \frac{1 m^2}{10^6 mm^2} = \underline{8.35 kN}$$

14-4 FIXED ENDS : $Le/r = (0.65)(800)/(20/4) = 104$: $C_c = 110$: SHORT COLUMN

$$P_{CR} = (314)(331)\left[1 - \frac{(331\times10^6)(104)^2}{4\pi^2(207\times10^9)}\right] = \underline{58.4 kN}$$

14-5 $A = 314 \text{ mm}^2 = b^2$: $b = \sqrt{314} = 17.7$ mm ; $r = b/\sqrt{12} = 5.12$ mm

$Le/r = (1.0)(800)/5.12 = 156$: $C_c = 138$: LONG COLUMN

$$P_{CR} = \frac{\pi^2(207\times10^9)(314)}{(156)^2(10^6)} = \underline{26.2 kN}$$

14-6 1 IN SCH 40 PIPE : $r = 0.421$ IN (25.4 mm/IN) = 10.69 mm

$A = 0.494 \text{ IN}^2 \times (25.4)^2 \text{ mm}^2/\text{IN}^2 = 318.7 \text{ mm}^2$

$S_y = 331$ MPa ; $C_c \approx 110$ – FIG 14-3.

(a) FIXED ENDS : $Le/r = (0.65)(2050 mm)/10.69 mm = 124.6$ – LONG

$$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2(207\times10^3 N/mm^2)(318.7 mm^2)}{(124.6)^2} = \underline{41.5 kN}$$

(b) FIXED-PINNED : $Le/r = (0.80)(2050)/10.69 = 153$ – LONG

$$P_{CR} = \frac{\pi^2(200\times10^3 N/mm^2)(318.7 mm^2)}{(153)^2} = \underline{27.8 kN}$$

(c) PINNED : $Le/r = 1.0(2050)/10.69 = 192$ – LONG

$$P_{CR} = \frac{\pi^2(207\times10^3)(318.7)}{(192)^2} = 17.7 kN$$

(d) FIXED-FREE : $Le/r = (2.0)(2050)/10.69 = 403$ LONG

$$P_{CR} = \frac{\pi^2(207\times10^3)(318.7)}{(403)^2} = \underline{4.01 kN}$$

14-7 $r_{MIN} = 12/\sqrt{12} = 3.46\,mm$; $L_e/r = 1.0(210)/3.46 = 60.6$

FOR $S_y = 469\,MPa$ - STEEL : $C_c = 90$ - SHORT COLUMN

$A = (12)(25) = 300\,mm^2$

$$P_{CR} = 300\,mm^2 (469\,N/mm^2)\left[1 - \frac{469\times10^6 Pa\,(60.6)^2}{4\pi^2(207\times10^9 Pa)}\right] = \underline{111\,kN}$$

14-8 FOR A36 STRUCTURAL STEEL : $S_y = 248\,MPa$ - $C_c = 122$; $E = 200\times10^3\,N/mm^2$

FOR S6×12.5 : $r_{MIN} = \sqrt{\frac{I_Y}{A}} = \sqrt{\frac{1.82\,in^4}{3.67\,in^2}} = 0.704\,in \times 25.4\,mm/in = 17.9\,mm$

$L_e/r = (0.65)(5450)/17.9 = 198 > C_c$ · LONG : $A = 3.67\,in^2 \frac{(25.4)^2 mm^2}{in^2} = 2368\,mm^2$

$$P_a = \frac{\pi^2 E A}{1.92(L_e/r)^2} = \frac{\pi^2(200\times10^3)(2368)}{1.92(198)^2} = \underline{62.1\,kN}$$

14-9 3 IN SCH 40 PIPE : $r = 1.163\,IN$: $L_e/r = (2.1)(8\,FT)(12\,IN/FT)/1.163\,IN = 173$

$S_y = 30\,KSI$: $C_c = 138$ - LONG COLUMN : $E = 30\times10^6\,PSI$ FOR STEEL.

$$P_a = \frac{P_{CR}}{N} = \frac{\pi^2 E A}{N(L_e/r)^2} = \frac{\pi^2(30\times10^6)(2.228)}{2(173)^2} = 7318\,LB/COLUMN$$

$$\text{No. OF COLUMNS} = \frac{\text{TOTAL LOAD}}{7318\,LB/COL.} = \frac{(75\,LB/FT^2)(20\times40)FT^2}{7318\,LB/COL.} = 8.20 - \underline{\text{USE } 9}$$

14-10 I 10×8.646 : $r_{MIN} = 1.42\,in \times 25.4\,mm/in = 36.1\,mm$

$A = 7.352\,in^2 \times (25.4)^2\,mm^2/in^2 = 4743\,mm^2$

$L_e/r = 1.0(2800)/36.1 = 77.6$: FOR $S_y = 276\,MPa$ ALUM., $C_c = 70$ - LONG COLUMN

$$\text{(EQ 14-18b)}\quad P_a = \frac{352000(A)}{(L_e/r)^2} = \frac{352000(4743)}{(77.6)^2} = \underline{277\,kN}$$

14-11 $L_e/r = 1400/36.1 = 38.8$ - INTERMED. : (EQ 14-17b)

$$P_a = A[139 - 0.869(L_e/r)] = 4743\,mm^2[139 - 0.869(38.8)]\,N/mm^2 = \underline{499\,kN}$$

14-12 W8×15 : $r_{MIN} = \sqrt{I_Y/A} = \sqrt{3.41\,in^4/4.44\,in^2} = 0.876\,IN$

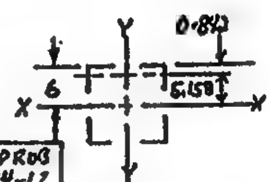
$$C_c = \sqrt{\frac{2\pi^2 E}{S_y}} = \sqrt{\frac{2\pi^2(29\times10^6\,PSI)}{36000\,PSI}} = 126$$

$L_e/r = (0.8)(12.5\,FT)(12\,IN/FT)/0.876\,IN = 137 > C_c$ - LONG

$$P_a = \frac{\pi^2 E A}{1.92(L_e/r)^2} = \frac{\pi^2(29\times10^6)(4.44)}{1.92(137)^2} = \underline{35265\,LB}$$

14-13

$I_{XX} = I_{YY} = 4[1.24 + 1.44(5.158)^2] = 158.2\,in^4$



$A = 4(1.44\,in^2) = 5.76\,in^2$

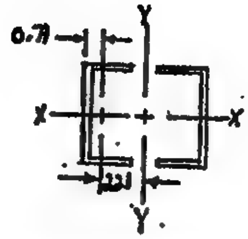
$r = \sqrt{I/A} = \sqrt{158.2/5.76} = 5.24\,in$

PROB 14-12

$L_e/r = (1.0)(18.4\,ft)(12\,in/ft)/5.24\,in = 42.1 < \boxed{C_c = 126}$ JOHNSON EQ.

$$P_a = \frac{P_{cr}}{N} = \frac{(5.76)(36000)}{3}\left[1 - \frac{36000(42.1)^2}{4\pi^2(29\times10^6)}\right] = \underline{65\,300\,lb}$$

14-14



$r_{xx} = 2.44 \text{ in}$

$I_{YY} = 2\left[1.53 + 2.41(2.21)^2\right] = 26.6 \text{ in}^4$

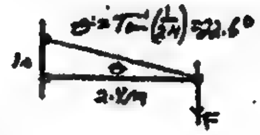
$r_{YY} = \sqrt{I/A} = \sqrt{26.6/(2 \times 2.41)} = 2.35 \text{ in}$

$L_e = L = 10.5 \text{ FT} (12 \text{ IN}/\text{FT}) = 126 \text{ IN.}$

$L_e/r = 126/2.35 = 53.6$ (INT.)

EQ (14-17a) $P_a = 2(2.41)\left[20.2 - 0.126(53.6)\right] = 64.8 \text{ kips} = \underline{64\,800 \text{ lb}}$

14-15

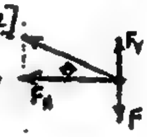


$F = mg = 1320 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 12950 \text{ N} = F_V$

$F_H = F/\tan\theta = 12950/\tan 22.6 = 31110 \text{ N}$

FOR S6×12.5, $r = 0.705 \text{ in} \times 25.4 \text{ mm/in} = 17.9 \text{ mm}$

$L_e/r = 2400/17.9 = 134$ – LONG [$C_c = 126$ FROM 14-12].



$$P_{cr} = \frac{\pi^2 EA}{(L_e/r)^2} = \frac{\pi^2 (207 \times 10^3 \text{ N/mm}^2)(2368 \text{ mm}^2)}{(134)^2} = 2.68 \times 10^5 \text{ N}$$

$A = 2.67 \text{ in}^2 \times 645.16 \text{ mm}^2/\text{in}^2 = 2368 \text{ mm}^2$

IF $P_a = P_{cr}/N$

$$N = \frac{P_{cr}}{P} = \frac{2.68 \times 10^5 \text{ N}}{31110 \text{ N}} = \underline{8.37 \text{ OK}}$$

14-16 $r_{MIN} = \frac{0.125}{\sqrt{12}} = 0.036 \text{ IN} : A = 0.25 \times 0.125 = 0.0313 \text{ IN}^2$

FOR 1040 CD: $S_y = 82 \text{ KSI} - C_c = 83 : \frac{L_e}{r} = \frac{8.40}{0.036} = 233 > C_c$ – LONG COLUMN

$P_{CR} = \frac{\pi^2 EA}{(L_e/r)^2} = \frac{\pi^2(30 \times 10^6)(0.0313)}{(233)^2} = \underline{171 \text{ LB}}$

$N = \frac{P_{CR}}{P} = \frac{171 \text{ LB}}{50 \text{ LB}} = \underline{3.41 \text{ OK}}$

14-17 FOR AISI 1141 OQT 1300: $S_y = 469 \text{ MPa} : C_c = 95 : A = \frac{\pi D^2}{4} = \frac{\pi(12)^2}{4} = 113 \text{ mm}^2$

$r = \frac{D}{4} = \frac{12}{4} = 3.0 \text{ mm} : \frac{L_e}{r} = \frac{(0.8)(190)}{3.0} = 50.7 < C_c$ – SHORT COLUMN

$P_{CR} = (113 \text{ mm}^2)(469 \text{ N/mm}^2)\left[1 - \frac{469 \text{ MPa}(50.7)^2}{4\pi^2(207 \times 10^3 \text{ MPa})}\right] = \underline{45.2 \text{ kN}}$

FOR $N = 3$: $P_a = \frac{P_{CR}}{3} = \frac{45.2 \text{ kN}}{3} = \underline{15.1 \text{ kN}}$

14-18 FOR AISI 1020 HR; $S_y = 48 \text{ ksi}$; $C_c \approx 105$

$r = \frac{D}{4} = \frac{0.800 \text{ IN}}{4} = 0.200 \text{ IN} : \frac{L_e}{r} = \frac{28.5}{0.20} = 142.5$ – LONG $: A = \frac{\pi(0.8)^2}{4} = 0.503 \text{ IN}^2$

$P_{CR} = \frac{\pi^2 EA}{(L_e/r)^2} = \frac{\pi^2(30 \times 10^6)(0.503)}{(142.5)^2} = \underline{7334 \text{ LB}} : N = \frac{P_{CR}}{P} = \frac{7334}{1375} = \underline{5.33 \text{ OK}}$

14-19 2 IN SCH 40 PIPE: $r = 0.787 \text{ IN} : A = 1.075 \text{ IN}^2$; FIXED/PINNED – $K = 0.8$

$\frac{L_e}{r} = \frac{(0.8)(14 \text{ FT})(12 \text{ IN/FT})}{0.787} = 171$: FOR AISI 1040 HR, $S_y = 48 \text{ ksi} - C_c = 105$ – LONG

$P_{CR} = \frac{\pi^2 EA}{(L_e/r)^2} = \frac{\pi^2(30 \times 10^6)(1.075)}{(171)^2} = \underline{10914 \text{ LB}}$

EACH COLUMN CARRIES 5000 LB : $\underline{N} = \frac{P_{CR}}{P} = \frac{10914 \text{ LB}}{5000 \text{ LB}} = \underline{2.18 \text{ – MARGINAL}}$

14-20 LACK OF RESTRAINT AT TOP OF COLUMNS MAKE THEM FREE.

FOR FIXED-FREE: $\frac{L_e}{r} = \frac{(2.0)(14 \text{ FT})(12 \text{ IN/FT})}{0.787 \text{ IN}} = 448$ – VERY LONG

$P_{CR} = \frac{\pi^2 EA}{(L_e/r)^2} = \frac{\pi^2(30 \times 10^6)(1.075)}{(448)^2} = \underline{1584 \text{ LB}} : P_{ACTUAL} = 5000 \text{ LB}$ – FAILURE

14-21

$r = 1.25 \text{ in}/\sqrt{12} = 0.361$; $A = (1.25 \text{ in})^2 = 1.563 \text{ in}^2$

COLUMNS ARE FIXED-FREE ; $L_e = 2(42) = 84 \text{ in.}$

$L_e/r = 84/0.361 = 233$ – LONG

ALUMINUM 6061-T6 ; $S_y = 40000 \text{ psi}$; $C_c \approx 70$

$\underline{P_{cr}} = \frac{\pi^2 EA}{(L_e/r)^2} = \frac{\pi^2(10 \times 10^6)(1.563)}{(233)^2} = \underline{2844 \text{ lb}}$

EACH COLUMN CARRIES 1500 LB

$\underline{N} = P_{cr}/P = 2844/1500 = \underline{1.90 \text{ – LOW}}$

14-22 ASSUME FIXED-PINNED ENDS : $r_{MIN} = \frac{2.00}{\sqrt{12}} = 0.577$ IN

$\frac{Le}{r} = \frac{(0.80)(12.75\,FT)(12\,IN/FT)}{0.577\,IN} = 212$: AISI 1040 OQT 1100; $S_y = 80$ KSI $- C_c = 85$ $\underline{LONG}$

$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2(30 \times 10^6\,LB/IN^2)(6.0\,IN^2)}{(212)^2} = \underline{39525\ LB}$

$N = \frac{P_{CR}}{P} = \frac{39525}{12500} = \underline{3.16\ OK}$

THIS IS PROBABLY CONSERVATIVE BECAUSE PIN MAY PROVIDE SOME RESTRAINT AGAINST BUCKLING WITH RESPECT TO VERTICAL AXIS. THUS Le/r WOULD BE SMALLER; P_{CR} AND N WOULD BE LARGER.

14-23 ASSUME A LONG COLUMN : EQ. 14-5 : $Le = (0.8)(12.75\,FT)(12\,IN/FT) = 122$ IN

REQD $I = \frac{NPLe^2}{\pi^2 E} = \frac{(4.0)(12500\,LB)(122\,IN)^2}{\pi^2(30 \times 10^6\,LB/IN^2)} = 2.53\,IN^4 = \frac{\pi D^4}{64}$

$\underline{D_{MIN}} = \left[\frac{64(2.53)}{\pi}\right]^{1/4} = \underline{2.68\,IN}$: $r = \frac{D}{4} = \frac{2.68}{4} = 0.670$; $\frac{Le}{r} = \frac{122}{0.67} = \underline{182 - LONG}$

14-24 LET $E = 29 \times 10^6$ PSI FOR STRUCTURAL STEEL : $S_y = 36$ KSI : $C_c \cong 130$

ASSUME COLUMN IS LONG : $I_{MIN} = \frac{NPLe^2}{\pi^2 E} = \frac{(4.0)(12500)(122)^2}{\pi^2(29 \times 10^6)} = 2.60\,IN^4$

USE 3 IN SCH. 40 : $I = 3.017\,IN^4$; $r = 1.163$ IN : $\frac{Le}{r} = \frac{122}{1.163} = 105 < C_c$ - SHORT

$P_a = \frac{AS_y}{N}\left[1 - \frac{S_y(Le/r)^2}{4\pi^2 E}\right] = \frac{(2.228\,IN^2)(36000\,LB/IN^2)}{4}\left[1 - \frac{(36000)(105)^2}{4\pi^2(29 \times 10^6)}\right] = \underline{13114\,LB}$ $\underline{OK}$

14-25 FOR BUCKLING ABOUT X-X AXIS - FIXED-PINNED : $Le = (0.8)(12.75\,FT)(12\,IN/FT) = 122$ IN

FOR Y-Y AXIS - FIXED-FIXED : $Le = 0.65(12.75)(12) = 99.5$ IN : ASSUME LONG COLUMN

$I_{X_{MIN}} = \frac{NPLe^2}{\pi^2 E} = \frac{4(12500)(122)^2}{\pi^2(10 \times 10^6)} = 7.54\,IN^4$ — I5X3.700 BEAM SHAPE REQD

$I_{Y_{MIN}} = \frac{4(12500)(99.5)^2}{\pi^2(10 \times 10^6)} = 5.02\,IN^4$ — $\underline{I\ 7 \times 5.800\ REQD.}$; $r_y = 1.08$ IN

$\frac{Le}{r_y} = \frac{99.5\,IN}{1.08\,IN} = 92.1$: FOR 6061-T6 : $S_y = 40$ KSI ; $C_c = 70$ - LONG COLUMN - OK

14-26 $Le = 0.8L = 0.8(16.5\,FT)(12\,IN/FT) = 158$ IN : $3 \times 3 \times \frac{1}{4}$ — $r = 1.10$ IN; $A = 2.59\,IN^2$

$Le/r = 158/1.10 = 144$: FOR ASTM A500, GRADE B : $S_y = 46$ KSI $- C_c = 110$ - LONG

$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2(29 \times 10^6)(2.59)}{(144)^2} = 35750$ LB : $P_a = \frac{P_{CR}}{N} = \frac{35750}{3} = \underline{11917\,LB}$

14-27 $Le = 0.8L = 0.8(16.5)(12) = 158$ IN : FOR $4 \times 2 \times \frac{1}{4}$, $r_{MIN} = 0.77$ IN; $A = 2.59\,IN^2$

$Le/r = 158/0.77 = 205$: $C_c = 110$ (PROB 14-28) - LONG

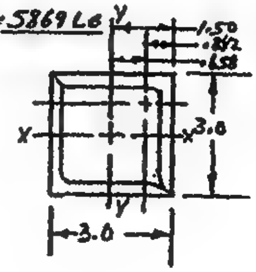
$P_{CR} = \frac{\pi^2(29 \times 10^6)(2.59)}{(205)^2} = 17606$ LB : $P_a = \frac{P_{CR}}{N} = \frac{17606}{3} = \underline{5869\,LB}$

14-28 FROM 14-26 : $Le = 158$ IN : $S_y = 36$ KSI $- C_c = 130$

$I_x = I_y = 2\left[I + Ad^2\right] = 2\left[1.24 + 1.44(0.658)^2\right] = 3.73\,IN^4$

$r = \sqrt{I/A} = \sqrt{3.73/2.88} = 1.138$ IN. ; $\frac{Le}{r} = \frac{158}{1.138} = 139$ - LONG

$P_a = \frac{P_{CR}}{N} = \frac{\pi^2(29 \times 10^6)(2.88)}{(3)(139)^2} = \frac{42730\,LB}{3} = \underline{14243\,LB}$



14-29 AISI 1020 HR: $S_y = 331\ \text{MPa}$: $C_c = 105$: $r = \frac{40}{\sqrt{12}} = 11.55\ \text{mm}$

FIXED-FIXED: $L_e = 0.65(750) = 488\ \text{mm}$: $\frac{L_e}{r} = \frac{488}{11.55} = 42.2$ – SHORT

$$P_{cr} = A S_y\left[1 - \frac{S_y(L_e/r)^2}{4\pi^2 E}\right] = (2400\ \text{mm}^2)(331\ \text{N/mm}^2)\left[1 - \frac{331\ \text{MPa}(42.2)^2}{4\pi^2(207\times10^3\ \text{MPa})}\right] = 737\ \text{kN}$$

LET $N = 3$: $P_a = \frac{P_{cr}}{N} = \frac{737\ \text{kN}}{3} = \underline{245\ \text{kN}}$

14-30 6061-T4: $S_y = 145\ \text{MPa}$ – $C_c = 98$: 4C×1.738 – $r_y = 0.64\ \text{IN}\ \frac{25.4\ \text{mm}}{\text{IN}} = 16.3\ \text{mm}$

$\frac{L_e}{r} = \frac{4250\ \text{mm}}{16.3\ \text{mm}} = 261$ – LONG: $A = 1.478\ \text{IN}^2\ \frac{(25.4)^2\ \text{mm}^2}{\text{IN}^2} = 954\ \text{mm}^2$

$$P_a = \frac{P_{cr}}{N} = \frac{\pi^2 E A}{N(L_e/r)^2} = \frac{\pi^2(69\times10^3\ \text{N/mm}^2)(954\ \text{mm}^2)}{4(261)^2} = \underline{2376\ \text{N}}$$

14-31 NO IMPROVEMENT BECAUSE COLUMN IS STILL LONG AND BUCKLING LOAD FROM EULER FORMULA IS INDEPENDENT OF STRENGTH. E IS THE SAME.

14-32 FOR ASTM A36: $S_y = 36\ \text{ksi}$, $E = 29\times10^6\ \text{psi}$: $C_c = \sqrt{\frac{2\pi^2 E}{S_y}} = 126$; $r_{min} = r_y = 3.02\ \text{in}$

$\frac{L_e}{r} = \frac{0.8(22.5\ \text{FT})(12\ \text{IN/FT})}{3.02\ \text{IN}} = 71.5 < C_c$ – USE EQS. 14-8, 14-9

$$FS = \frac{5}{3} + \frac{3(71.5)}{8(126)} - \frac{(71.5)^3}{8(126)^3} = 1.86$$

$$P_a = \left[1 - \frac{(L_e/r)^2}{2C_c^2}\right]\frac{S_y A}{FS} = \left[1 - \frac{71.5^2}{2(126)^2}\right]\frac{36000(19.1)}{1.86} = \underline{310{,}721\ \text{LB}}$$

COLUMN ANALYSIS - SUMMARY OF RESULTS OF PROBLEMS 14-33 TO 14-43

Prob. No.	*L*	*K*	L_e	s_y	*E*	C_c	*A*	*r*	*SR*	*N*	P_{cr}	*Eqn.*	P_a
14-33	163.2 in	0.65	106.2 in	36 ksi	29E06 psi	126	2.09 in^2	0.742	143	3	29267 lb	Euler	9756 lb
14-34	83.2 in	0.80	66.6 in	36 ksi	29E06 psi	126	2.09 in^2	0.742	89.7	3	56202 lb	Johnson	18734 lb
14-35	163.2 in	0.80	130.6 in	36 ksi	29E06 psi	126	2.09 in^2	0.742	176	3	19321 lb	Euler	6440 lb
14-36	163.2 in	0.65	106.1 in	36 ksi	29E06 psi	126	2.09 in^2	1.1	96.4	3	65974 lb	Johnson	21991 lb
14-37	2650 mm	1.00	2650 mm	248 MPa	200 GPa	126	1671 mm^2	19.6	135.5	3	179.7 kN	Euler	59.9 kN
14-38	2650 mm	1.00	2650 mm	248 MPa	200 GPa	126	1671 mm^2	27.9	9408	3	297 kN	Johnson	99.1 kN

L→3x3x1/4 STEEL TUBE

Problem 14-39 Truss Analysis + Design of Compression Members
Example data only. Many possible designs   Listing of compression members only
Design factor on load = 3.0

Member	*Load*	*L*	*Shape*	*Size*	*Material*	P_a
AC	1925 lb	40 in	Circular	15/16 in	ASTM A36	2253 lb
CD	750	25 in	Circular	9/16 in	ASTM A36	750 lb
CE	650	40 in	Circular	11/16 in	ASTM A36	654 lb

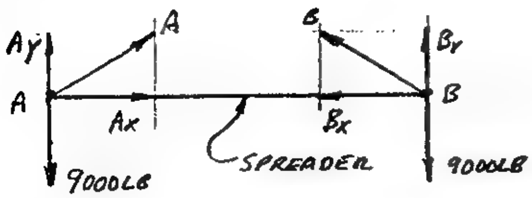
Problem 14-40 Truss Analysis + Design of Compression Members
Example data only. Many possible designs   Listing of compression members only
Design factor on load = 2.5

Member	*Load*	*L*	*Shape*	*Size*	*Material*	P_a
AC	4609 lb	120 in	Steel tube	4x3x1/4	ASTM A501	5694 lb
CG	3625 lb	120 in	Steel tube	3x3x1/4	ASTM A501	4012 lb
EH	1101 lb	154 in	Steel tube	3x3x1/4	ASTM A501	2446 lb
EG	3625 lb	120 in	Steel tube	3x3x1/4	ASTM A501	4012 lb
BE	4141 lb	120 in	Steel tube	4x3x1/4	ASTM A501	5694 lb

Problem 14-41 Truss Analysis + Design of Compression Members
Example data only. Many possible designs   Listing of compression members only
Design factor on load = 2.5

Member	*Load*	*L*	*Shape*	*Size*	*Material*	P_a
DE	2300 N	.25 m	Square	10 mm	Alum. 6061-T6	3600 N
BD	2597 N	.297 m	Square	10 mm	Alum. 6061-T6	2600 N
EF	2300 N	.20 m	Square	8 mm	Alum. 6061-T6	2324 N
FG	550 N	.15 m	Square	5 mm	Alum. 6061-T6	630 N
CF	800 N	.16 m	Square	6 mm	Alum. 6061-T6	1149 N

14-42 The support cables for the sling act at 30° to the horizontal and exert a direct axial compressive force on the spreader as shown below. Assume central loading of a straight column. The horizontal (axial) component of the cable force is 15 588 lb.



$A_y = B_y = 9000\,LB$

$A = B = 9000\,LB/SIN30° = 18000\,LB$

$A_x = B_x = 18000\,LB(COS30°) = 15588\,LB$

Design decision: Use a hollow steel tube made from ASTM A501 structural steel. The column buckling analysis spreadsheet (Figure 14-9) was used to determine that the lightest size with adequate capacity is a 3x3x1/4 hollow steel tube. Other results are summarized below.

$L = L_e = 96$ in; $r = 1.10$ in; SR = 87.3; $A = 2.59$ in^2; $s_y = 36\,000$ psi; $E = 29\times10^6$ psi; C_c = 126; Column is long; Use $N = 2.5$ (design decision); Column is short; Use Johnson formula; $P_{cr} = 70\,900$ lb; $P_a = 28\,364$ lb.

14-43 The analysis is similar to Problem 14-42. With the angle of 15°, the axial force on the tube is 33 588 lb. The spreader now must be a 4x4X1/4 steel tube with: $A = 3.59$ in^2; r = 1.51 in; SR = 63.6; Short column; From the Johnson formula, $P_{cr} = 112\,814$ lb; P_a = 45 126 lb..

Crooked Columns

For Problems 14-44 to 14-49, loading data were taken from earlier problems as listed in the problem statements. The amount of initial crookedness is given. The Crooked Column Analysis spreadsheet (Figure 14-14) was used to determine the critical buckling load and the allowable load for a design factor of 3.0. The spreadsheet solves Equation 14-18. Results are summarized in the table on the following page.

Eccentrically-Loaded Columns

For Problems 14-50 to 14-58, data from the problem statements were entered into the Eccentric Column Analysis spreadsheet (Figure 14-15). Where the problem asks for the maximum stress and deflection, the design factor $N = 1.0$ was entered at the lower left column. For design problems, the requested design factor (typically $N = 3.0$) was entered. Results are summarized in the table on the following page.

Problem 14-59

Straight and crooked column analysis required for the 2-in schedule 40 steel pipe, 156 in long. The spreadsheets in Figures 14-9 (Straight columns) and 14-14 (crooked columns) were used to determine the following results.

a) Straight pipe: SR = 198; $C_c = 126$; Long column; $P_{cr} = 7831$ lb; $P_a = 2610$ lb.
b) Crooked pipe: $a = 1.25$ in; C_1 in Eqn. 14-18 = -21 766; $C_2 = 3.36\times10^7$; Euler buckling load = 7831 lb; $P_a = 1676$ lb.

CROOKED COLUMN ANALYSIS - SUMMARY OF RESULTS OF PROBLEMS 14-44 TO 14-49

Prob.	a	L	K	L_e	s_y	E	C_c	A	r	SR	N	P_{cr} (Equation 14-18)	C_1 (Equation 14-18)	C_2 (Equation 14-18)	P_a
14-44	4.00 mm	800 mm	1.00	800 mm	331 MPa	207 GPa	111	314 mm^2	5.00 mm	160	3	25.06 kN	-56362	2.89E08	5.71 kN
14-45	1.60 mm	210 mm	1.00	210 mm	469 MPa	207 GPa	93.3	300 mm^2	3.46 mm	60.6	3	111 kN	-146962	2.61E09	20.6 kN
14-46	14.0 mm	2800 mm	1.00	2800 mm	276 MPa	69 GPa	70.2	4743 mm^2	36.1 mm	77.6	3	537 kN	-761829	7.81E10	122 kN
14-47	0.75 in	150 in	0.80	120 in	36 ksi	29E06 psi	126	4.44 in^2	0.876 in	137	3	67778 lb	-120164	1.204E09	11030 lb
14-48	1.25 in	163.2 in	0.65	106.1 in	36 ksi	29E06 psi	126	2.09 in^2	0.742 in	143	3	29267 lb	-56985	2.48E08	4678 lb
14-49	32.0 mm	2650 mm	1.00	2650 mm	248 MPa	200 GPa	126	1671 mm^2	19.6 mm	135	1.1	180 kN	-875101	6.00E10	75.04 kN

—> Iterated to find N for P_a = 75 kN

ECCENTRICALLY LOADED COLUMN ANALYSIS - SUMMARY OF RESULTS OF PROBLEMS 14-50 TO 14-58

Value of N in Eqn. 14-20 set equal to 1.0 to find maximum stress in column.

Prob.	e	L	K	L_e	s_y	E	C_c	A	r	SR	N	*Value of secant for: Stress*	*Value of secant for: Defl.*	*Stress*	y_{max}
14-50	0.60 in	42.0 in	1.00	42.0 in	21.0 ksi	10E06 psi	97	1.563 in^2	0.361 in	116	1	1.1527	1.1527	3456 psi	0.092 in
14-51	150 mm	3200 mm	1.00	3200 mm	331 MPa	207 GPa	111	1437 mm^2	29.5 mm	108	1	1.1719	1.1719	211 MPa	25.8 mm
14-52	0.30 in	14.75 in	1.00	14.75 in	40.0 ksi	28E06 psi	118	0.063 in^2	0.072 in	204	1	1.1511	1.1511	6687 psi	0.045 in

Problems 14-53 to 14-58 Eccentrically Loaded Column Analysis
Value of N = 3 used to evaluate safety

Prob.	e	L	K	L_e	s_y	E	C_c	A	r	SR	N	*Value of secant for: Stress*	*Value of secant for: Defl.*	*Reqd* s_y	y_{max}
14-53	0.50 in	40.0 in	1.00	40.0 in	50.0 ksi	29E06 psi	107	3.59 in^2	1.51 in	26.5	3	1.225	1.0667	96.3 ksi	0.033 in

Design is not safe because reqd s_y is greater than given s_y. Prop is redesigned to find lightest steel tube that is safe.

Prob.	e	L	K	L_e	s_y	E	C_c	A	r	SR	N	*Stress*	*Defl.*	*Reqd* s_y	y_{max}
14-53	0.50 in	40.0 in	1.00	40.0 in	50.0 ksi	29E06 psi	107	7.59 in^2	3.15 in	12.7	3	1.021	1.0069	35.7 ksi	0.003 in

Square steel tube 8x8x1/4. Assumed pinned ends. Would be safer if ends are flat and parallel to the press ram.

Prob.	e	L	K	L_e	s_y	E	C_c	A	r	SR	N	*Stress*	*Defl.*	*Reqd* s_y	y_{max}
14-54a	0.90 in	72 in	1.00	72.0 in	90.0 ksi	30E06 psi	81.1	1.28 in^2	0.231 in	312	1	1.4289	1.4289	8319 psi	0.386 in

This is the maximum stress in the column for an assumed design factor of N = 1.0.

Prob.	e	L	K	L_e	s_y	E	C_c	A	r	SR	N	*Stress*	*Defl.*	*Reqd* s_y	y_{max}
14-54b	0.9	72 in	1.00	72.0 in	90.0 ksi	30E06 psi	81.1	1.28 in^2	0.231 in	312	3	5.2212	1.4289	84.9 ksi	0.386 in

This gives the required yield strength (84.9 ksi) of the material for N = 3. Specify AISI 1040 WQT 900 steel; s_y = 90.0 ksi.

Problems 14-53 to 14-58 Eccentrically Loaded Column Analysis (Continued)

Value of N = 3 used to evaluate safety

												Value of secant for:			
Prob.	e	L	K	L_e	s_y	E	C_c	A	r	SR	N	Stress	Defl.	Reqd s_y	y_{max}
14-55	0.4645 in	112 in	1.00	112 in	36.0 ksi	29E06 psi	126	2.64 in^2	0.489 in	229	3	7.2758	1.4774	94.8 ksi	0.222 in

The design is not safe using a desired value of N = 3. Required yield strength is over 2.5 times higher that given yield strength.

The eccentricity is the distance from the centoidal axis and the middle of the flange width.

14-56	3.00 in	126 in	0.80	100.8	46.0 ksi	29E06 psi	112	6.36 in^2	1.39 in	72.5	3	1.2579	1.0753	46.0 ksi	0.226 in

The load for these data is 19,870 lb, found by iterating Equation 14-20 using the spreadsheet until the required yield strength became less than the given 46,000 psi.

14-57	1.75 in	40.0 in	1.00	40.0 in	40.0 ksi	10E06 psi	70.2	0.600 in^2	0.433 in	92.4	3	1.5127	1.1333	39.1 ksi	0.233 in

The load for these data is 675 lb, found by iterating Equation 14-20 using the spreadsheet until the required yield strength became less than the given 40,000 psi.

The analysis was done assuming that the loading in the plane of the drawing was critical. IT IS NOT! See the analysis below.

14-57b *Buckling about the thickness of the bar is now checked assuming that the load is centrally applied.*

Column analysis spreadsheet is used to determine allowable load for N = 3.

0	40.0 in	1.00	40.0 in	40.0 ksi	10E06 psi	70.2	0.60 in^2	0.1154 in	347	3	-	-	-	

The allowable load for buckling about this axis is only 164 lb. This is the limiting load.

14-58	20 mm	750 mm	1.00	750 mm	931 MPa	200 GPa	65.1	314 mm^2	7.29 mm	103	3	1.4512	1.1205	389 MPa	2.41 mm

The design is safe because the required yield strength is less than the actual yield strength of the given material.

Problem 14-59 has two-parts. The first analysis is for a straight pipe. The second analysis is for the crooked pipe.

												Equation 14-18			
Prob.	a	L	K	L_e	s_y	E	C_c	A	r	SR	N	P_{cr}	C_1	C_2	P_a
14-59a	0	156 in	1.00	156 in	36.0 ksi	29E06 psi	126	1.075 in^2	0.787 in	198	3	7831	-	-	2610 lb
14-59b	1.25 in	156 in	1.00	156 in	36.0 ksi	29E06 psi	126	1.075 in^2	0.787 in	198	3	7831	-21766	3.37E07	1676 lb

The allowable load decreases from 2610 lb to 1676 lb when the pipe is crooked.

CHAPTER 15 Pressure Vessels

15-1

$t = (200-184)/2 = 8.0\text{ mm}$

LET D = MEAN DIA. $= (D_o + D_i)/2 = (200+184)/2 = 192\text{ mm}$

$D/t = 192/8.0 = 24$ — THIN-WALLED SPHERE

$$\sigma = \frac{pD}{4t} = \frac{(19.2\text{ N/mm}^2)(192\text{ mm})}{4(8.0\text{ mm})} = \underline{115\text{ MPa}}$$

15-2

$D_m = D_o - t = 10\,500 - 12 = 10\,488\text{ mm}$

$\frac{D_m}{t} = \frac{10488\text{ mm}}{12\text{ mm}} = 874$ — VERY THIN WALL ; AISI 1040 HR

$\sigma_d = \frac{S_y}{4} = \frac{414\text{ MPa}}{4} = 103.5\text{ MPa}$; $\sigma_{max} = \frac{pD}{4t}$

$$p = \frac{4t\sigma_d}{D_m} = \frac{4(12\text{ mm})(103.5\text{ MPa})}{10488\text{ mm}} = \underline{0.474\text{ MPa}} = \underline{474\text{ kPa}}$$

15-3 $D_o = 1200\text{ mm}$ — ASSUME THIN WALL ; FOR Ti-6Al-4V, $S_y = 1070\text{ MPa}$

$\sigma_d = \frac{S_y}{4} = \frac{1070\text{ MPa}}{4} = 267\text{ MPa}$; $\sigma = \frac{pD_m}{4t} = \frac{p(D_o - t)}{4t} = \frac{pD_o - pt}{4t}$

$t = \frac{pD_o}{4\sigma_d + p} = \frac{(4.20\text{ MPa})(1200\text{ mm})}{4(267\text{ MPa}) + 4.2\text{ MPa}} = \underline{4.70\text{ mm}}$ USE $t = 5.00\text{ mm}$ CONVENIENT SIZE

$D_m/t \approx \frac{1195}{5} = 239$ — VERY THIN WALL

15-4 $\sigma_d = \frac{S_y}{4} = \frac{414\text{ MPa}}{4} = 103.5\text{ MPa}$

$t = \frac{pD}{4\sigma_d} = \frac{(4.20\text{ MPa})(1200\text{ mm})}{4(103.5\text{ MPa})} = \underline{12.2\text{ mm}}$; $D/t = 1200/12.2 = 98$ — THIN

MASS = DENSITY × VOLUME OF SPHERE

VOLUME $= 0.5236(D^3 - d^3)$; $D = 1200\text{ mm}$; $d = D - 2t$

Ti : $d = 1200 - 2(4.71\text{ mm}) = 1190\text{ mm}$

$V = 0.5236(1200^3 - 1190^3) = 2.114\times10^7\text{ mm}^3$

ALUM: $d = 1200 - (12.2)(2) = 1176\text{ mm}$

$V = 0.5236(1200^3 - 1176^3) = 5.39\times10^7\text{ mm}^3$

MASS OF Ti $= 4430\frac{\text{kg}}{\text{m}^3} \times 2.114\times10^7\text{ mm}^3 \times \frac{1\text{ m}^3}{(10^3\text{ mm})^3} = 93.6\text{ kg}$ LIGHTER

MASS OF ALUM: $2770\frac{\text{kg}}{\text{m}^3} \times 5.39\times10^7\text{ mm}^3 \times \frac{1\text{ m}^3}{(10^3\text{ mm})^3} = 149\text{ kg}$

15-5

$D_o = 10.750\text{ in}$; $D_i = 10.020\text{ in}$; $D_m = (D_o + D_i)/2 = 10.385\text{ in}$

$t = 0.365\text{ in}$; $D_m/t = 10.385/0.365 = 28.5$ (THIN)

$$\sigma = \frac{pD}{2t} = \frac{(50\text{ psi})(10.385\text{ in})}{2(0.365\text{ in})} = \underline{2134\text{ psi}}$$

15-6 $D_m = D_i + t = 80 + 3.5 = 83.5\ MM$; $D_m/t = 83.5/3.5 = 23.9$ THIN

$$\sigma = \frac{pD_m}{2t} = \frac{(1.85\ MPa)(83.5\ mm)}{2(3.5\ mm)} = \underline{34.0\ MPa}$$

15-7 ASSUME THIN WALL: $\sigma_d = \frac{s_y}{4} = \frac{565\ MPa}{4} = 141.3\ MPa = \frac{pD}{2t}$

$$t = \frac{pD}{2\sigma_d} = \frac{(1.7\ MPa)(300\ mm)}{2(141.3\ MPa)} = \underline{1.80\ mm}$$

$D_m = D_o - t = 300 - 1.81 = 298.2$; $D_m/t = 298.2/1.80 = 164$ - VERY THIN

15-8 $t = \frac{pD}{2\sigma_d} = \frac{(15.2\ MPa)(250\ mm)}{2(141.3\ MPa)} = \underline{13.45\ mm}$

$D_m = D_o - t = 250 - 13.45 = 236.5\ mm$; $D_m/t = 236.5/13.45 = 17.6$ - THICK WALL

USE EQ. FOR σ_2 FROM TABLE 15-1. TRY $t = 14.0\ mm$

$D_i = D_o - 2t = 250 - 2(14) = 222\ mm$; $a = 111\ mm$, $b = 125\ mm$.

$$\sigma_2 = \frac{p(b^2+a^2)}{b^2-a^2} = \frac{(15.2\ MPa)(125^2+111^2)}{(125^2-111^2)} = 128.6\ MPa$$

OK FOR $\sigma_d = 141.3\ MPa$ BUT SOMEWHAT LOW.

FOR $t = 13.0\ mm$; $D_i = 224\ mm$; $a = 112\ mm$, $b = 125\ mm$

$\sigma = \underline{139.0\ MPa}$ $\underline{OK}$ USE $t = 13.0\ mm$

15-9 $D_m = D_o - t = 450 - 2.20 = 447.8\ mm$; $D_m/t = 447.8/2.20 = 204$ - THIN

$$\sigma_{max} = \frac{pD_m}{2t} = \frac{(750 \times 10^3\ Pa)(447.8\ mm)}{2(2.20\ mm)} = \underline{76.3\ MPa}$$

$$N = \frac{s_y}{\sigma} = \frac{290\ MPa}{76.3\ MPa} = \underline{3.80}$$

15-10 ASSUME THIN WALL: $\sigma_d = s_y/4 = 414/4 = 103\ MPa = \frac{pD}{2t}$

$$t = \frac{pD}{2\sigma_d} = \frac{(750 \times 10^3\ Pa)(1800\ mm)}{2(103 \times 10^6\ MPa)} = \underline{6.55\ mm}$$ LET $t = 7.0\ mm$

$D_m = D_o - t = 1800 - 7.0 = 1793\ mm$; $D_m/t = 1793/7.0 = 256$ VERY THIN

15-11 $D_m = D_o - t = 250 - 18 = 232\ mm$; $D_m/t = 232/18 = 12.9 < 20$ - THICK WALL

$b = 250/2 = 125\ mm$; $a = b - t = 125 - 18 = 107\ mm$

$$\sigma_{2max} = \frac{p(b^3+2a^3)}{2(b^3-a^3)} = \frac{(70.0\ MPa)(125^3+2(107)^3)}{2(125^3-107^3)} = \underline{212\ MPa}\ \underline{\text{TANGENTIAL}}$$

$\sigma_{3max} = -p = \underline{-70.0\ MPa}$ $\underline{\text{RADIAL}}$

15-12

$D_o = 0.840\ in$; $D_i = 0.622$; $D_m = (D_o + D_i)/2 = 0.731$; $D_m/t = \frac{0.731}{0.109} = 6.71$ (THICK)

$b = D_o/2 = 0.420\ in$; $a = D_i/2 = 0.311\ in$

$$\sigma_1 = \frac{pa^2}{b^2-a^2} = \frac{(250\ psi)(0.311)^2}{(0.420)^2-(0.311)^2} = \underline{303\ psi\ \text{LONGITUDINAL}}$$

$$\sigma_{2max} = \frac{p(b^2+a^2)}{b^2-a^2} = \frac{(250\ psi)(0.420^2+0.311^2)}{0.420^2-0.311^2} = \underline{857\ psi\ \text{HOOP}}$$

$\sigma_3 = -p = \underline{-250\ psi}$ $\underline{\text{RADIAL}}$

15-13

$$\frac{D_m}{t} = \frac{(300+220)/2}{(300-220)/2} = 6.5 \quad (\text{THICK})\ a = 110\text{ mm};\ b = 150\text{ mm}$$

$$\sigma_t = \frac{p a^2 (b^2 + r^2)}{r^2 (b^2 - a^2)} = \frac{b^2 + r^2}{r^2} \times \frac{p a^2}{b^2 - a^2} = \frac{b^2 + r^2}{r^2} \times \frac{(50\text{ MPa})(110)^2}{150^2 - 110^2} = 58.17 \left[\frac{b^2 + r^2}{r^2}\right]$$

r	σ_t HOOP STRESS
110 mm	166 MPa — INNER SURFACE — MAXIMUM
120	149 MPa
130	136 MPa
140	125 MPa
150	116 MPa — OUTER SURFACE

15-14

$D_m = (D_o + D_i)/2 = (1.900 + 1.610)/2 = 1.755 \quad D_m/t = 1.755/0.145 = 12.1$ (THICK)

USING THICK WALLED EQN. $b = 1.90/2 = 0.95$; $a = 1.61/2 = 0.805$

$$\sigma_{t_{MAX}} = \frac{p(b^2 + a^2)}{b^2 - a^2} = \frac{(10.0\text{ MPa})(.95^2 + .805^2)}{(.95^2 - .805^2)} = \underline{60.9\text{ MPa}}$$

USING THIN-WALLED EQN.

$$\sigma_t = \frac{pD}{2t} = \frac{(10.0\text{ MPa})(1.755\text{ mm})}{2(0.145\text{ mm})} = \underline{60.5\text{ MPa}}$$

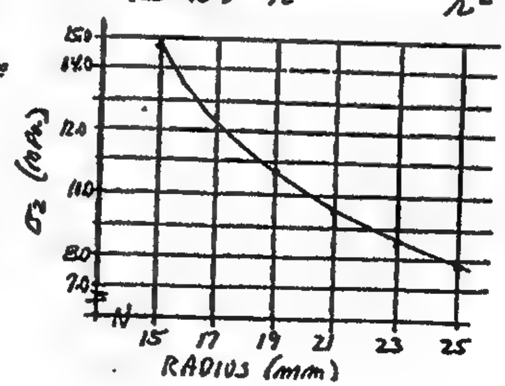
15-15 $D_m = (D_o + D_i)/2 = (50 + 30)/2 = 40\text{ mm}$: $t = (D_o - D_i)/2 = (50 - 30)/2 = 10\text{ mm}$

$D_m/t = 40/10 = 4.0$ — THICK : $b = D_o/2 = 25\text{ mm}$: $a = D_i/2 = 30/2 = 15\text{ mm}$

$$\sigma_{t_{MAX}} = \frac{p(b^2 + a^2)}{b^2 - a^2} = \frac{(0.0\text{ MPa})(25^2 + 15^2)}{(25^2 - 15^2)} = \underline{14.88\text{ MPa}} \text{ TANGENTIAL}$$

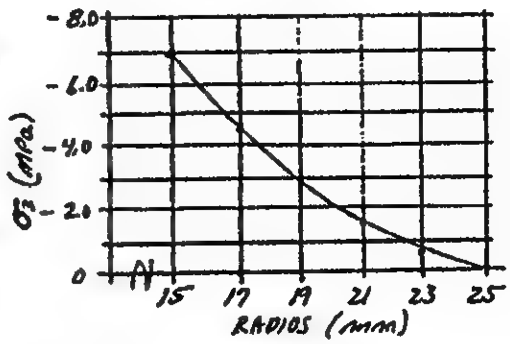
15-16 $$\sigma_t = \frac{p a^2 (b^2 + r^2)}{r^2 (b^2 - a^2)} = \frac{p a^2}{b^2 - a^2} \times \frac{(b^2 + r^2)}{r^2} = \frac{7.0(15)^2}{(25^2 - 15^2)} \times \frac{(b^2 + r^2)}{r^2} = 3.94 \times \frac{(b^2 + r^2)}{r^2}$$

r	σ_t (MPa)
15	14.88 INNER SURFACE
17	12.45
19	10.75
21	9.52
23	8.59
25	7.88 OUTER SURFACE



15-17 $\sigma_3 = \frac{-p a^2 (b^2 - r^2)}{r^2 (b^2 - a^2)} = \frac{-p a^2}{b^2 - a^2} \times \frac{b^2 - r^2}{r^2} = \frac{-(7.0)(15^2)}{(25^2 - 15^2)} \times \frac{b^2 - r^2}{r^2} = 3.94 \frac{b^2 - r^2}{r^2}$

r	σ_3 (MPa) RADIAL
15	−7.00 INNER SURFACE
17	−4.58
19	−2.88
21	−1.64
23	−0.71
25	0 OUTER SURFACE



15-18 ASSUMING THIN WALL THEORY

$\sigma_2 = \frac{p D_m}{2t} = \frac{(7.0\,MPa)(40\,mm)}{2(10\,mm)} = \underline{14.0\,MPa}$ (5.9% LOW)

FROM PROB. 15-15: ACTUAL $\sigma_{2_{MAX}} = 14.88\,MPa$

15-19 $D_m = D_o - t = 500 - 40 = 460\,mm$; $D_m/t = 460/40 = 11.5 < 20$ - THICK

$b = D_o/2 = 250\,mm$; $a = D_i/2 = 210\,mm$

$\sigma_d = \frac{S_y}{4} = \frac{931\,MPa}{4} = 232\,MPa$ AISI 501 OQT 1000

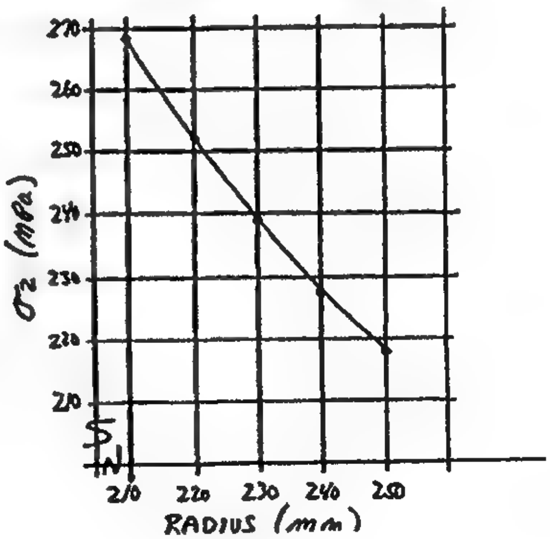
$\sigma_{MAX} = \sigma_{2_{MAX}} = \frac{p(b^3 + 2a^3)}{2(b^3 - a^3)}$; $p = \frac{2\sigma_d (b^3 - a^3)}{(b^3 + 2a^3)} = \frac{2(232\,MPa)(250^3 - 210^3)}{[250^3 + 2(210^3)]}$

$\underline{p_{MAX} = 86.8\,MPa}$

15-20 FROM PROB. 15-19: THICK WALL: $b = 250\,mm$, $a = 210\,mm$

$\sigma_1 = \sigma_2 = \frac{p a^3 (b^3 + 2r^3)}{2 r^3 (b^3 - a^3)} = \frac{p a^3}{2(b^3 - a^3)} \times \frac{(b^3 + 2r^3)}{r^3} = \frac{100\,MPa (210)^3}{2(250^3 - 210^3)} \times \frac{(b^3 + 2r^3)}{r^3}$

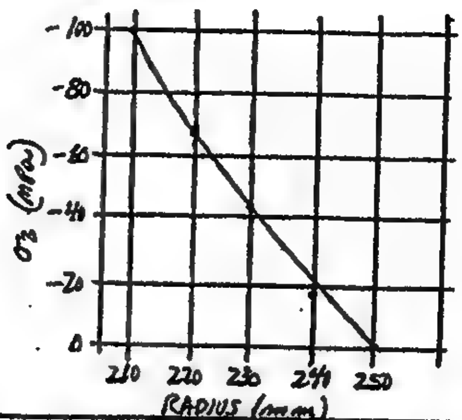
r	σ_2 (MPa) TANGENTIAL
210	268 INNER SURFACE
215	260
220	252
225	245
230	239
235	233
240	228
245	223
250	218 OUTER SURFACE



15-21 FROM PROB. 15-17: THICK WALL SPHERE: $b = 250$ mm, $a = 210$ mm

$$\sigma_3 = \frac{-pa^3(b^3 - r^3)}{r^3(b^3 - a^3)} = \frac{-pa^3}{(b^3 - a^3)} \times \frac{(b^3 - r^3)}{r^3} = \frac{-(100\,MPa)(210^3)}{(250^3 - 210^3)} \times \frac{(250^3 - r^3)}{r^3}$$

r	σ_3 (MPa) RADIAL
210	-100 INNER SURFACE
215	-83.3
220	-68.0
225	-54.1
230	-41.4
235	-29.7
240	-19.0
245	-9.1
250	0.0 OUTER SURFACE

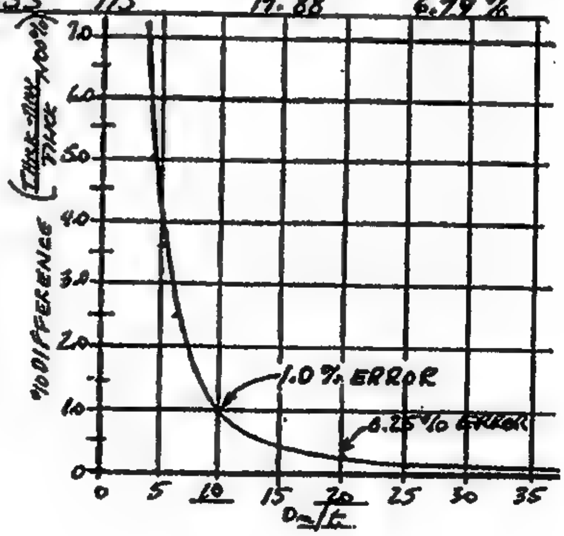


15-22 $D_o = 400$ mm

t (mm)	$D_m = D_o - t$ mm	D_m/t	THIN $\sigma_2 = \frac{pD_m}{2t}$	$a = \frac{D_o - 2t}{2}$	THICK $\sigma_2 = \frac{p(b^2+a^2)}{(b^2-a^2)}$	% = $\frac{\text{THICK}-\text{THIN}}{\text{THICK}} \times 100\%$
5	395	79	395 MPa	195	395.06 MPa	0.015%
15	385	25.67	128.3	185	128.53	0.179%
19.05	380.95	20.0	100.0	180.95	100.25	0.25%
25	375	15.0	75.0	175	75.33	0.44%
35	365	10.43	52.14	165	52.62	0.91%
36.36	363.63	10.0	50.0	163.63	50.50	0.99%
45	355	7.89	39.44	155	40.08	1.60%
55	345	6.27	31.36	145	32.16	2.49%
65	335	5.15	25.77	135	26.74	3.63%
75	325	4.33	21.67	125	22.82	5.04%
85	315	3.71	18.53	115	19.88	6.79%

NOTES:

- $D_m/t = 20.0$ ADDED TO SHOW THAT ARBITRARY DIVISION BETWEEN THICK AND THIN WALL CYLINDERS RESULTS IN LESS THAN 0.25% ERROR FOR THIN-WALLED THEORY.
- FOR $D_m/t > 10$, ERROR IS LESS THAN 1.0%
- ERROR INCREASES RAPIDLY FOR $D_m/t < 10$.



15-23 $D_o = 400\,mm$: $D_i = 325\,mm$: $D_m = 362.5\,mm$: $D_m/t = 9.67$ – THICK

$t = 37.5\,mm$: $b = D_o/2 = 200\,mm$: $a = D_i/2 = 162.5\,mm$

$$\sigma_2 = \frac{p a^3 (b^3 + 2r^3)}{2r^3(b^3 - a^3)} \; : \; p = 10.0\,MPa$$

r (mm)	σ_2 (MPa)	σ_3 (MPa) PROB 15-24
162.5 = a	22.35	10.0
170.0	20.99	7.27
177.5	19.84	4.98
185.0	18.88	3.05
192.5	18.06	1.41
200.0 = b	17.35	0.00

15-24 SAME DATA AS 15-23 : $\sigma_3 = \dfrac{-p a^3 (b^3 - r^3)}{r^3 (b^3 - a^3)}$

15-25 SCHEDULE 40 PIPE : APP. A-12 : $D_m = (D_o + D_i)/2$

NOM. SIZE	D_m (IN)	t (IN)	D_m/t	
1/8	.337	.068	4.96	THICK
1/4	.452	.088	5.14	
3/8	.584	.091	6.42	
1/2	.731	.109	6.71	
3/4	.937	.113	8.29	
1	1.182	.133	8.89	
1 1/4	1.520	.140	10.86	
1 1/2	1.755	.145	12.10	
2	2.221	.154	14.42	
2 1/2	2.672	.203	13.16	

NOM SIZE	D_m (IN)	t (IN)	D_m/t	
3	3.284	0.216	15.20	THICK
3 1/2	3.774	.226	16.70	
4	4.263	.237	17.99	
5	5.305	.258	20.56	THIN
6	6.345	.280	22.66	
8	8.303	.322	25.79	
10	10.385	.365	28.45	
12	12.344	.406	30.40	
16	15.50	.500	31.00	
18	17.438	.562	31.03	

16-1 (a) FIG. 16-9. (a) A502, GR1 RIVETS, A36 PLATE.

$F_s = \tau_a \cdot A_s = (17500\ LB/IN^2)(2)(\pi(0.5IN)^2/4) = \underline{6872\ LB} = \underline{F_{ALLOWABLE}}$

$F_b = \sigma_{ba} \cdot A_b = 1.2(58000)(2)(0.5)(0.375) = 26\ 100\ LB$

$F_t = \sigma_{ta} \cdot A_t = 0.6(36000)[3-2(0.5+0.063)](0.375) = \underline{15179\ LB}$

16-1(b) FIG. 16-9 (b)

$F_s = (17500)(3)(\pi(.375)^2/4) = \underline{5798\ LB = F_{ALL}}$

$F_b = (1.20)(58000)(3)(0.375)(0.375) = \underline{29363\ LB}$

$F_t = (0.6)(36000)[3-3(0.375+0.063)](0.375) = \underline{13657\ LB}$

16-1 (c) FIG 16-9 (c)

$F_s = (17500)(4)(\pi(0.375)^2/4) = \underline{7732\ LB = F_{ALL}}$

$F_b = (1.20)(58000)(2)(0.375)(0.375) = \underline{19575\ LB}$

$F_t = (0.6)(36000)[3-2(0.375+0.063)](0.375) = \underline{17204\ LB}$

16-1(d) FIG. 16-9 (d)

SAME AS 16-1(c): $\underline{F_s = 7732\ LB = F_{ALL}}$

16-2 (a) FIG. 16-10(a) A502, GR2 RIVETS, A242 PLATE

$F_s = (22000)(4)(\pi(0.5)^2/4) = \underline{17279\ LB = F_{ALL}}$

$F_b = (1.20)(70000)(4)(0.5)(0.5) = \underline{84000\ LB}$

$F_t = (0.6)(50000)[4-2(0.5+0.163)](0.5) = \underline{43110\ LB}$

16-2 (b) FIG. 16-10(b)

$F_s = (22000)(6)(2)(\pi(0.375)^2/4) = \underline{29158\ LB = F_{ALL}}$

$F_b = (1.20)(70000)(6)(0.375)(0.50) = \underline{94500\ LB}$

$F_t = (0.6)(50000)[4-3(0.375+0.063)](0.50) = \underline{40290\ LB}$

16-2 (c) FIG 16-10 (c)

$F_s = (22000)(4)(2)(\pi(0.5)^2/4) = \underline{34558\ LB = F_{ALL}}$

$F_b = (1.20)(70000)(4)(0.5)(0.5) = \underline{84000\ LB}$

$F_t = (0.6)(50000)[4-2(0.5+0.063)](0.50) = \underline{43110\ LB}$

16-2(d) FIG. 16-10 (d)

$F_s = (22000)(2)(2)(\pi(0.75)^2/4) = \underline{38877\ LB}$

$F_b = (1.20)(70000)(2)(0.75)(0.5) = \underline{63000\ LB}$

$F_t = (0.6)(50000)[4-2(0.75+0.063)](0.5) = \underline{35610\ LB = F_{ALL}}$

16-3 (a) FIG. 16-9. (a) A325 BOLTS; A441 STEEL; BEARING TYPE

$F_s = (30000)(2)(\pi(0.5)^2/4) = \underline{11780\ LB} = F_{ALLOW.}$

$F_b = (1.20)(70\,000)(2)(0.50)(0.375) = \underline{31500\ LB}$

$F_t = 0.6(50\,000)(3.0 - 2(0.5 + 0.063))(0.375) = \underline{21083\ LB}$

16-3 (b) FIG. 16-9 (b)

$F_s = (30000)(3)(\pi(0.375)^2/4) = \underline{9940} = F_{ALLOW.}$

$F_b = (1.20)(70\,000)(3)(0.375)(0.375) = \underline{35450\ LB}$

$F_t = 0.6(50\,000)(3.0 - 3(0.375 + 0.063))(0.375) = \underline{18968\ LB}$

16-3 (c) FIG. 16-9. (c)

$F_s = (30\,000)(2)(2)(\pi(0.375)^2/4) = \underline{13253\ LB} = F_{ALLOW.}$

$F_b = (1.20)(70000)(2)(0.375)(0.375) = \underline{23\,600\ LB}$

$F_t = 0.6(50000)(3.0 - 2(0.375 + 0.063))(0.375) = \underline{23895\ LB}$

16-3 (d) FIG. 16-9 (d) SAME AS 16-3 (c): $F_s = 13253\ LB = F_{ALLOW.}$

16-4 (a) FIG. 16-10 (a) A490 BOLTS; A514 STEEL; FRICTION TYPE

$F_s = (22000)(4)(\pi(0.5)^2/4) = \underline{17279\ LB} = F_{ALLOW.}$

$F_t = 0.6(100\,000)[4.0 - 2(0.50 + 0.063)](0.50) = \underline{86220\ LB}$

BEARING STRESS NOT CONSIDERED IN FRICTION-TYPE JOINT.

16-4 (b) FIG. 16-10 (b)

$F_s = (22000)(6)(2)(\pi(0.375)^2/4) = \underline{29158\ LB} = F_{ALLOW.}$

$F_t = 0.6(100\,000)[4.0 - 3(0.375 + 0.063)](0.5) = \underline{80580\ LB}$

16-4 (c) FIG. 16-10 (c)

$F_s = (22\,000)(4)(2)(\pi(0.5)^2/4) = \underline{34558\ LB} = F_{ALLOW.}$

$F_t = 0.6(100\,000)(4.0 - 2(0.5 + 0.063))(0.5) = \underline{86220\ LB}$

16-4 (d) FIG 16-10 (d)

$F_s = (22000)(2)(2)(\pi(0.75)^2/4) = \underline{38877\ LB} = F_{ALLOW.}$

$F_t = 0.6(100\,000)(4.0 - 2(0.75 + 0.063))(0.5) = \underline{71220\ LB}$

16-5

$P = 6500\,lb$; $M = 6500\,lb(48\,in) = 312\,000\,lb\cdot in$

$R_p = P/6 = 1083\,lb$

$\Sigma(x^2+y^2) = 6(2.0)^2 + 4(2.5)^2 = 49\,in^2$

FOR UPPER RIGHT BOLT

$$R_{1x} = \frac{M y_1}{\Sigma(x^2+y^2)} = \frac{(312\,000\,lb\cdot in)(2.5\,in)}{49\,in^2} = 15\,918\,lb$$

$$R_{1y} = \frac{M x_1}{\Sigma(x^2+y^2)} = \frac{(312\,000\,lb\cdot in)(2.0)}{49\,in^2} = 12\,735\,lb$$

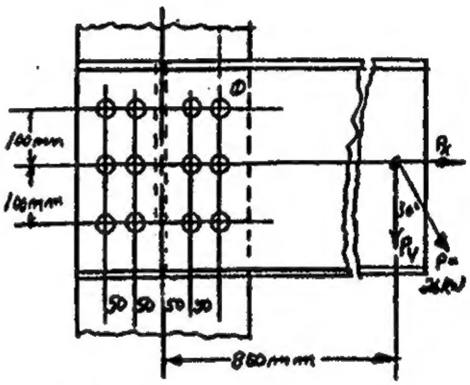
$R_{R1} = \sqrt{15918^2 + (1083+12735)^2} = 21079\,lb$

REQ'D $A = R_{R1}/\tau_A = 21079\,lb/17500\,psi = 1.205\,in^2$

REQ'D DIA $= \sqrt{\frac{4A}{\pi}} = \sqrt{4(1.205)/\pi} = 1.24\,in$ – USE $1\frac{1}{4}$ IN.

16-6

POSSIBLE SOLUTION –



$P_x = P\sin 30° = 13.0\,kN$

$P_y = P\cos 30° = 22.5\,kN$

$M = (22.5)(850) = 19125\,kN\cdot mm$

12-ASTM A325 BOLTS – FRICTION TYPE CONNECTION

ON BOLT ①

$\frac{P_x}{12} = \frac{13.0\,kN}{12} = 1.083\,kN \leftarrow$

$\frac{P_y}{12} = \frac{22.5\,kN}{12} = 1.875\,kN \uparrow$

FORCES DUE TO MOMENT:

$\Sigma(x^2+y^2) = 6(50)^2 + 6(100)^2 + 8(100)^2 = 155000\,mm^2$

$$R_{1x} = \frac{M y_1}{\Sigma(x^2+y^2)} = \frac{19125\,kN\cdot mm(100\,mm)}{155000\,mm^2} = 12.34\,kN \leftarrow$$

$$R_{1y} = \frac{M x_1}{\Sigma(x^2+y^2)} = \frac{19125\,kN\cdot mm(100\,mm)}{155000\,mm^2} = 12.34\,kN \uparrow$$

$R_1 = \left[(1.083+12.34)^2 + (1.875+12.34)^2\right]^{1/2} = 19.55\,kN$

REQ'D $A = \frac{R_1}{\tau_a} = \frac{19550\,N}{121\,N/mm^2} = 162\,mm^2 = \pi D^2/4$

$D = \sqrt{4A/\pi} = \sqrt{4(162)/\pi} = 14.3\,mm$ – USE 16mm

16-7 FIG. 16-9(a)

$P = \tau_a L t = (18000\ lb/in^2)(6\ in)(0.707)(0.3125\ in) = \underline{23860\ lb\ \text{ON WELDS}}$

ON STRAP: $P = \sigma_{ta}(3)(3/8) = (0.6)(36000\ lb/in^2)(3\ in)(0.375\ in) = 24300\ lb$

WELDS GOVERN JOINT STRENGTH

16-8 FIG. 16-10(c)

$P = \tau_a L t = (21000\ lb/in^2)(8\ in)(0.707)(0.250\ in) = \underline{29700\ lb\ \text{ON WELDS}}$.

ON STRAP: $P = (0.6)(50000\ lb/in^2)(0.5\ in)(4\ in) = 60000\ lb$

THE SOLUTIONS SHOWN BELOW FOR PROBLEMS 16-9, 16-10, AND 16-11 ARE JUST SAMPLES OF MANY POSSIBLE SOLUTIONS. THE GENERAL CONFIGURATION AND NUMBER OF FASTENERS SHOWN IN FIG. 16-1 ARE USED, BUT OTHERS COULD BE USED.

16-9

FORCE ON JOINT $= (\frac{1}{4})(54.4 \times 10^3\ kg)(9.81\ m/s^2) = 133\,400\ N$

USE 6 RIVETS – DESIGN FOR SHEAR STRENGTH

$F_s = \tau_a A_s$ – USE ASTM A502, GRADE 1 RIVETS

REQ'D. $A_s = F_s/\tau_a = 133400\ N/(121\ N/mm^2) = 1102\ mm^2$

DOUBLE SHEAR: $A_s = (6)(2)(\pi D^2/4) = 3\pi D^2$

REQ'D DIA. $= \sqrt{A_s/(3\pi)} = \sqrt{1102\ mm^2/3\pi} = \underline{10.8\ mm - \text{USE } 12\ mm}$

CHECK BEARING ON WEB OF TEE

$F_b = \sigma_{ba} A_b = 1.50(400\ N/mm^2)(6)(12\ mm)(10.6\ mm) = \underline{458\ kN\ \text{OK}}$

FIND REQ'D THICKNESS OF STRAP, t

$F_t = \sigma_{ta} A_t = \sigma_{ta}[w - ND_H](2)t$

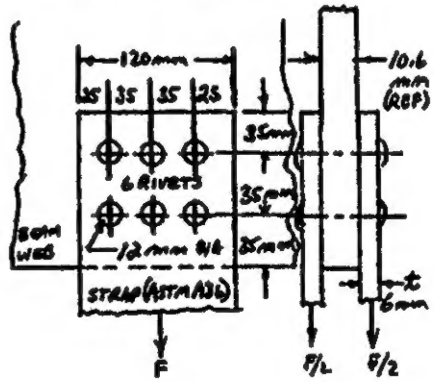
$t = \dfrac{F_t}{\sigma_{ta}(w - ND_H)(2)}$

USE $D_H = 12 + 2 = 14\ mm$

$t = \dfrac{133400\ N}{0.6(248\ N/mm^2)(120 - 3(14))(2)} = 5.75\ mm$

USE TWO STRAPS, 6 mm THICK EACH

USE ASTM A36 STEEL



16-10 FORCE ON JOINT = 133.4 kN (FROM PROB. 16-9)

USE 4 BOLTS - ASTM A490 - BEARING TYPE CONNECTION

REQ'D. $A_s = F_s/\tau_a = 133400\ N/(152\ N/mm^2) = 878\ mm^2$

$A_s = (2)(4)\pi D^2/4 = 2\pi D^2$

REQ'D. DIA $= \sqrt{A_s/2\pi} = \sqrt{878\ mm^2/2\pi} = 11.8\ mm$ USE 12 mm

CHECK BEARING ON STRAPS - TWO, 8 mm THICK, A36 STEEL

$F_b = \sigma_{ba} A_b = 1.50(400\ N/mm^2)(12\ mm)(8\ mm)(2)(4) = 461\ kN$ - OK

USE $D_H = 12 + 2 = 14\ mm$

$F_t = \sigma_{ta} A_t = 0.6(248\ N/mm^2)(120 - 2(14))(2)(8)\ mm^2 = 219\ kN$ - OK

TAB DESIGN - MAKE TAB 10 mm THICK TO MATCH BEAM WEB

REQ'D WIDTH, w, USING A36 STEEL

$$A_t = F_t/\sigma_{ta} = \frac{133400\ N}{0.6(248\ N/mm^2)} = 896.5\ mm^2$$

$A_t = (w - 2D_H)t$ THEN $w = \frac{A_t}{t} + 2D_H$

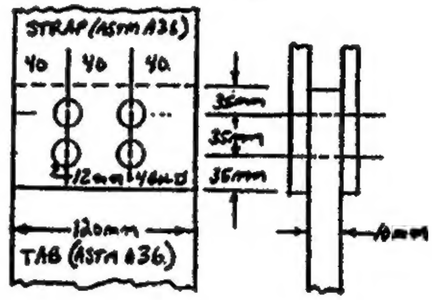
$$w = \frac{896.5\ mm^2}{10\ mm} + 2(14\ mm) = 118\ mm \quad \text{OK} - w = 120\ mm$$

CHECK BEARING ON TAB

$F_b = \sigma_{ba} A_b = 1.5 S_u A_b$

$F_b = 1.5(400)(4)(12)(10) = 288\ kN$

OK



16-11 FORCE ON ONE TAB = 133.4 kN (FROM PROB 16-9)

TAB - A36 STEEL, 120 mm WIDE by 10 mm THICK (PROB. 16-10)

WELD WITH E60 ELECTRODE

$\tau_a = 124\ MPa$

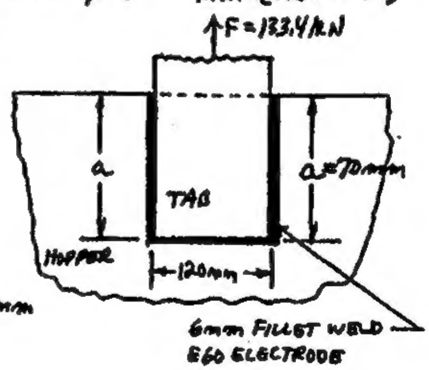
$F = \tau_a L t$

$$\text{REQ'D. } L = \frac{F}{\tau_a t}$$

USE 6 mm FILLET WELD

$t = 0.707(6\ mm) = 4.24\ mm$

$$L = \frac{133\,400\ N}{(124\ N/mm^2)(4.24\ mm)} = 254\ mm$$



BUT $L = 2a + 120\ mm$

$$\text{REQ'D } a = \frac{L - 120\ mm}{2} = \frac{254 - 120}{2} = 67\ mm \text{ - USE } 70\ mm = a$$